EXAMPLES
IN
GEOMETRICAL DRAWING

BY
V. LE NEVE FOSTER, M.A.

ETON
AT THE COLLEGE PRESS
1904


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อโยคา ภูริสังชโย

EXAMPLES

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GEOMETRICAL DRAWING

BY

V. LE NEVE FOSTER, M.A.

ASSISTANT MATHEMATICAL MASTER AT ETON

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PREFACE.

This book forms a classified collection of Examples in Geometrical Drawing.

It is not suggested that the Exercises should be taken in order; but a thorough grounding in Part I. is a necessary preliminary to Parts II. and III.

No question from Exercises XLV. and XLVII. should be set until the pupil has already mastered the principle which governs its solution.

Exercise XLVI. may be taken concurrently with the course of Pure Geometry.

I am fortunate in being able to include a collection of "Patterns" that has been in use at Eton for a year.

My thanks are due to the Controller of H.M. Stationery Office for permission to reprint some questions set in Military Entrance Examinations, and to several of my colleagues for advice.

Answers are published separately, and may be obtained from the Publishers by *bona-fide* teachers.

V. LE NEVE FOSTER.

Eton College,
January 1904.

CONTENTS.

PART I.—(*continued*).

PART II.

PART III.

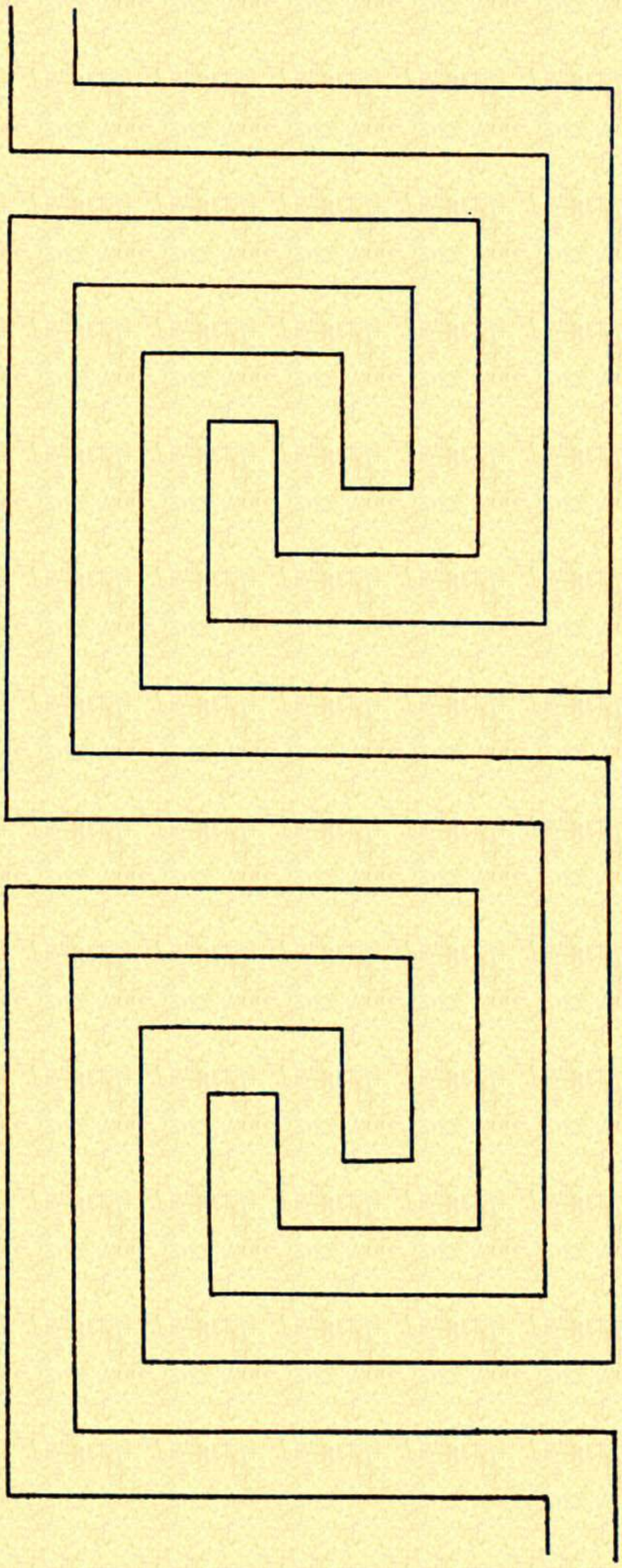
PRACTICAL SOLID GEOMETRY.

PART IV.

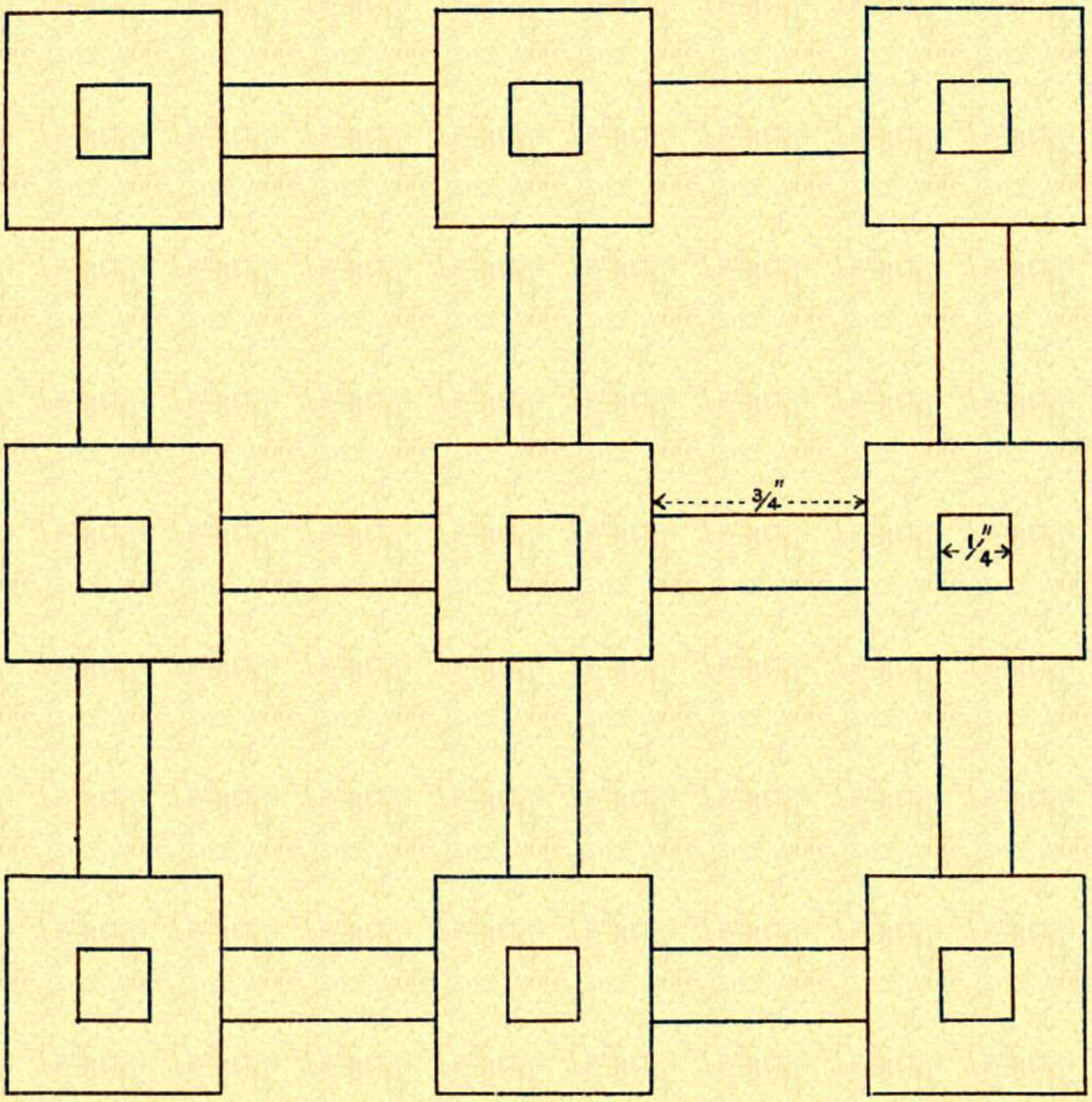
MILITARY EXAMINATION PAPERS.

GEOMETRICAL PATTERNS.

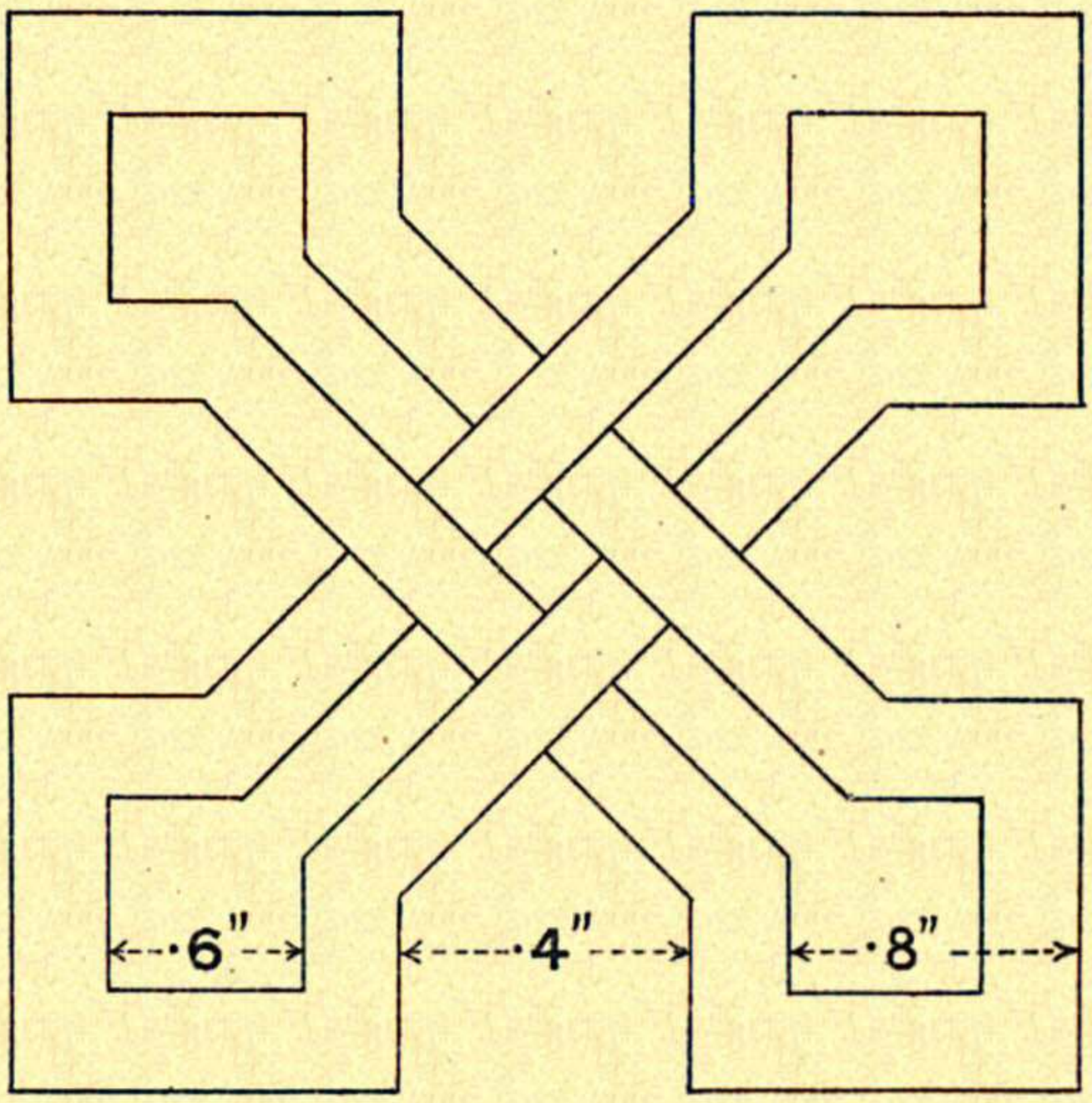
ARMY CLASS.



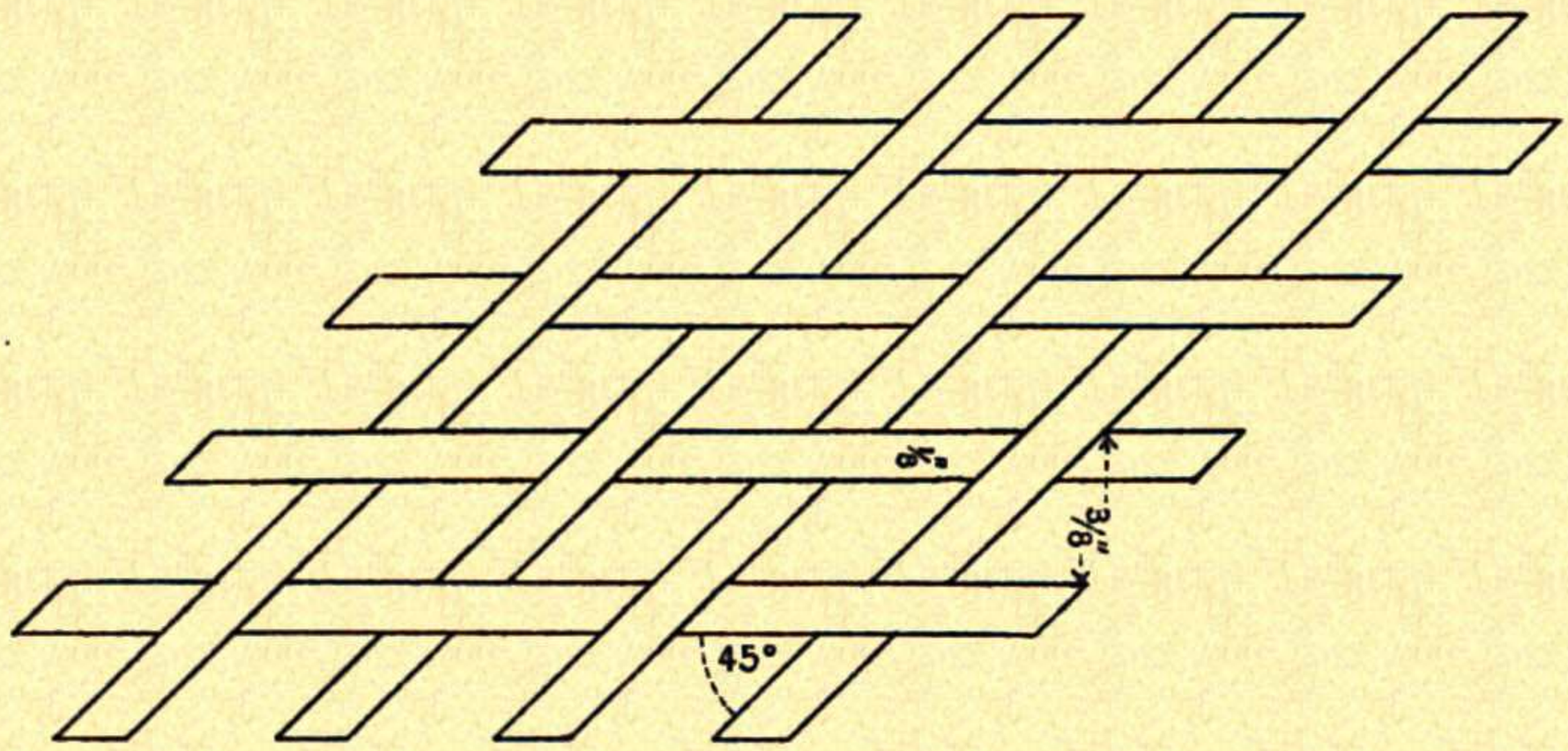
Distance between lines ¼"

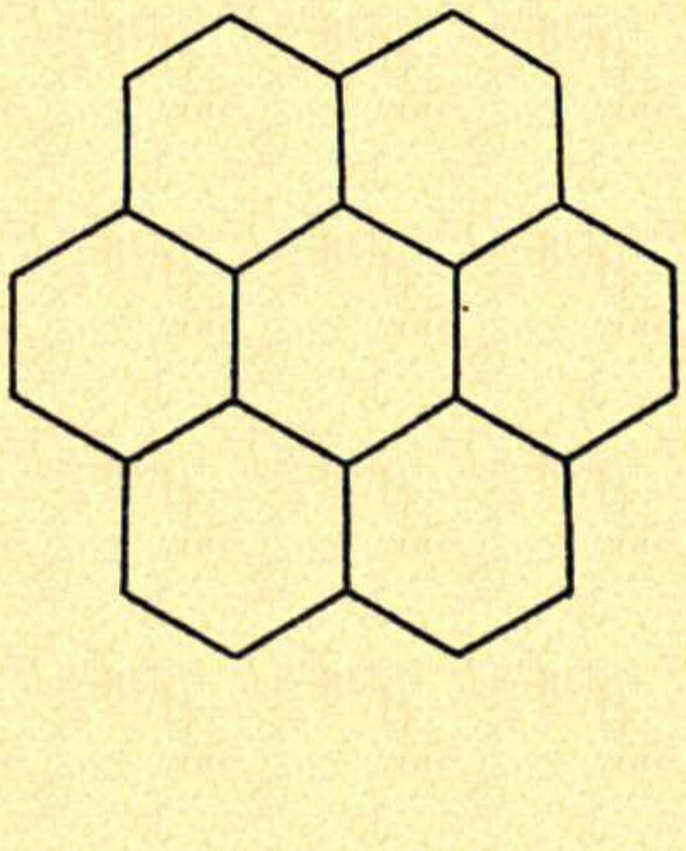

3/4"
1/4"

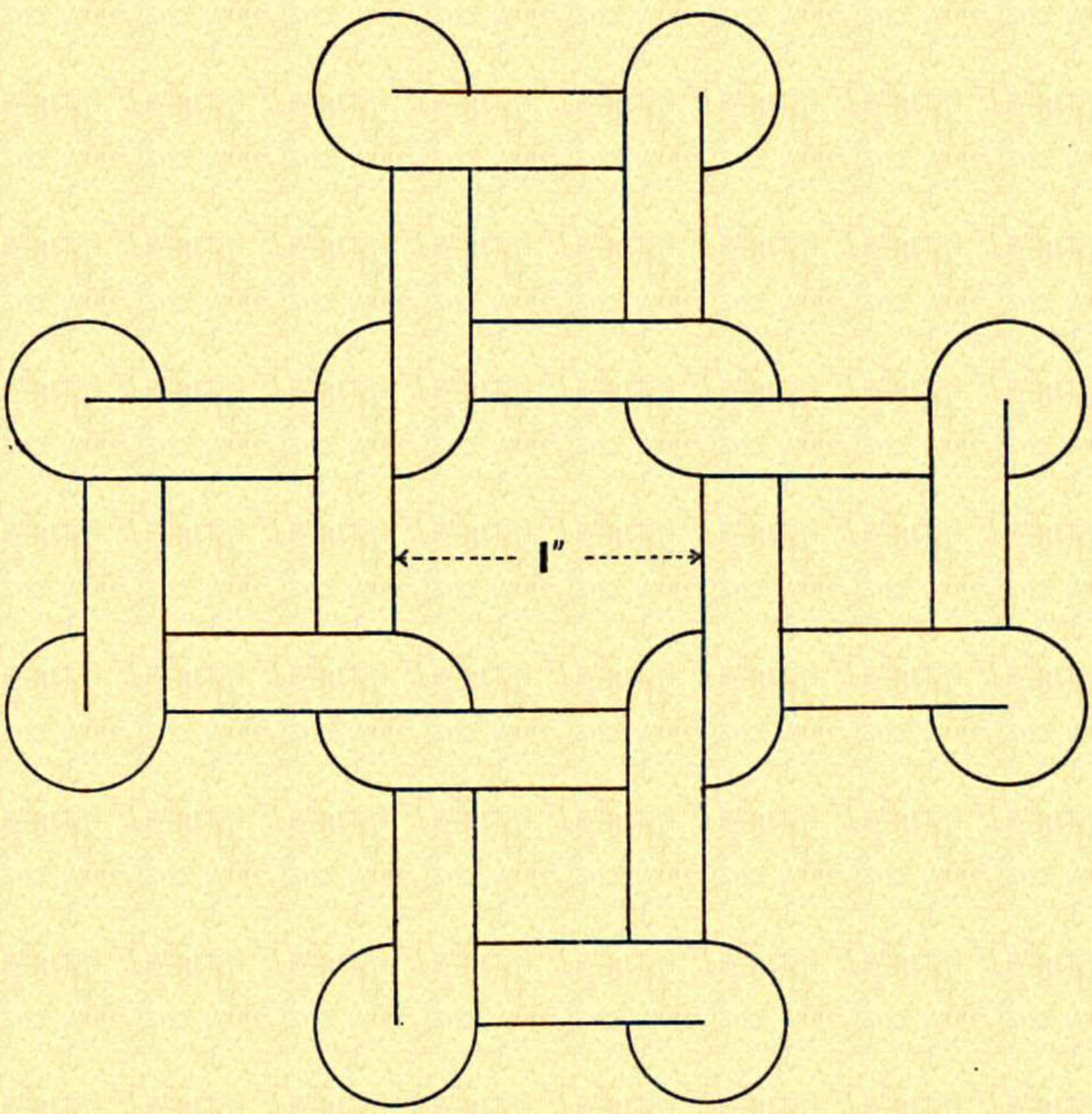
GEOMETRICAL PATTERNS.



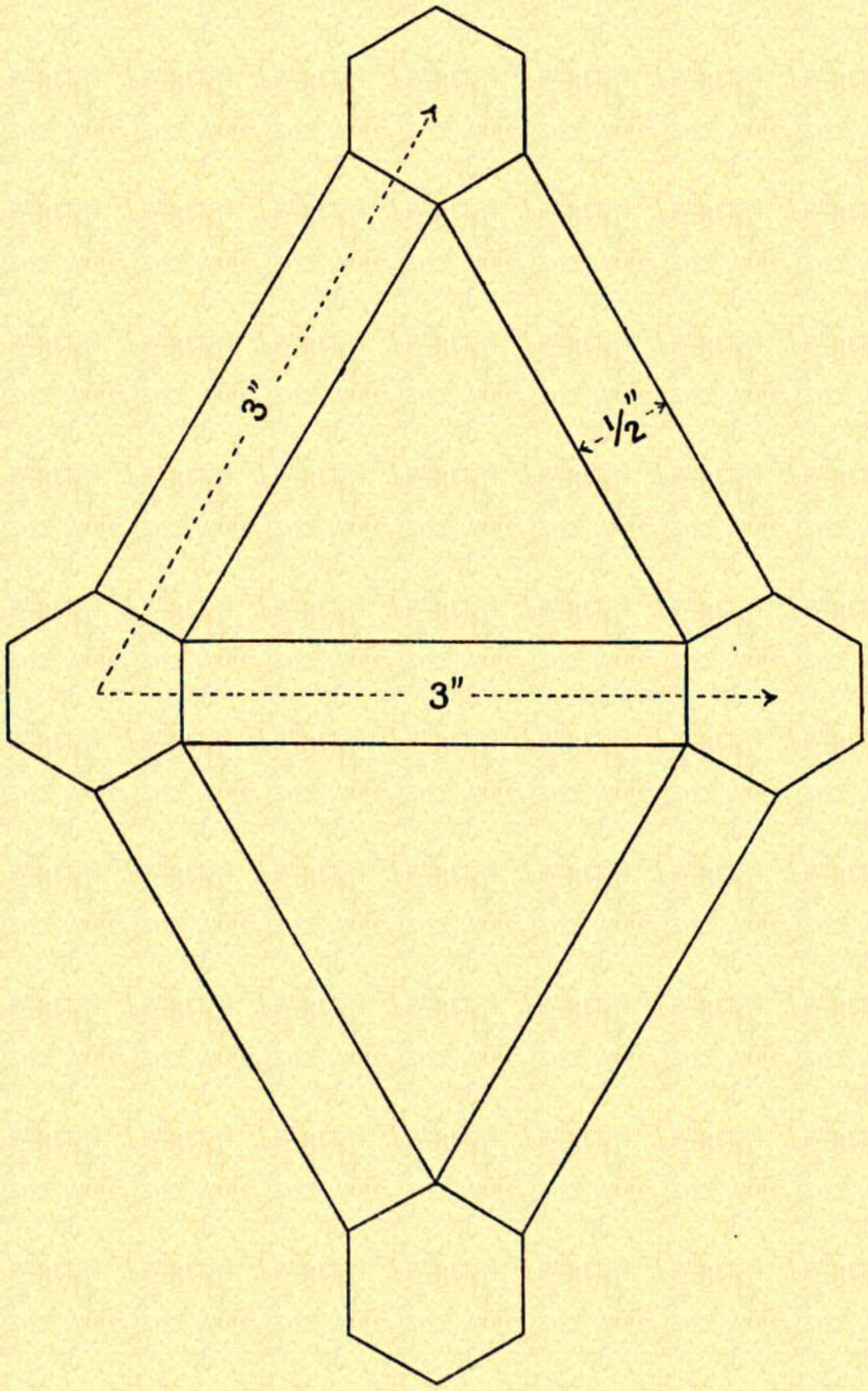
GEOMETRICAL PATTERNS.

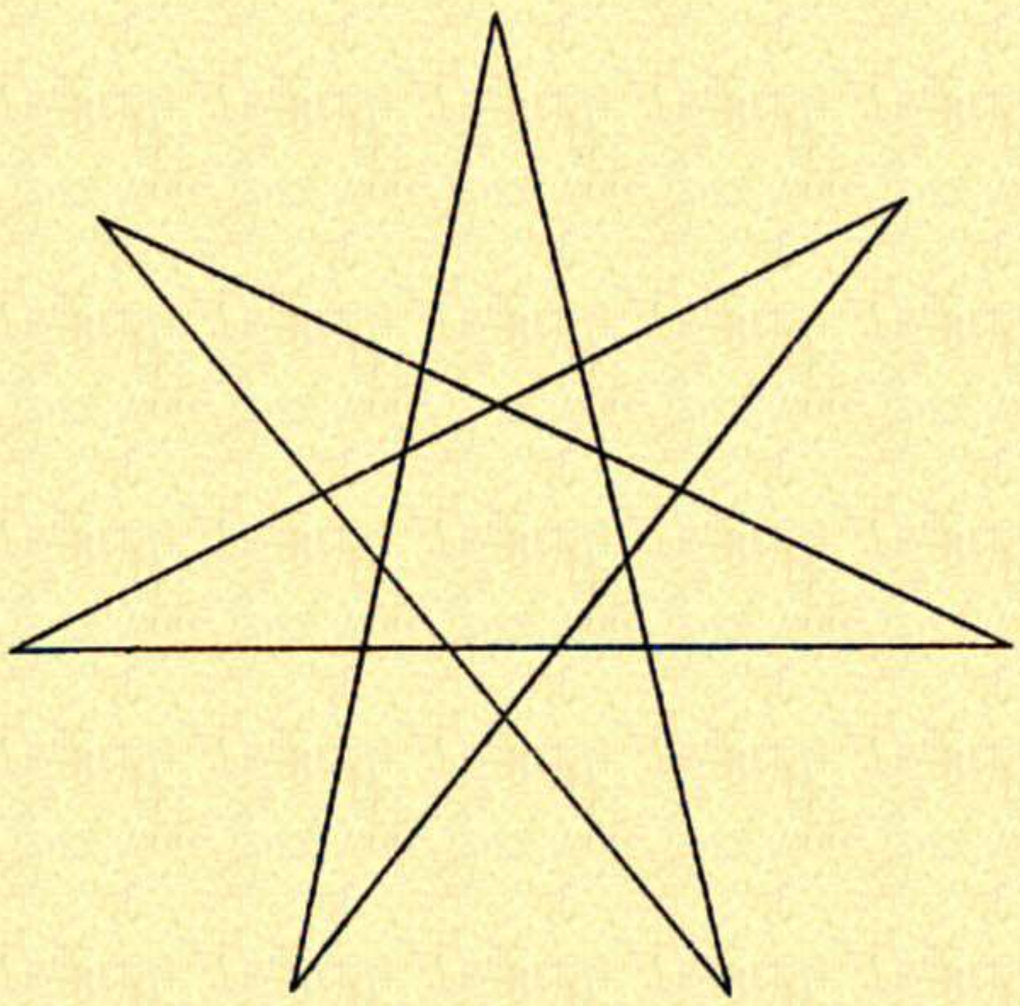


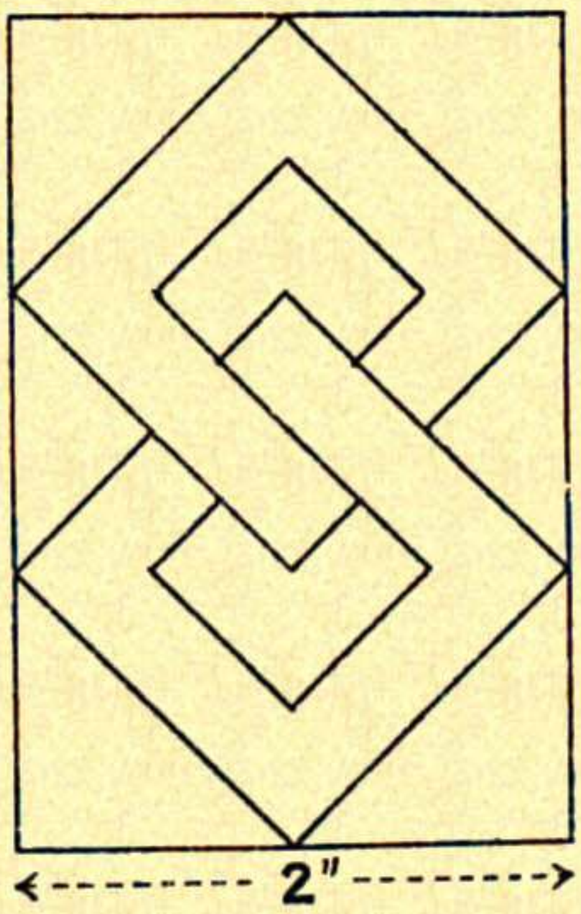


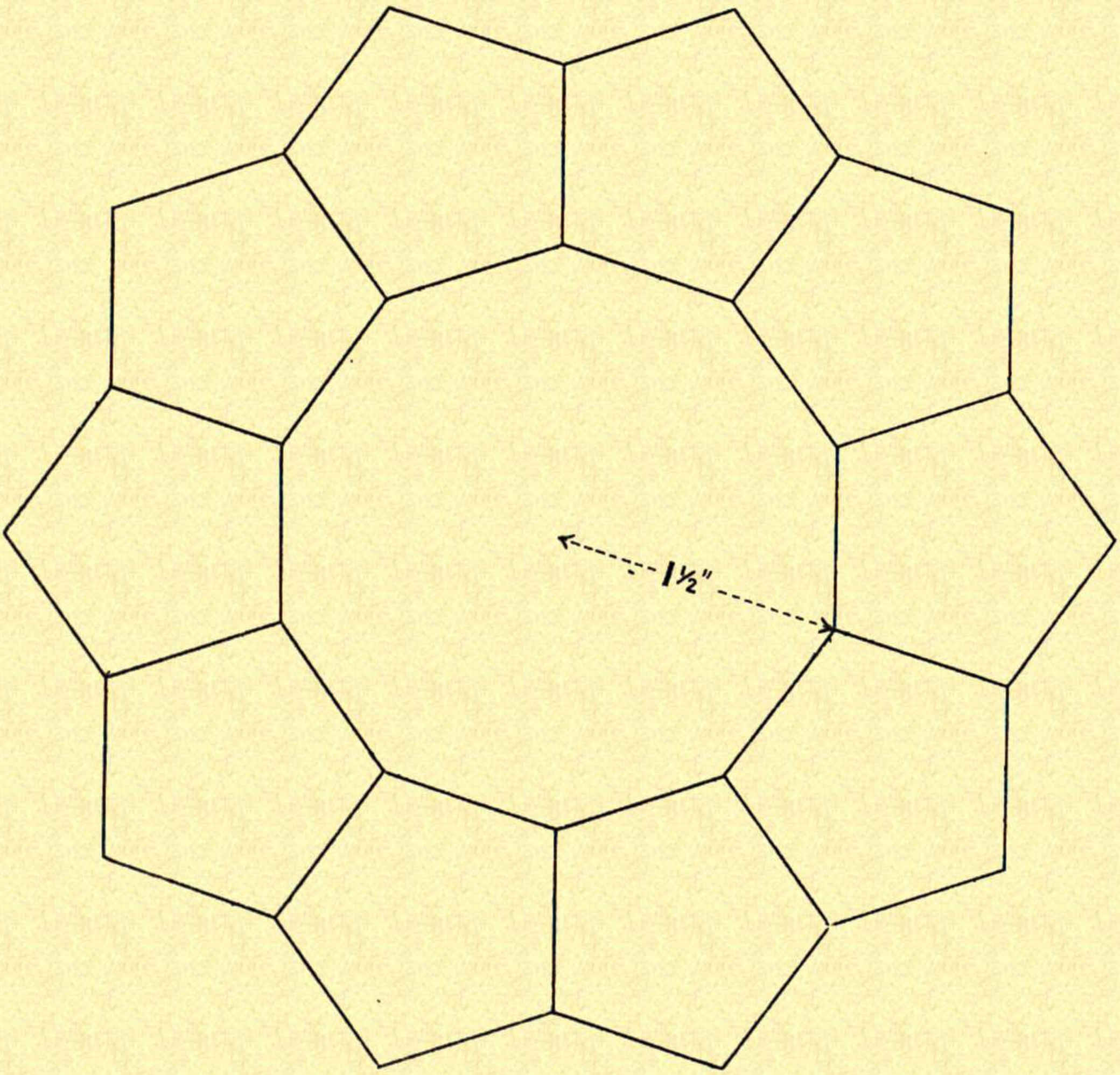

1"

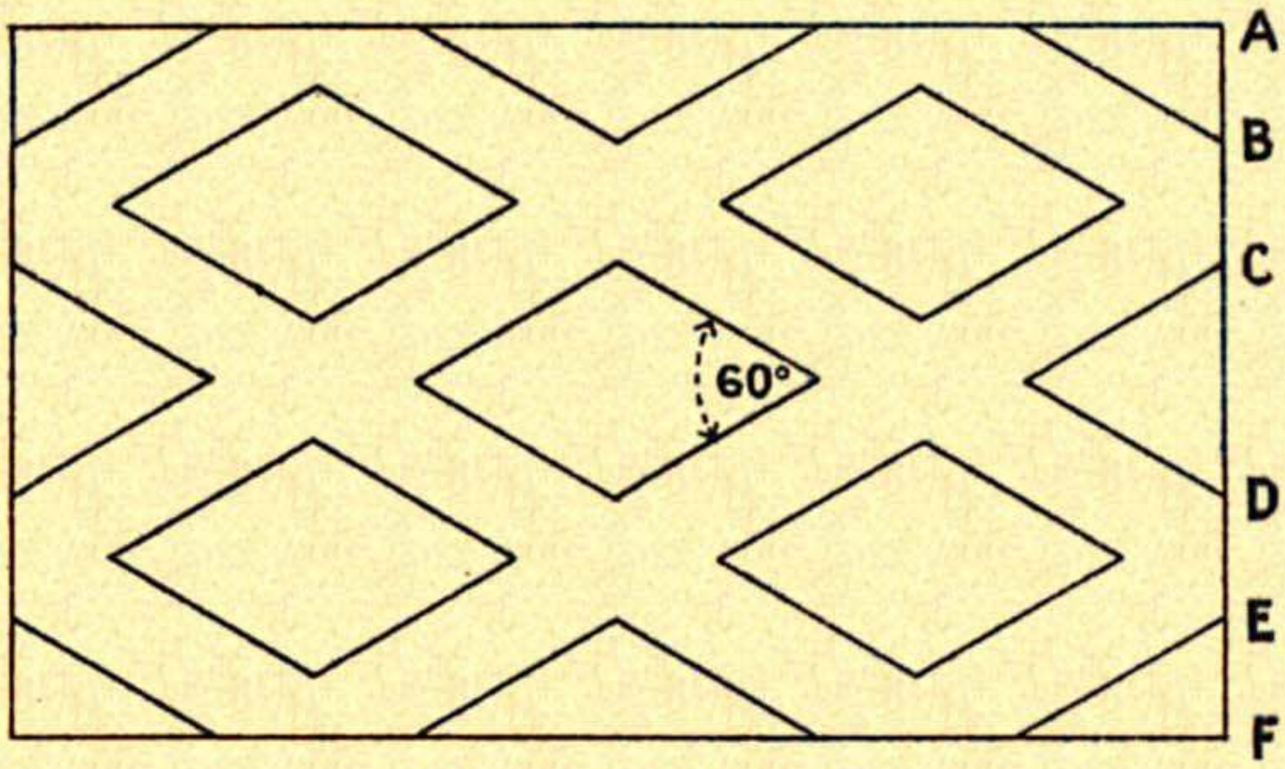
GEOMETRICAL PATTERNS.





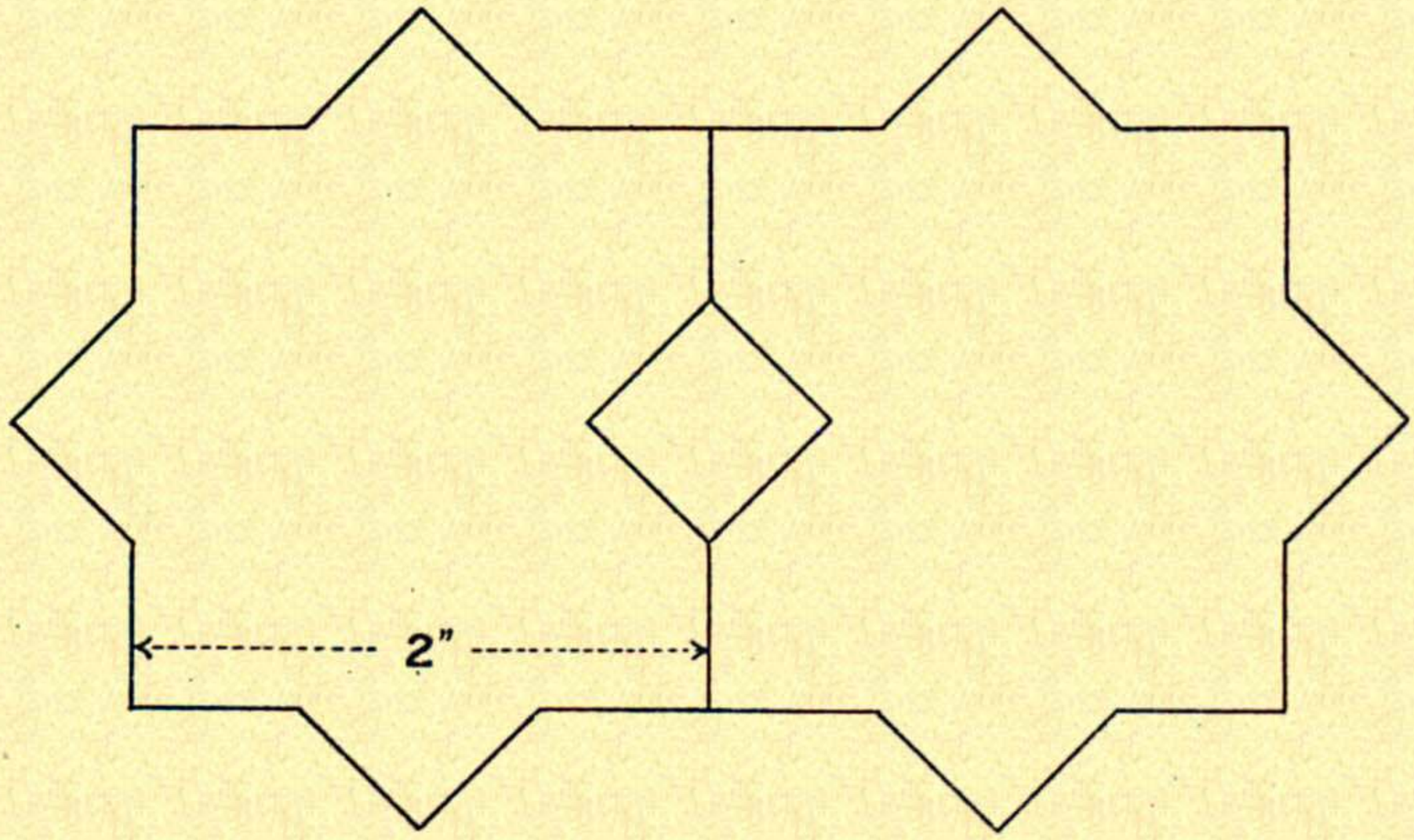

2"

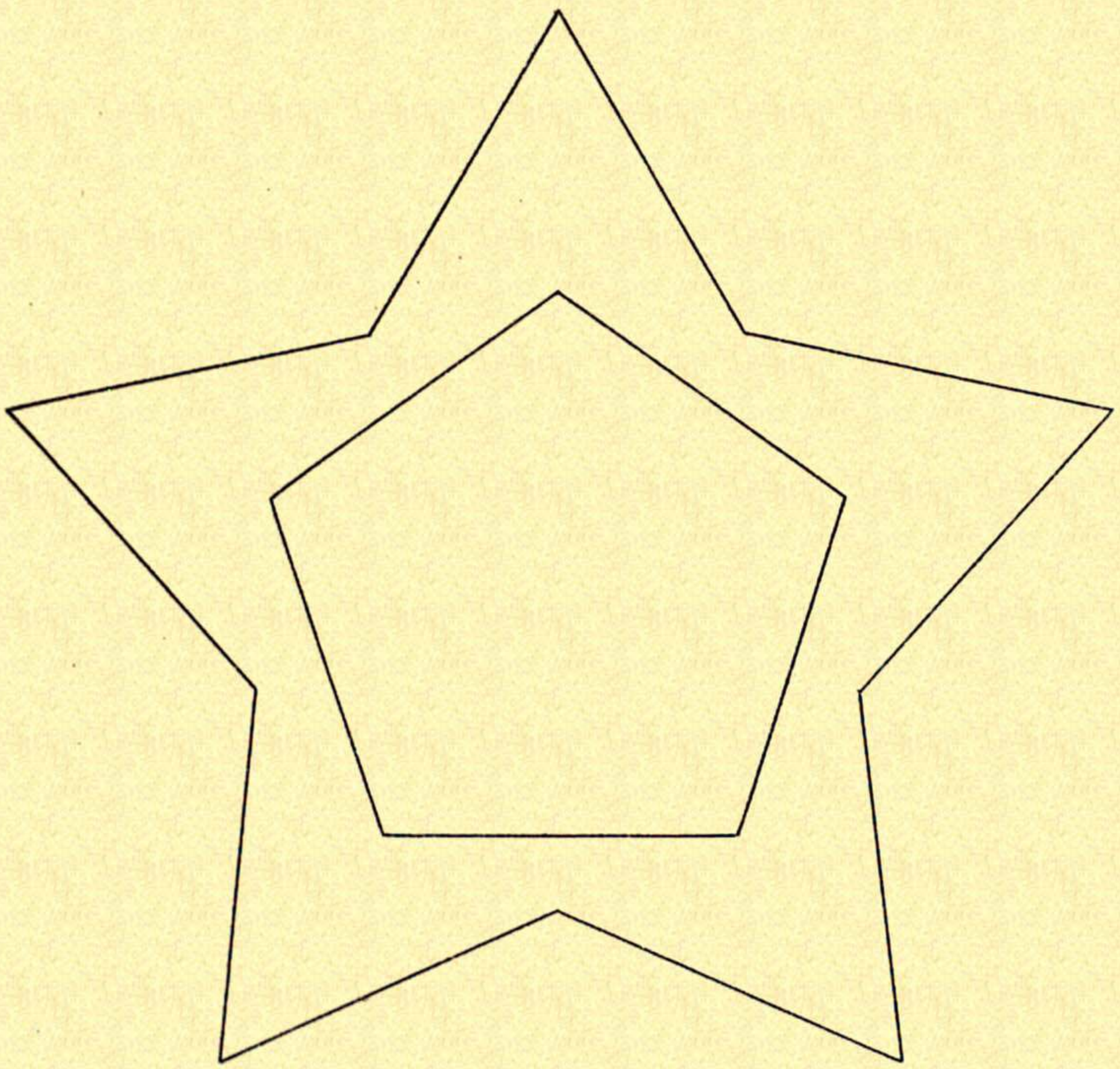

1½"

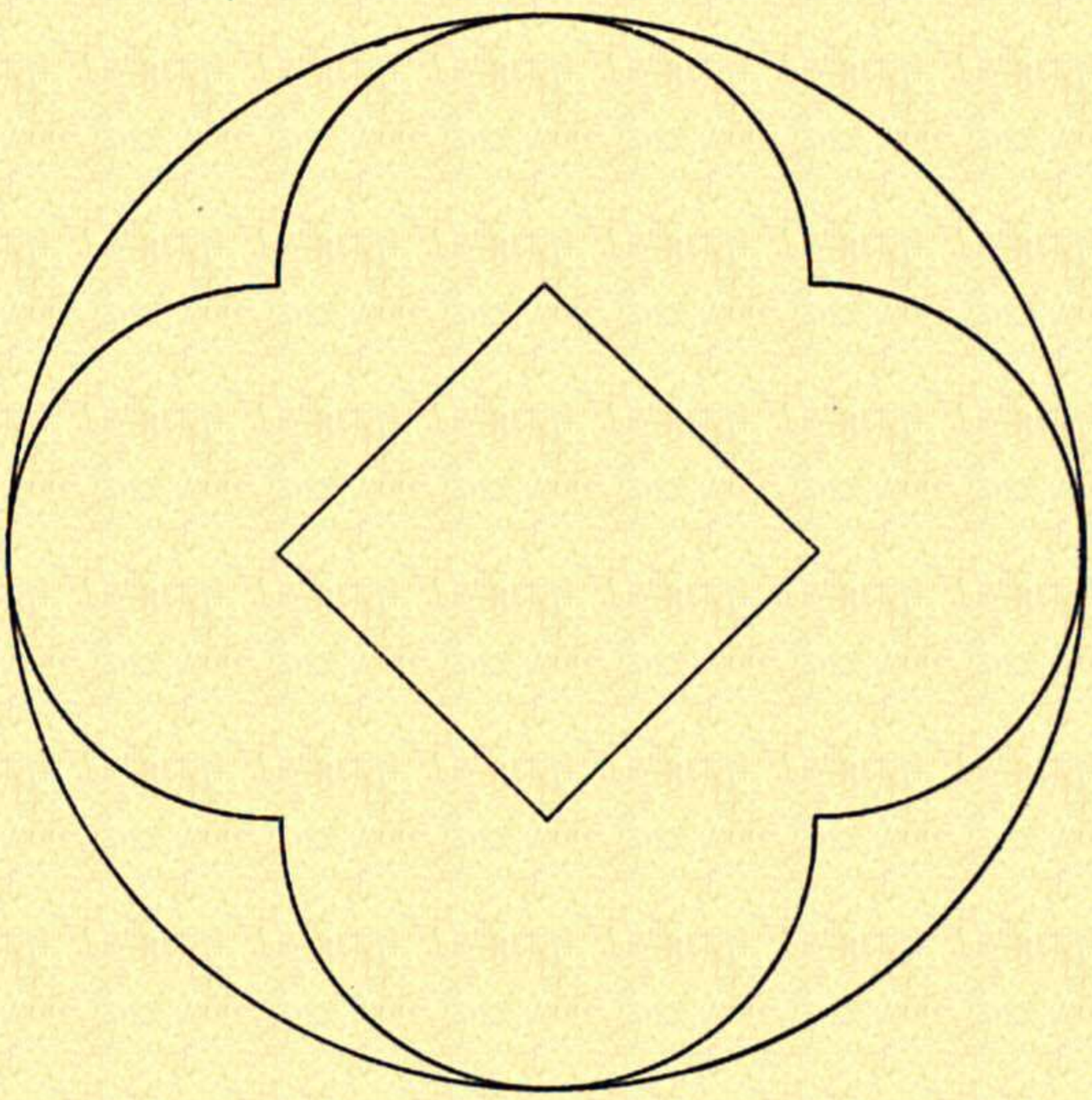


AB=BC=DE=EF

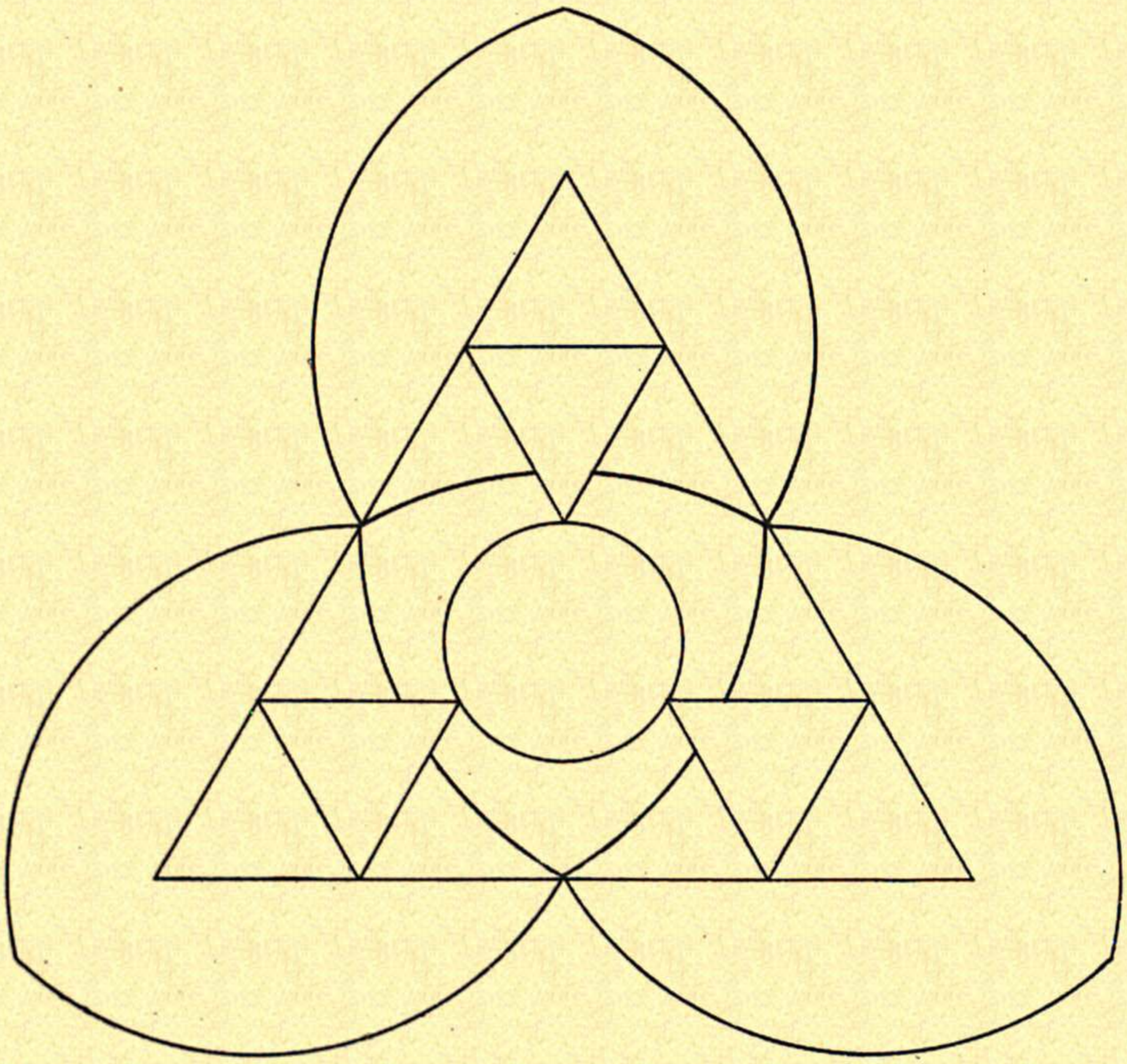
AF=1½" CD=½"

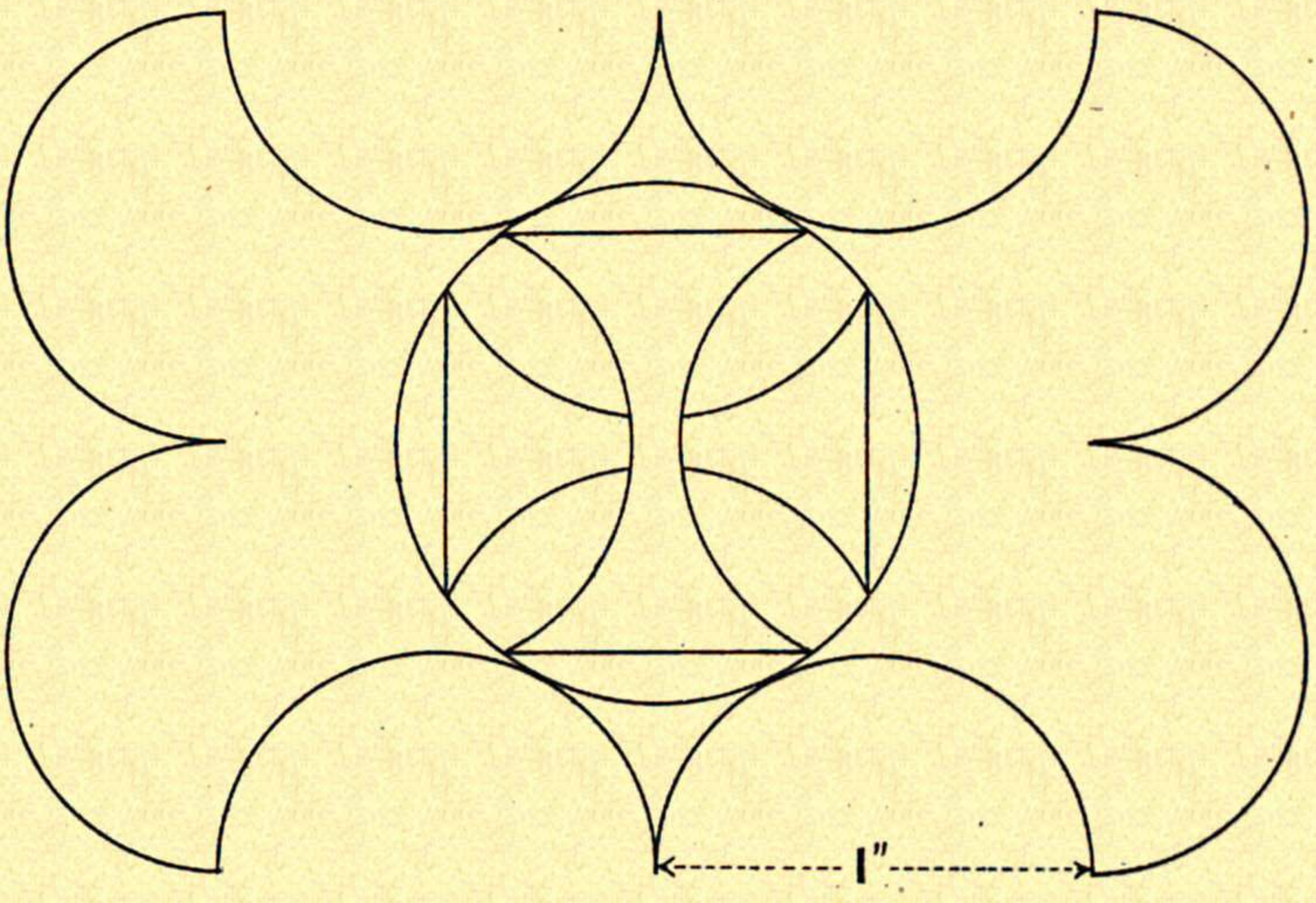


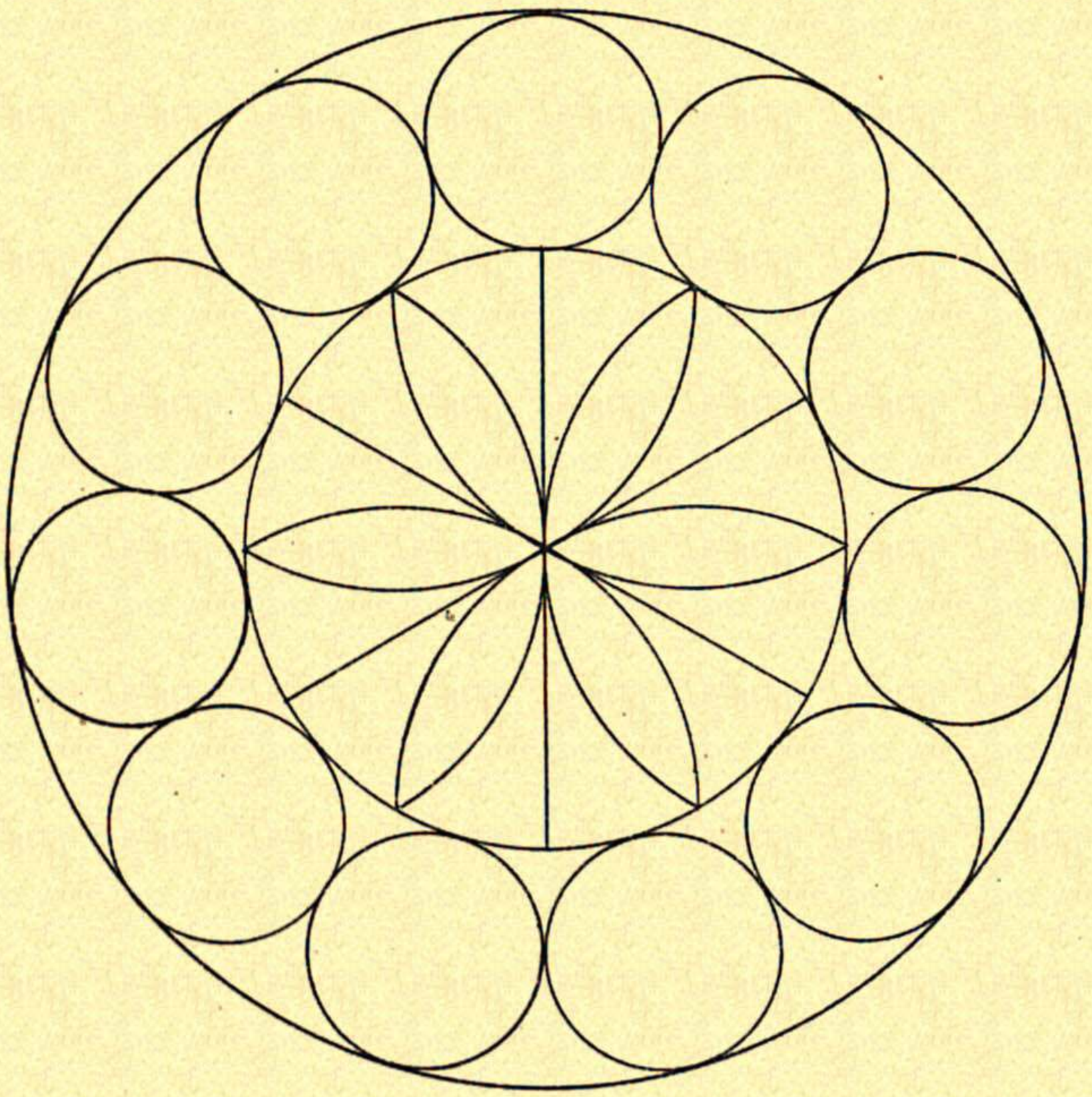


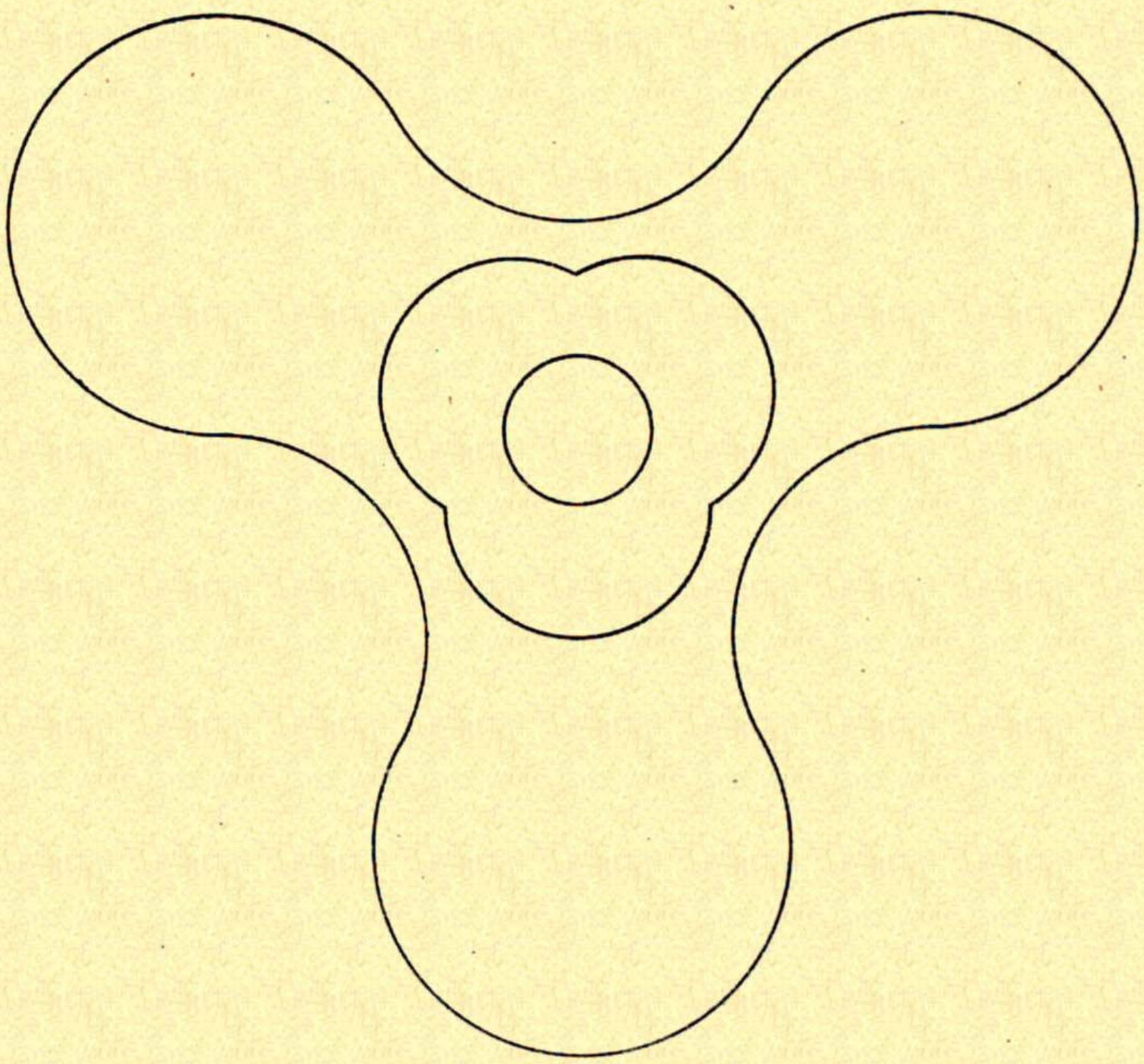


Radius of outer circle 1½"

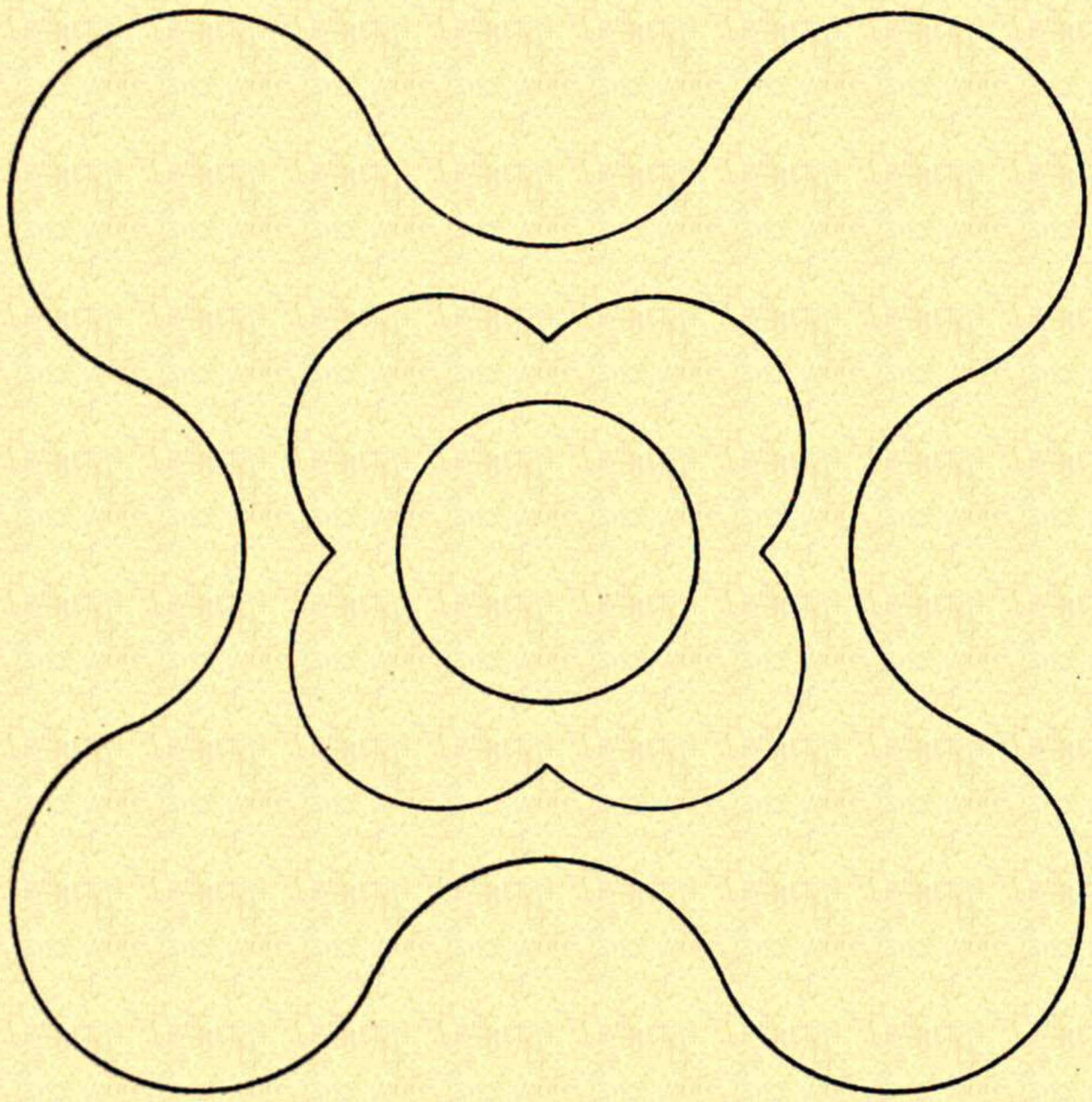



1"

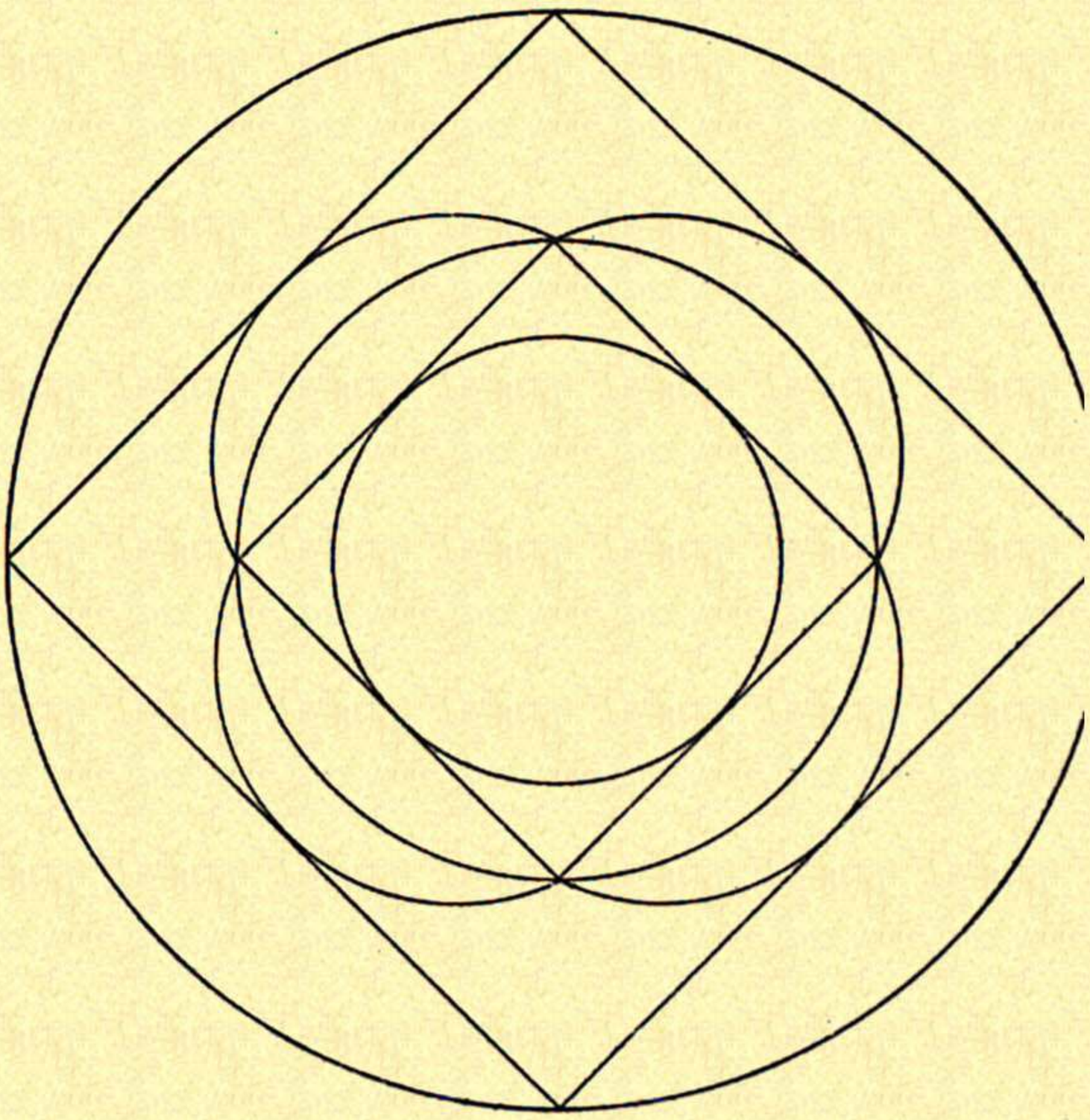




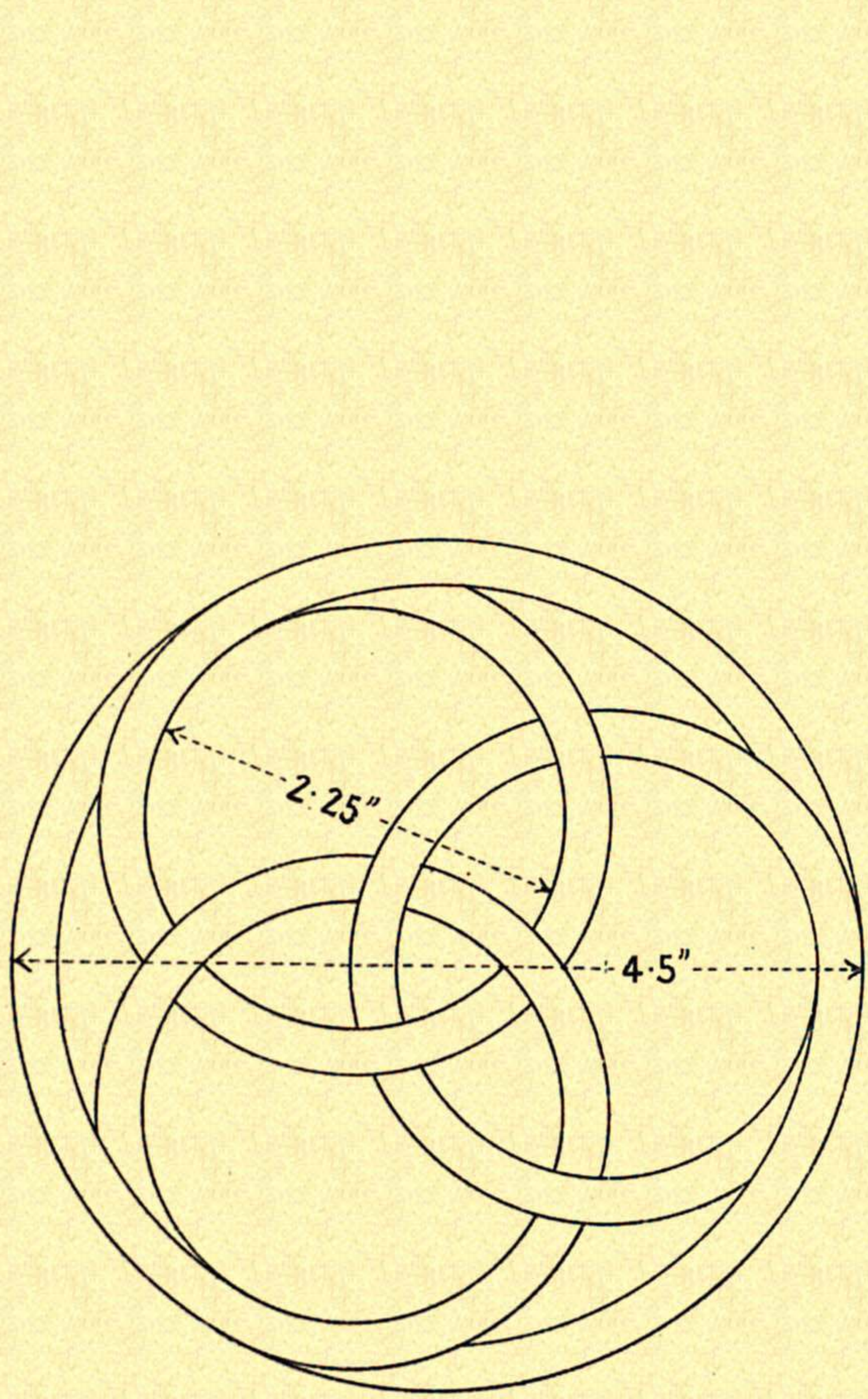
Radius of outer curve 8"

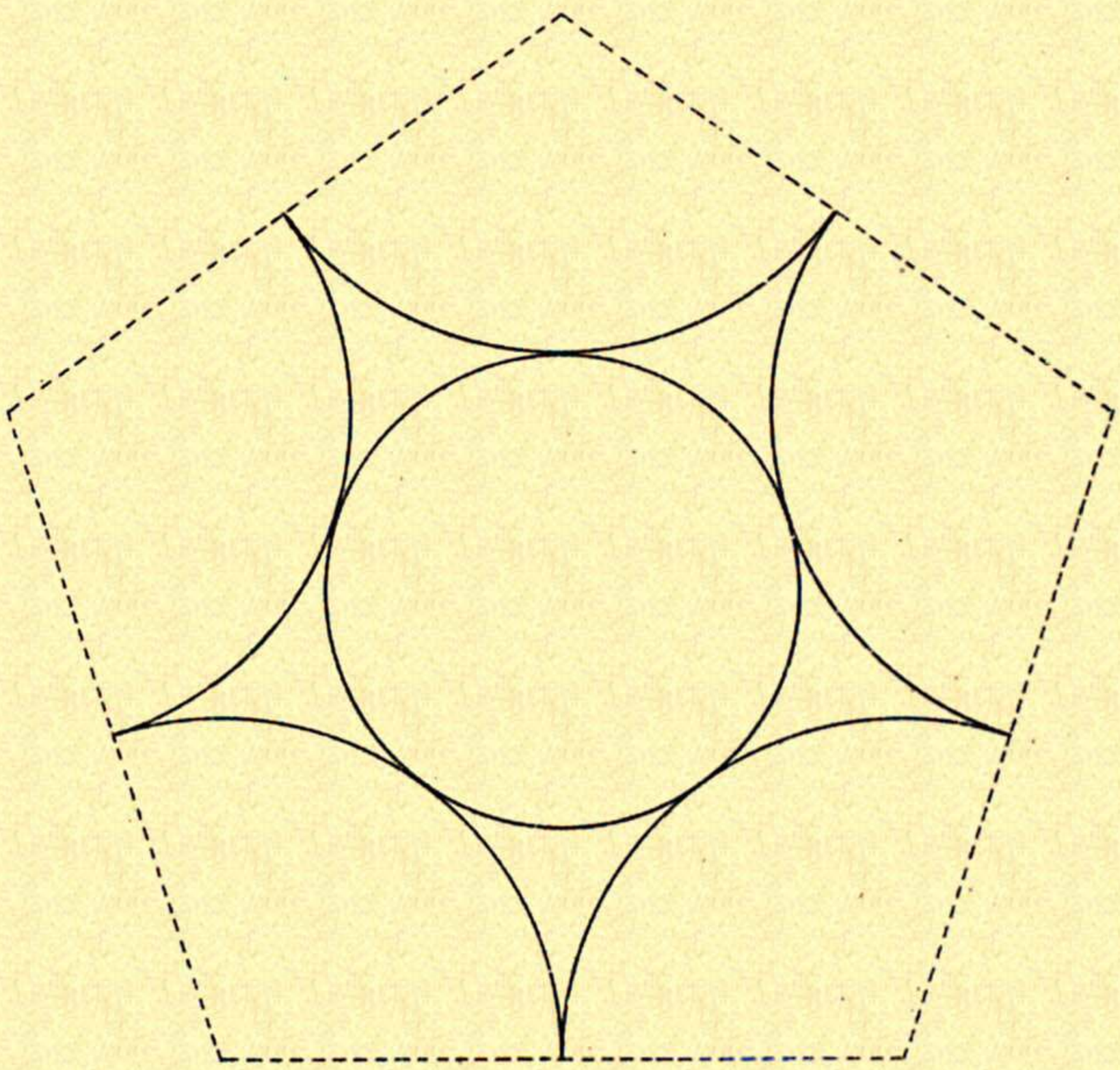


Radius of inner circle ½"



GEOMETRICAL PATTERNS.

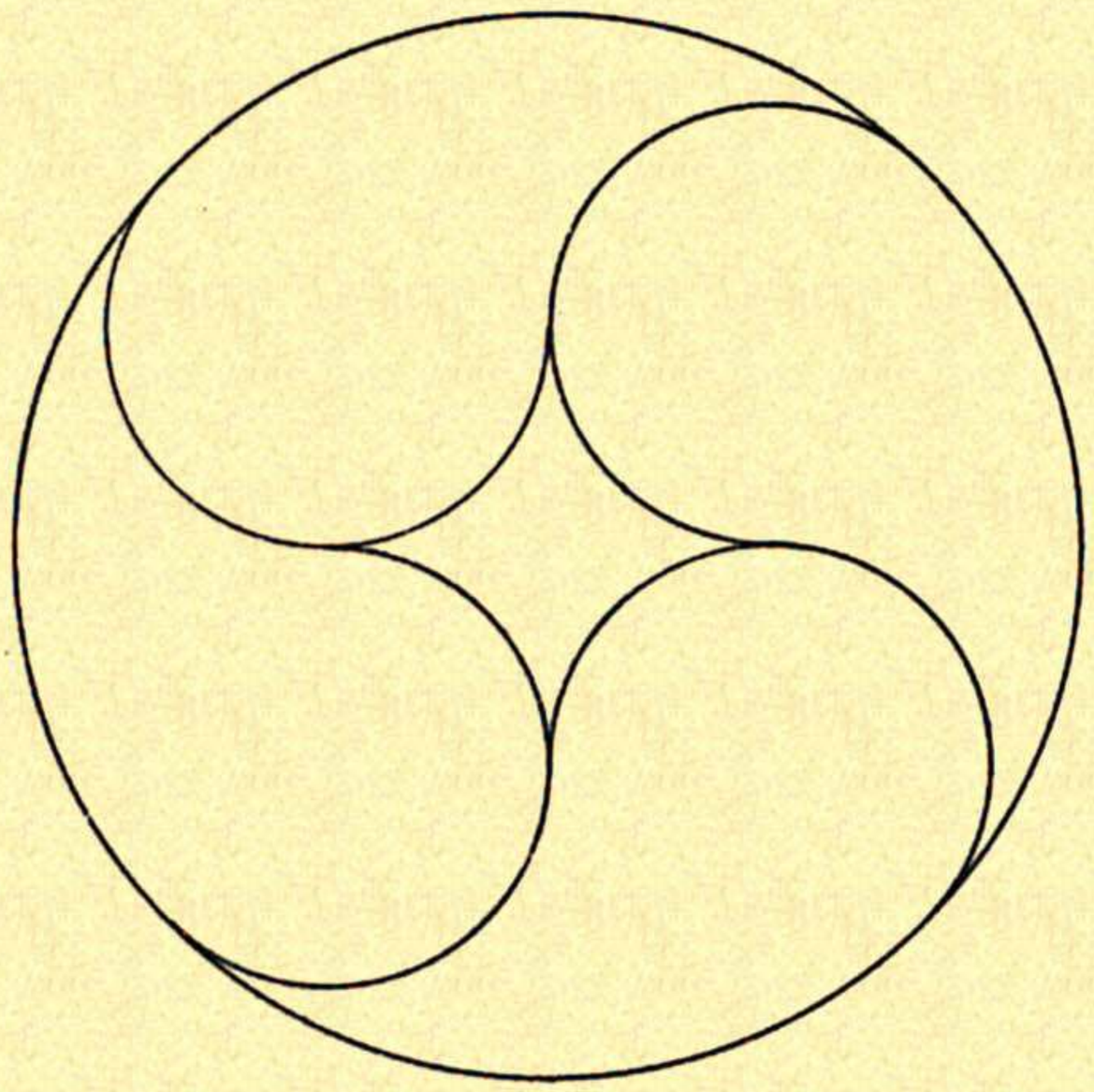


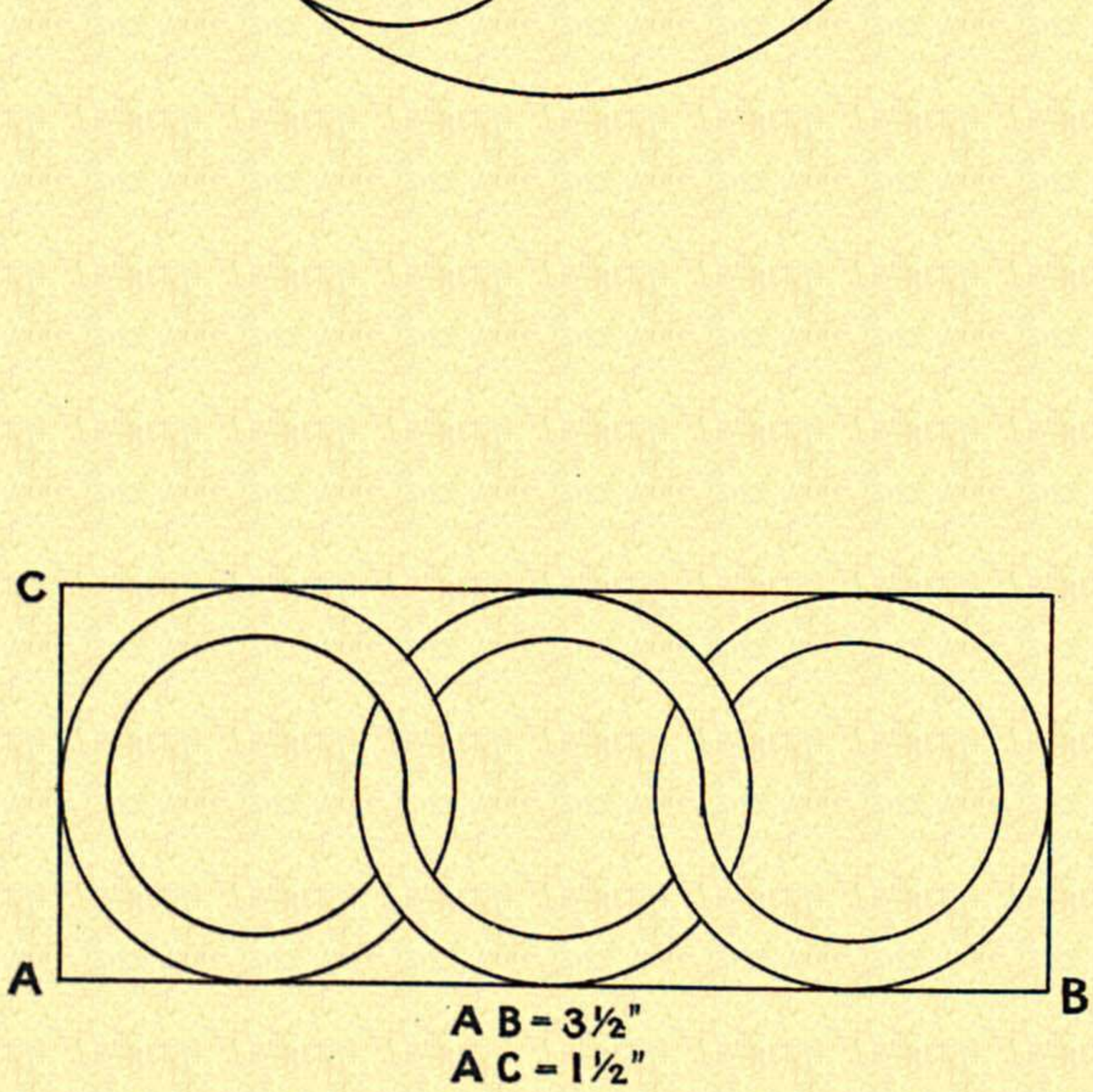


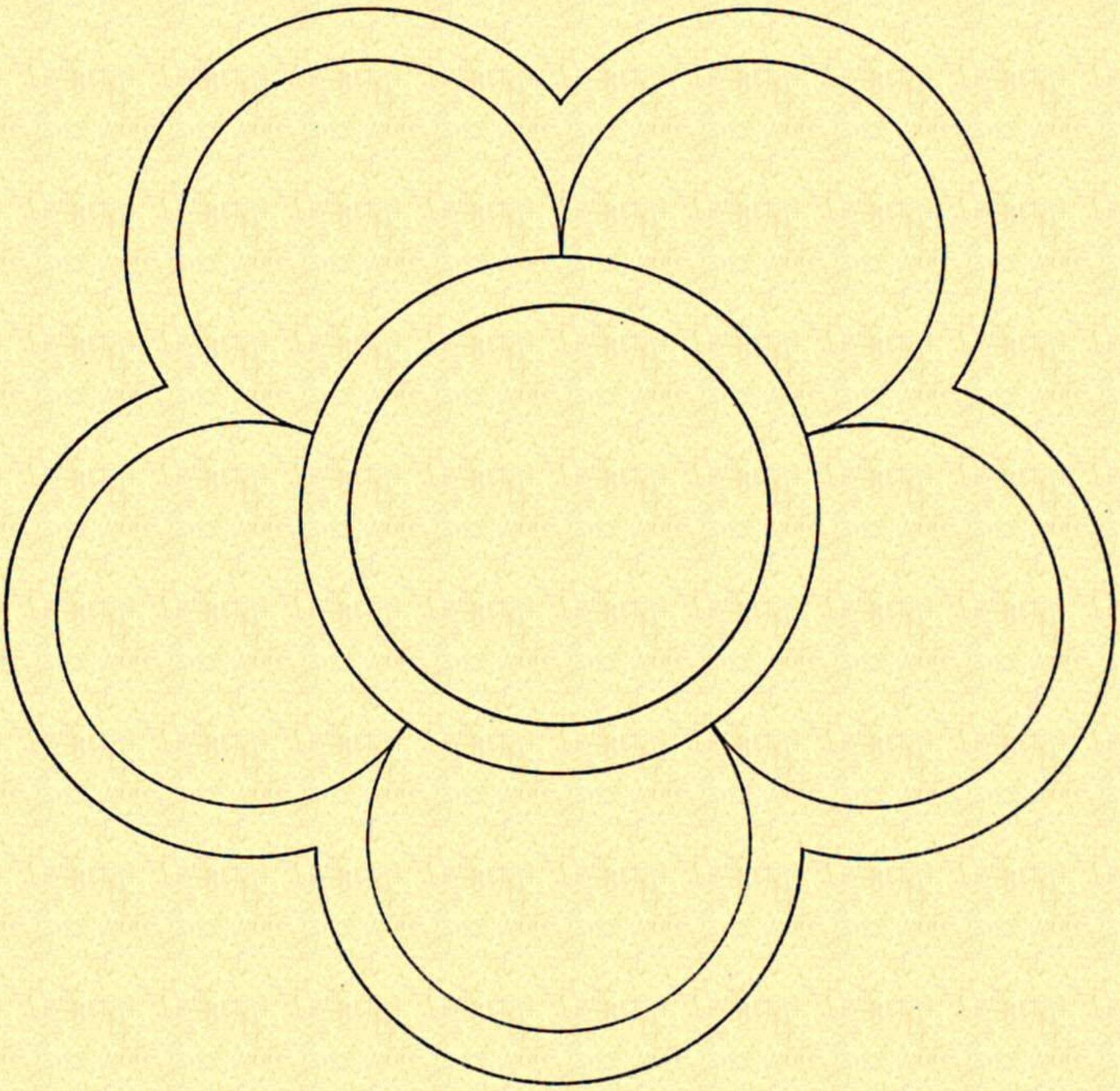
Side of pentagon 2"

<u>Dont</u> ink in the dotted lines.

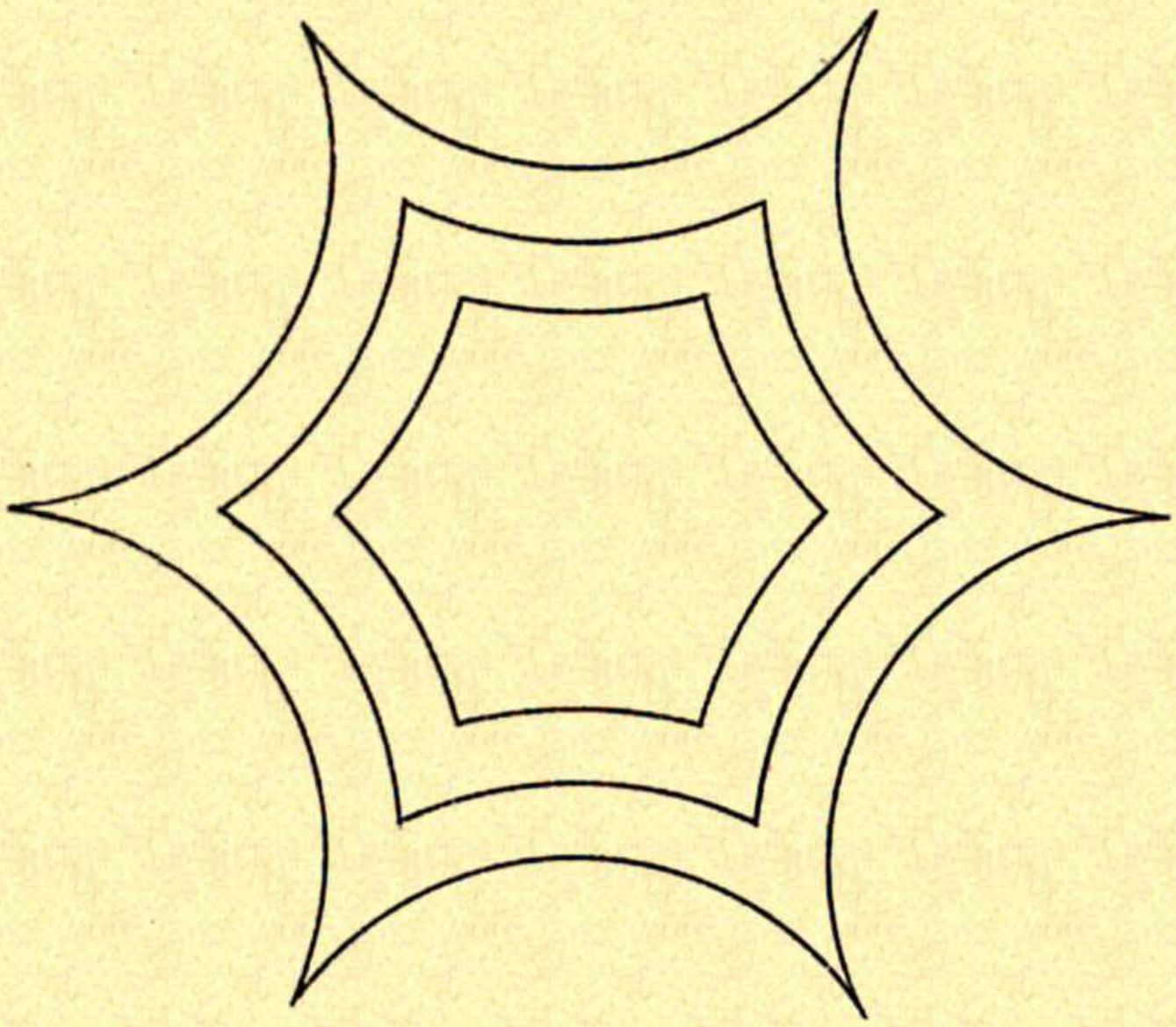
GEOMETRICAL PATTERNS.



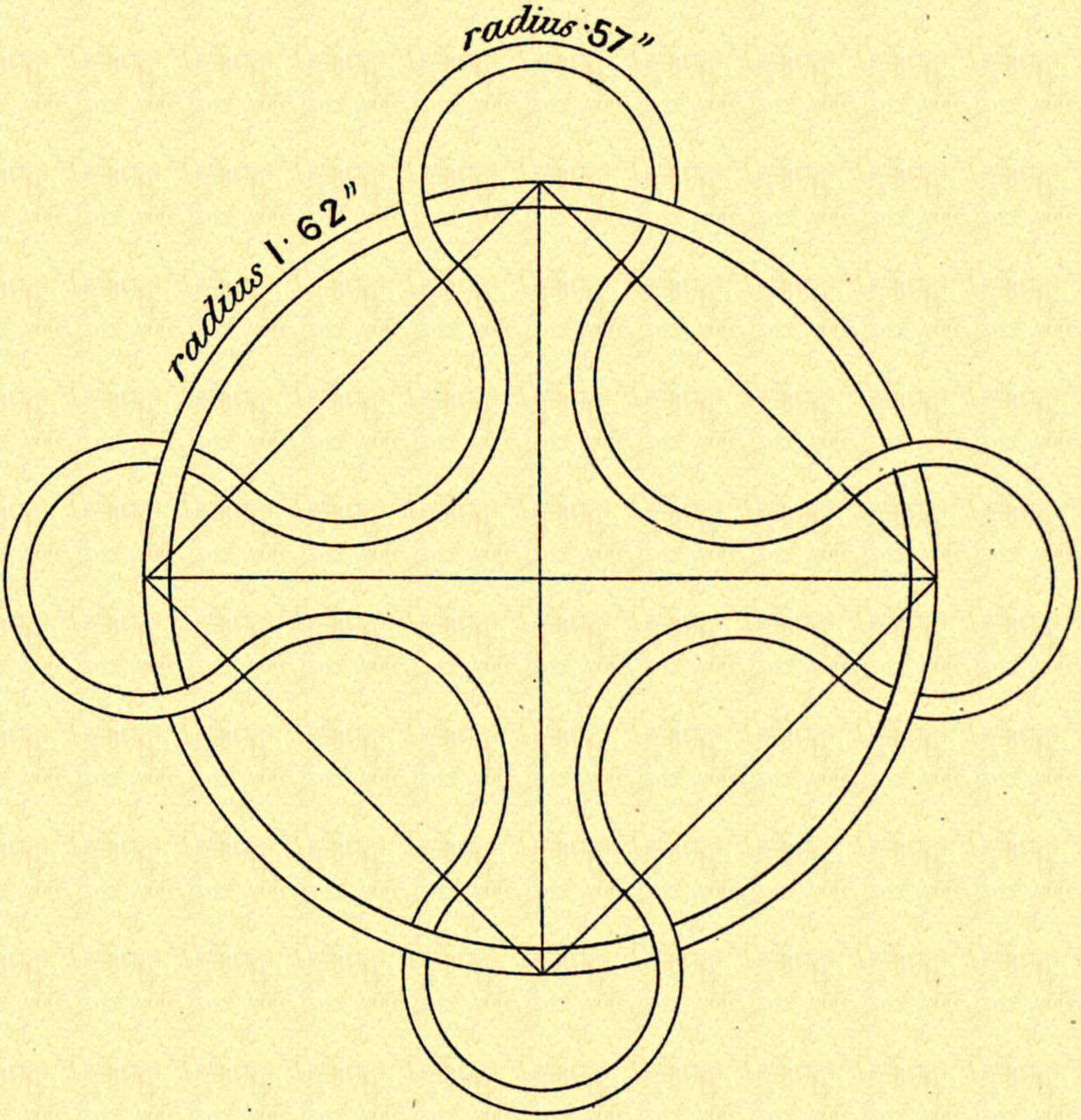




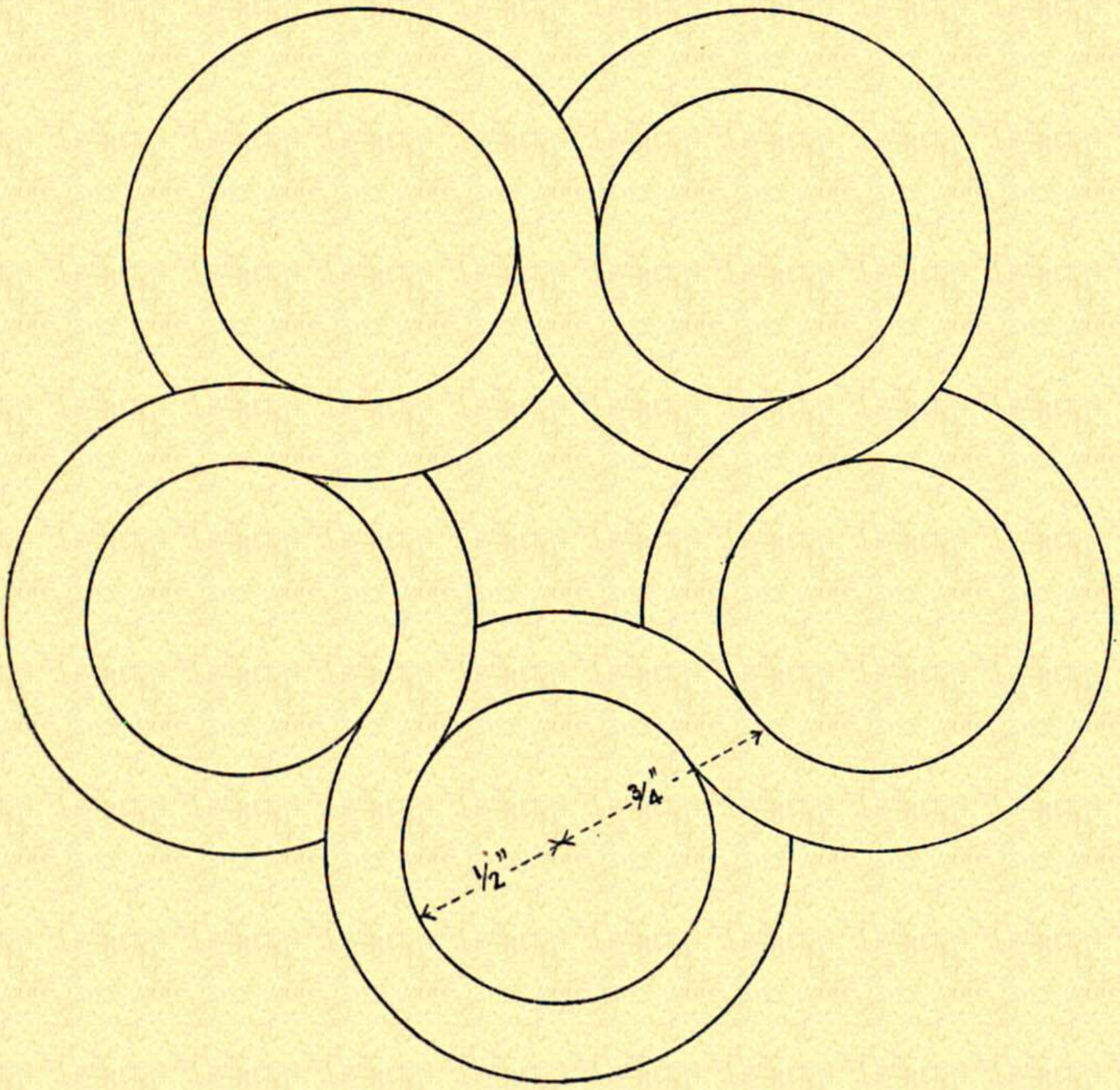
Inner circles 1", ¾".

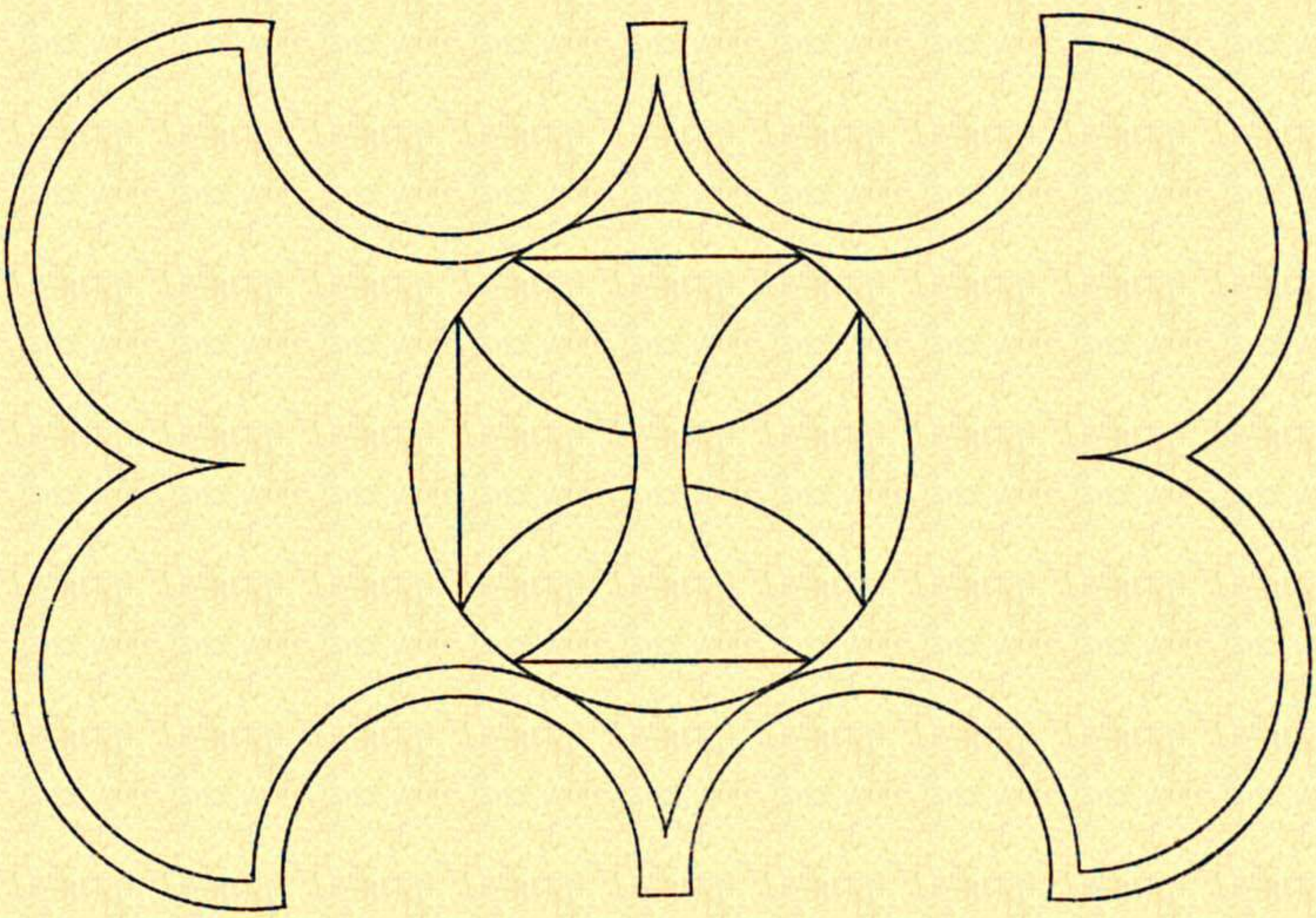


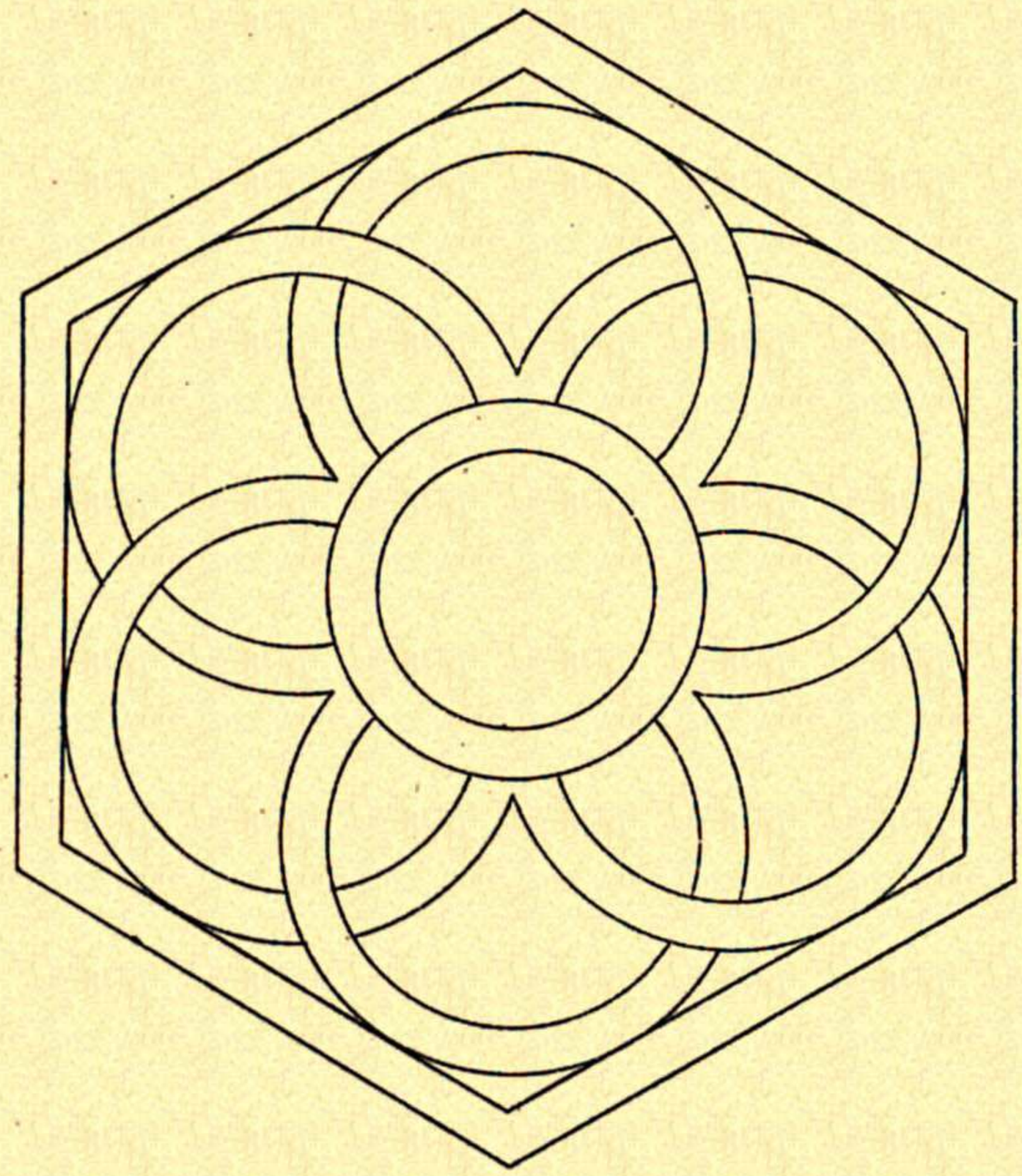
Ribs 1½"
Bands ⅕"



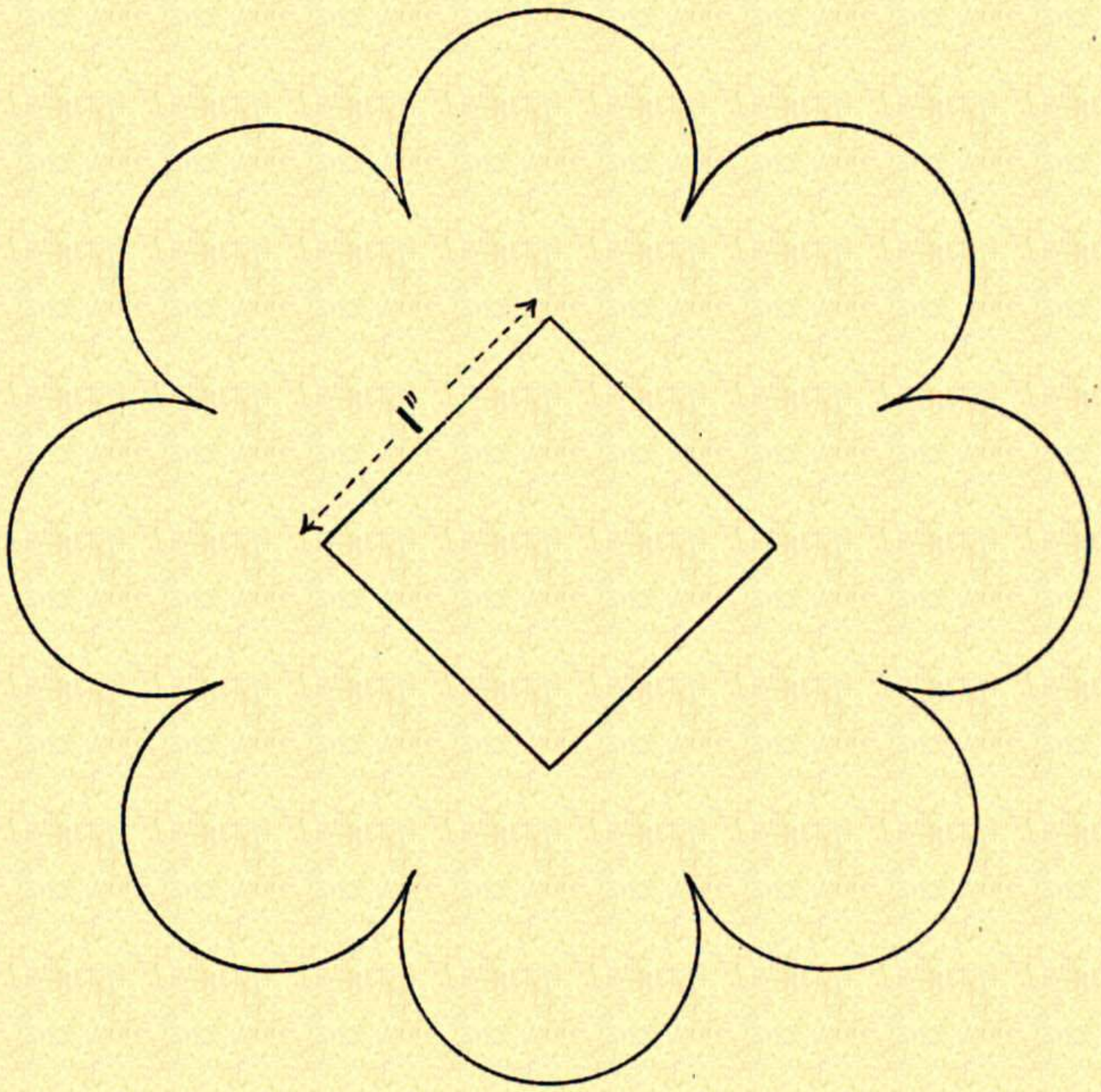
Bands ·1" wide

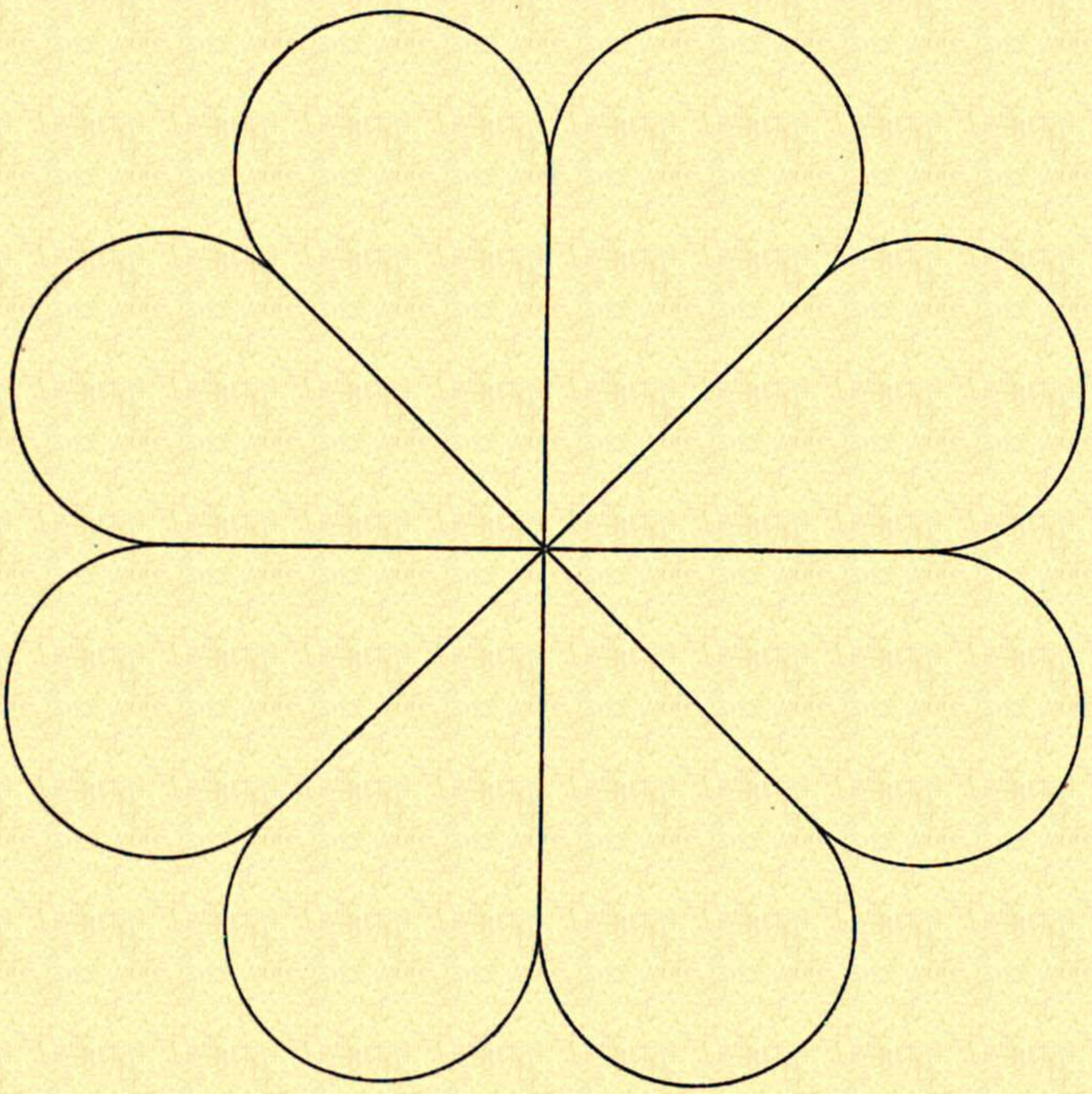

1/2"
3/4"



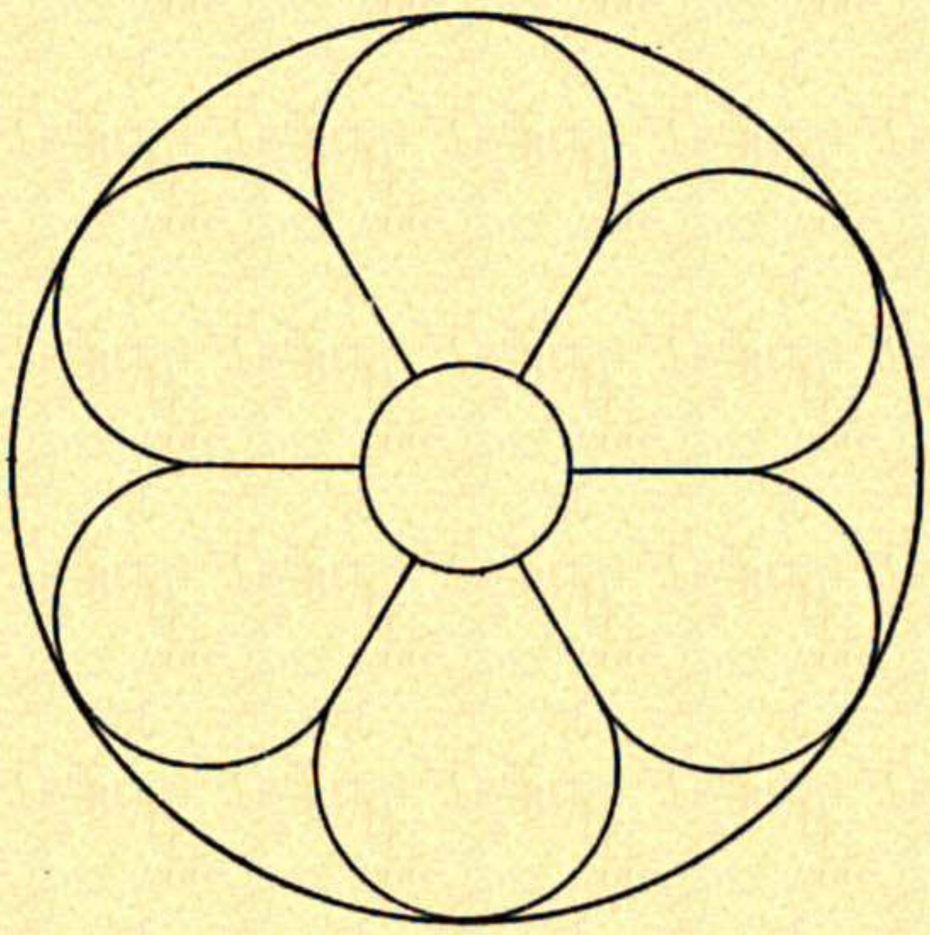


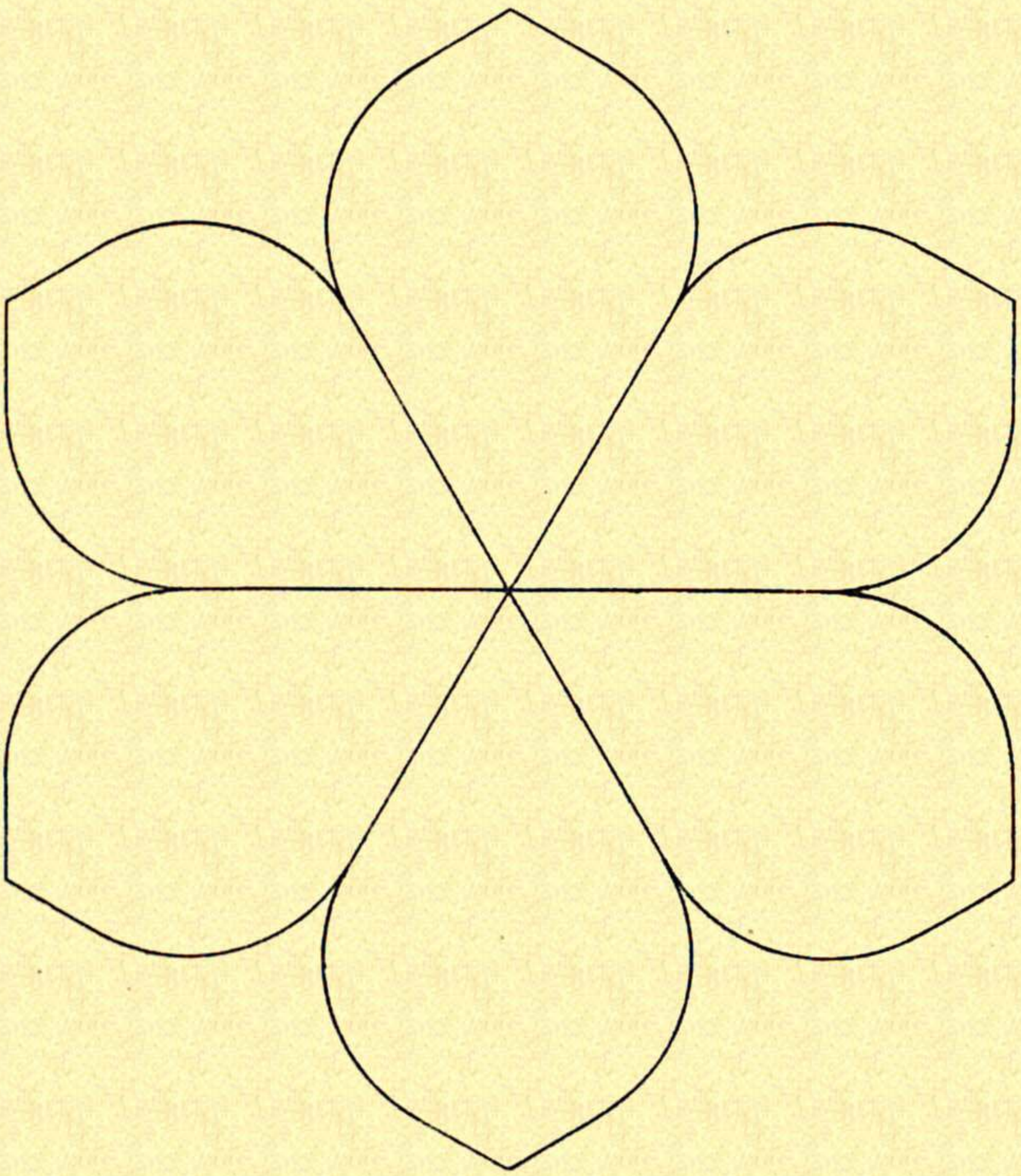
Side of inner hexagon 1"
Band ⅕"


1"

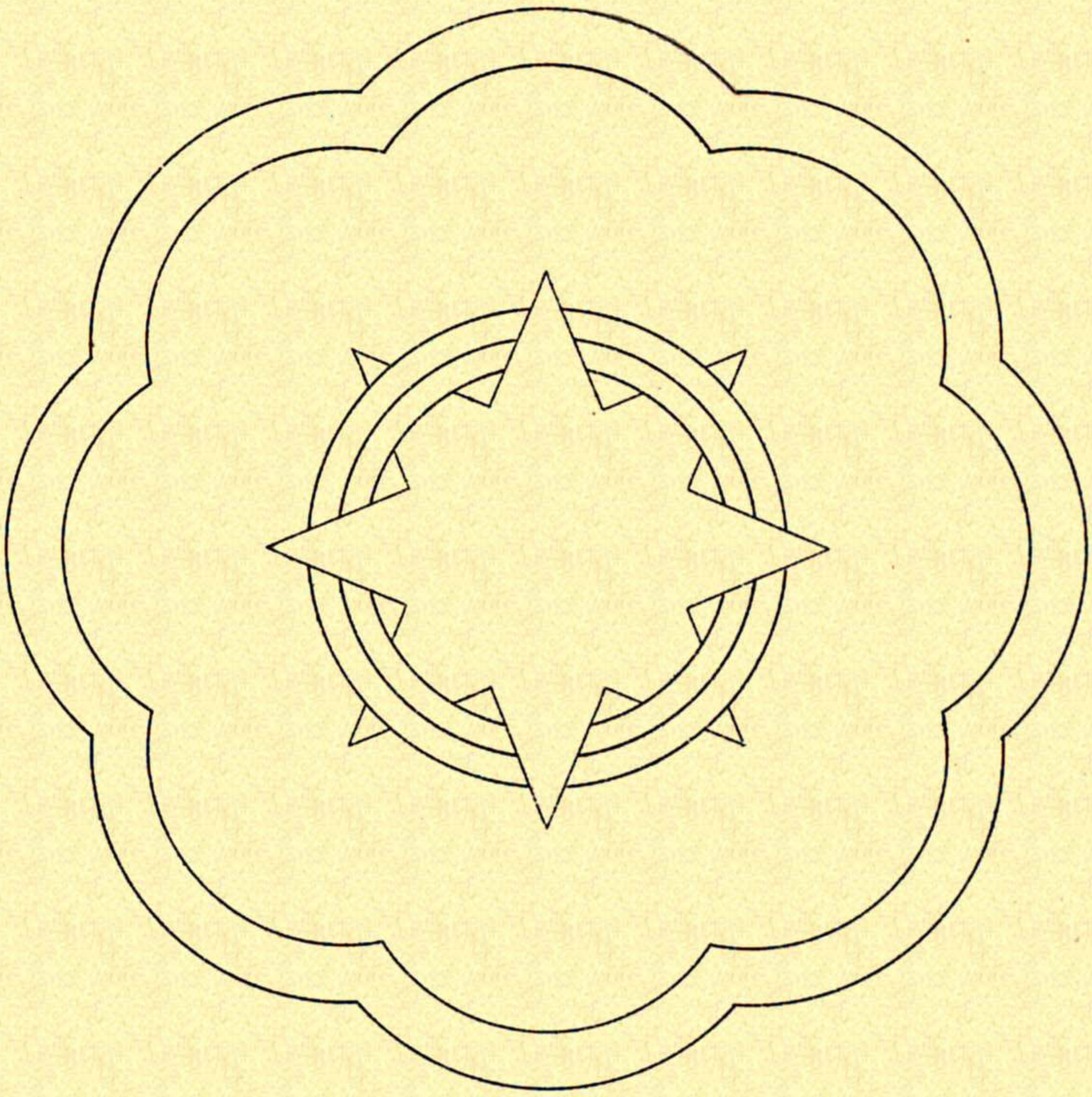


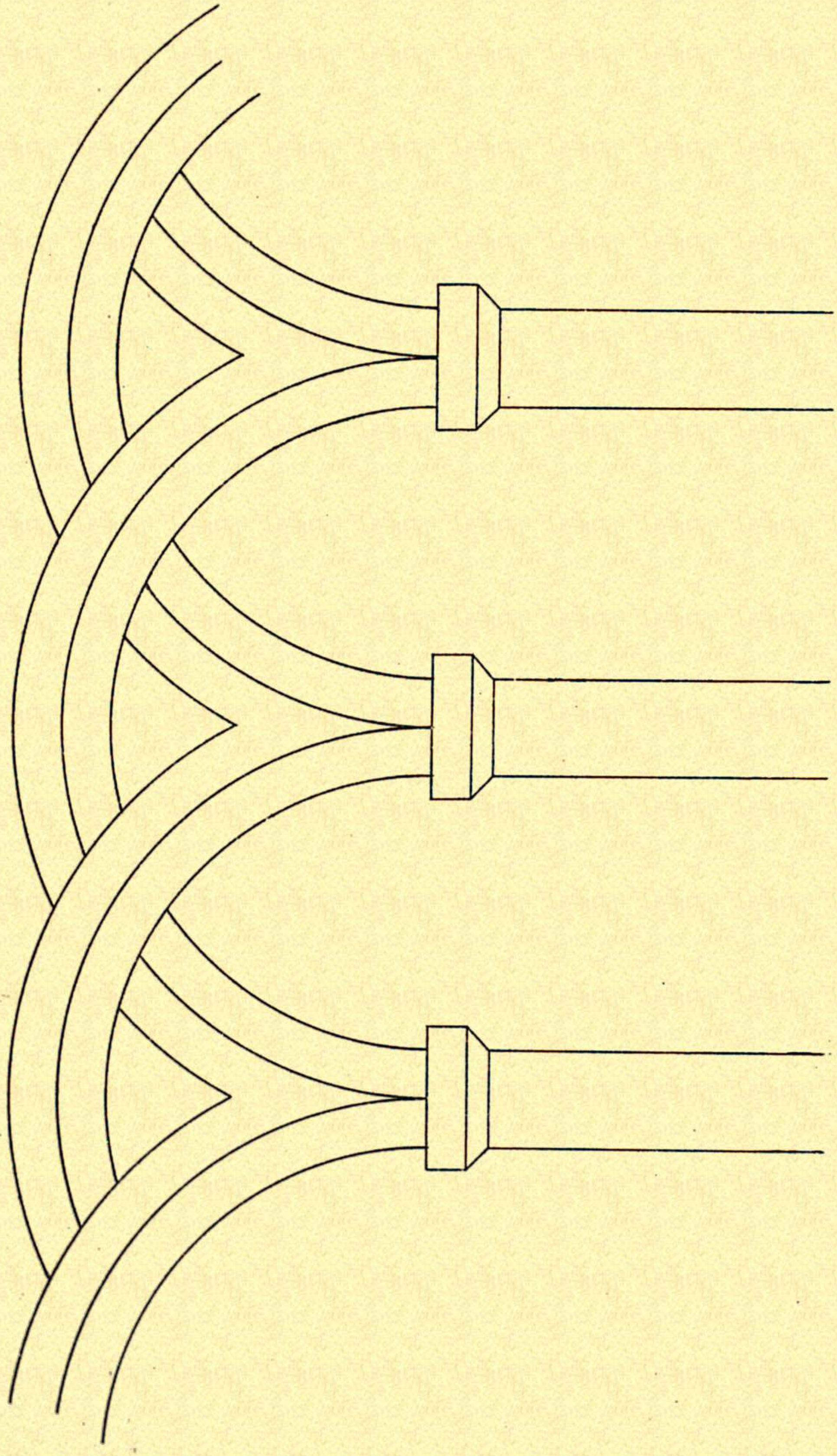
Radius of Long! arcs ½"

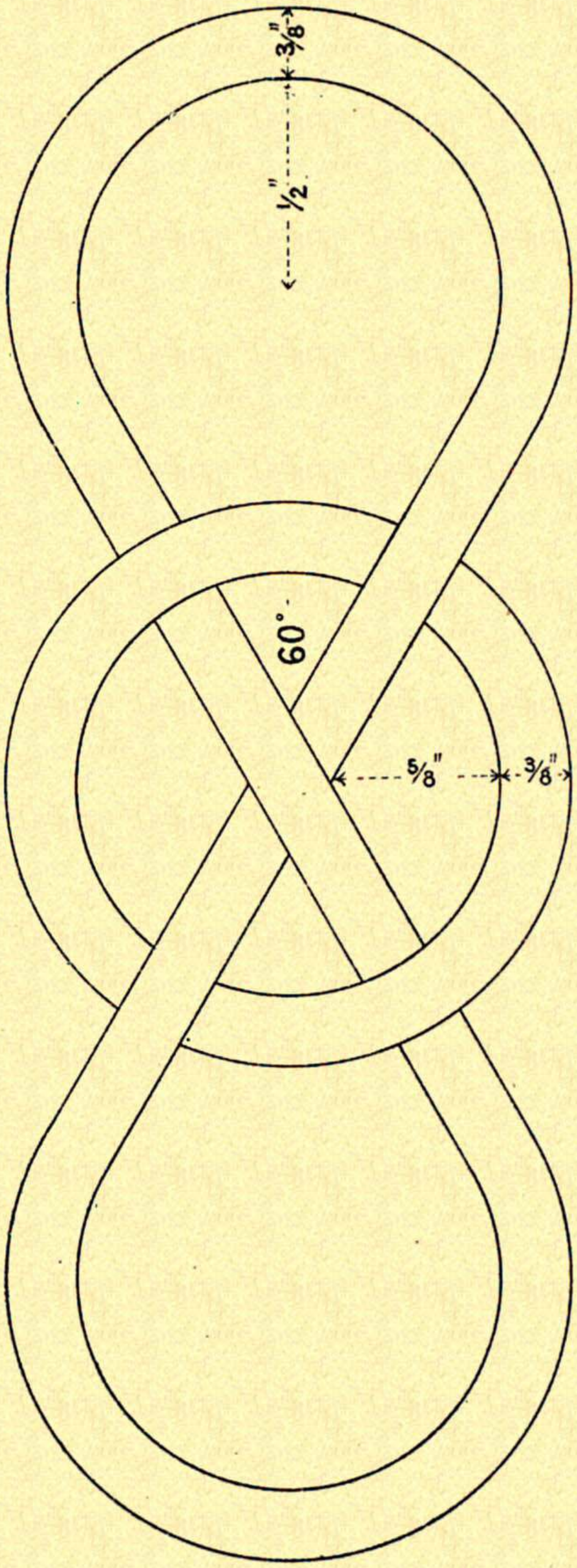


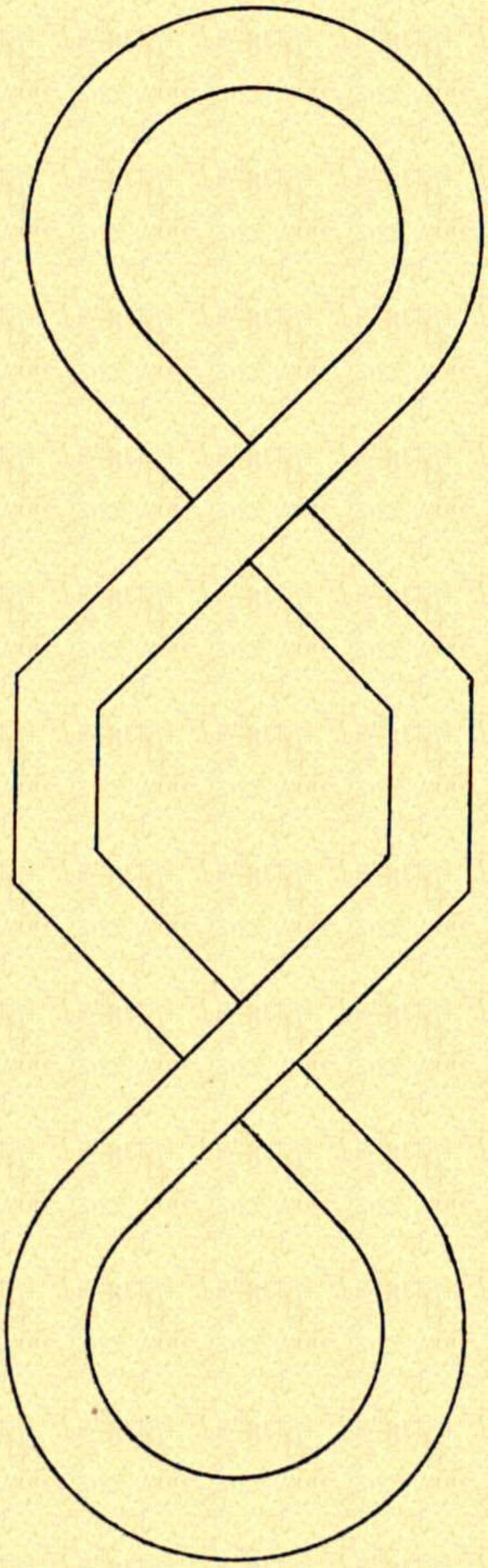


Side of outer hex. 2"

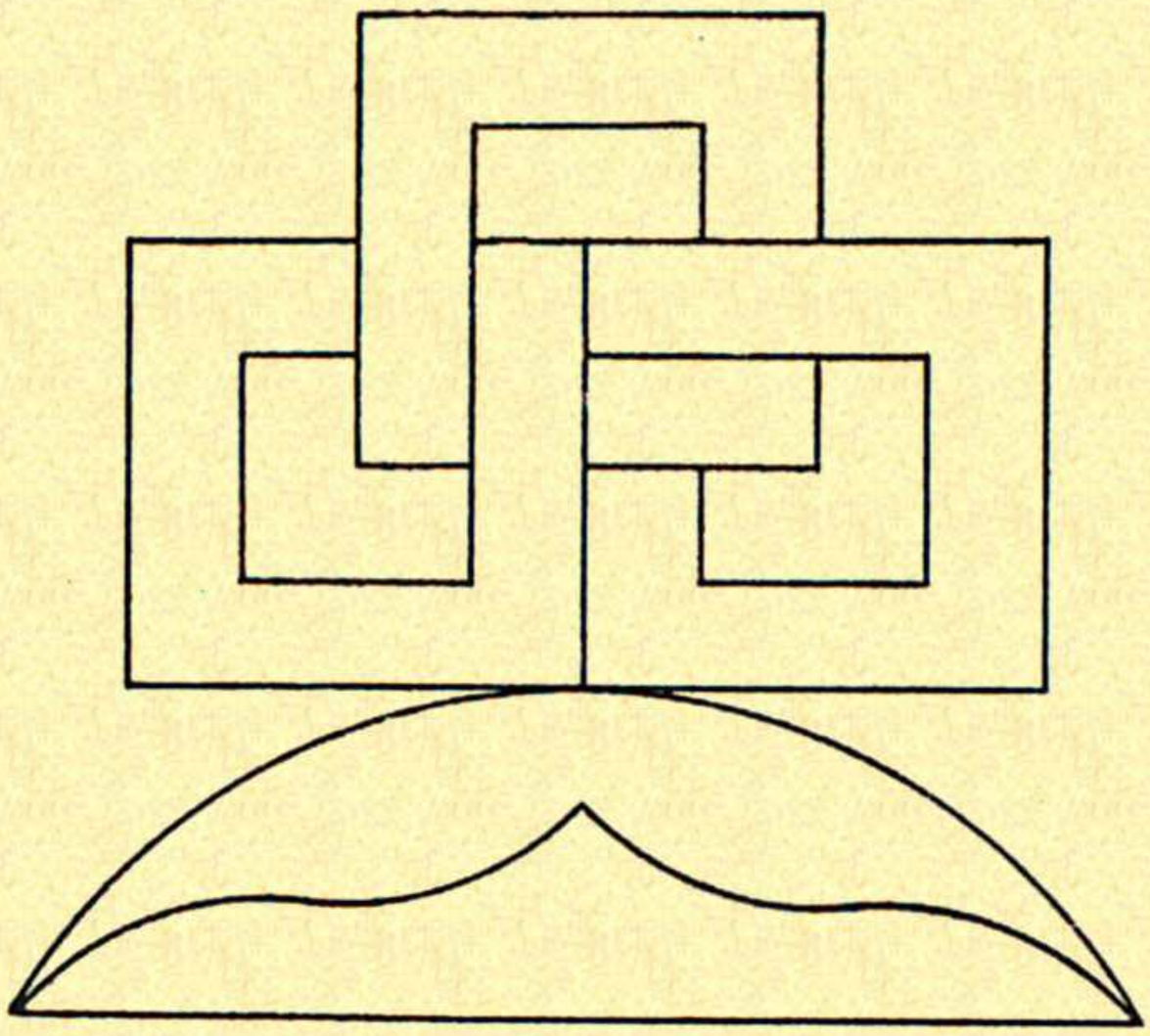


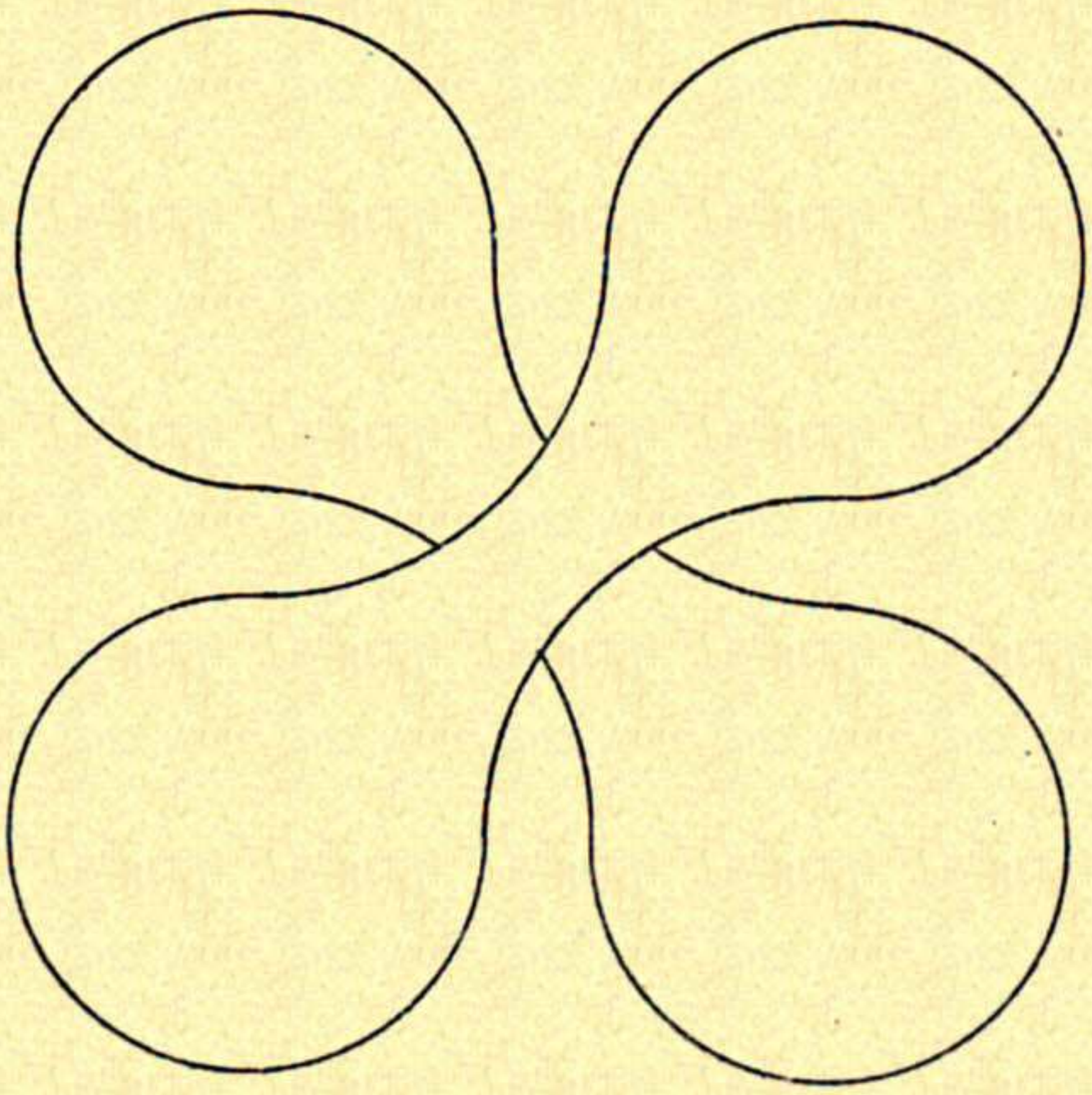



3/8"
1/2"
60°
5/8"
3/8"

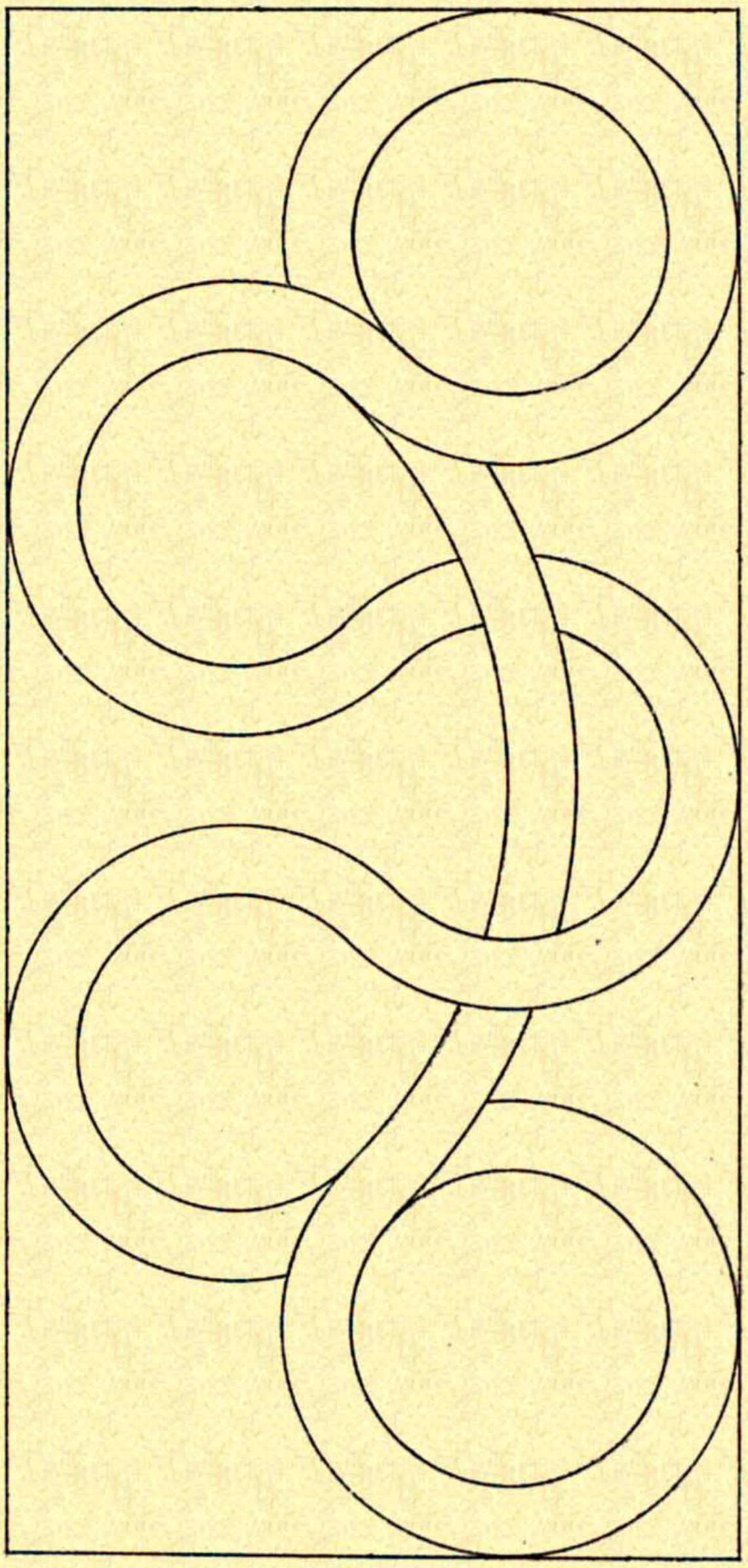


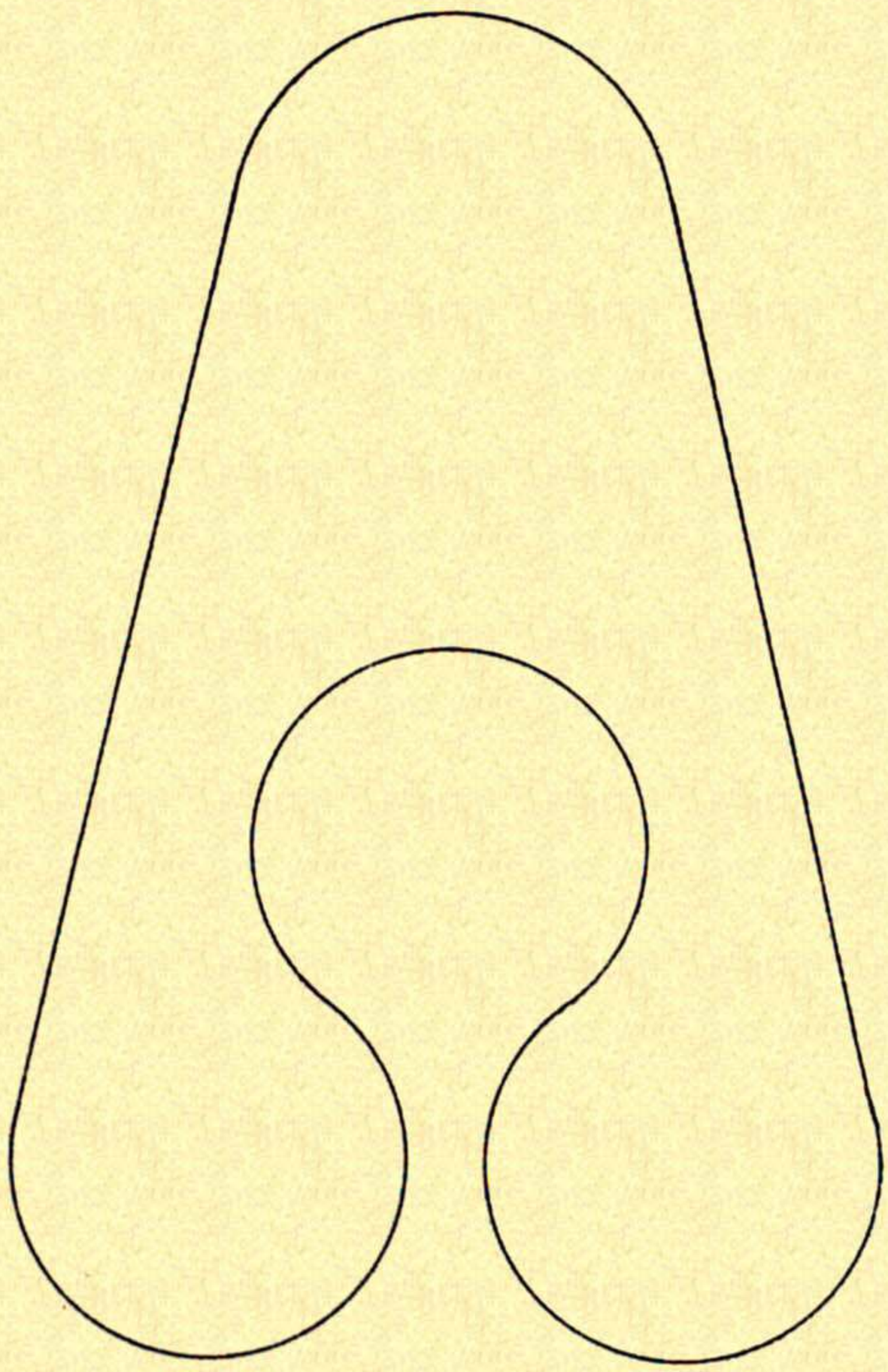
Radii ½" ¾"





Radii ·6". ·9".





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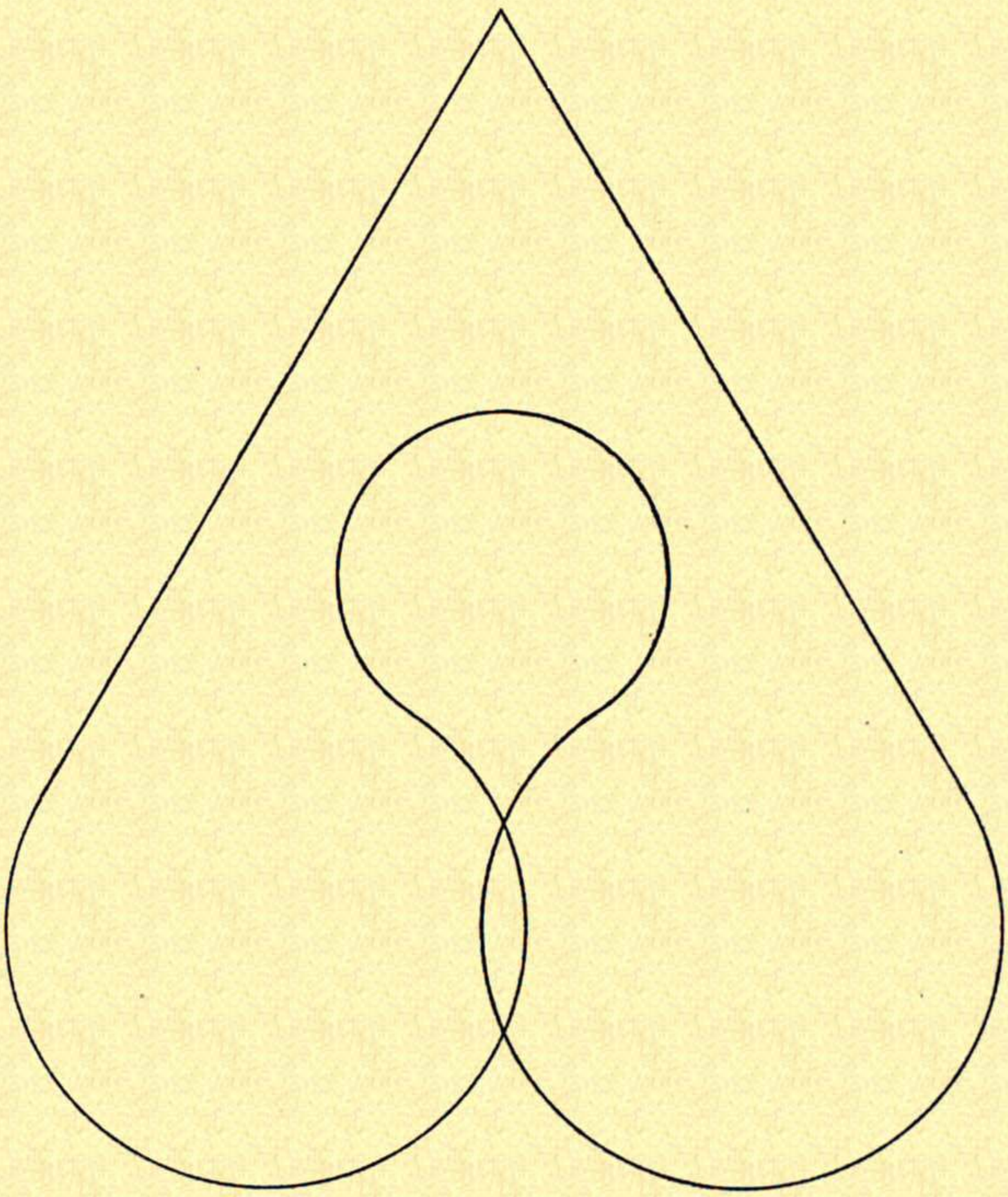
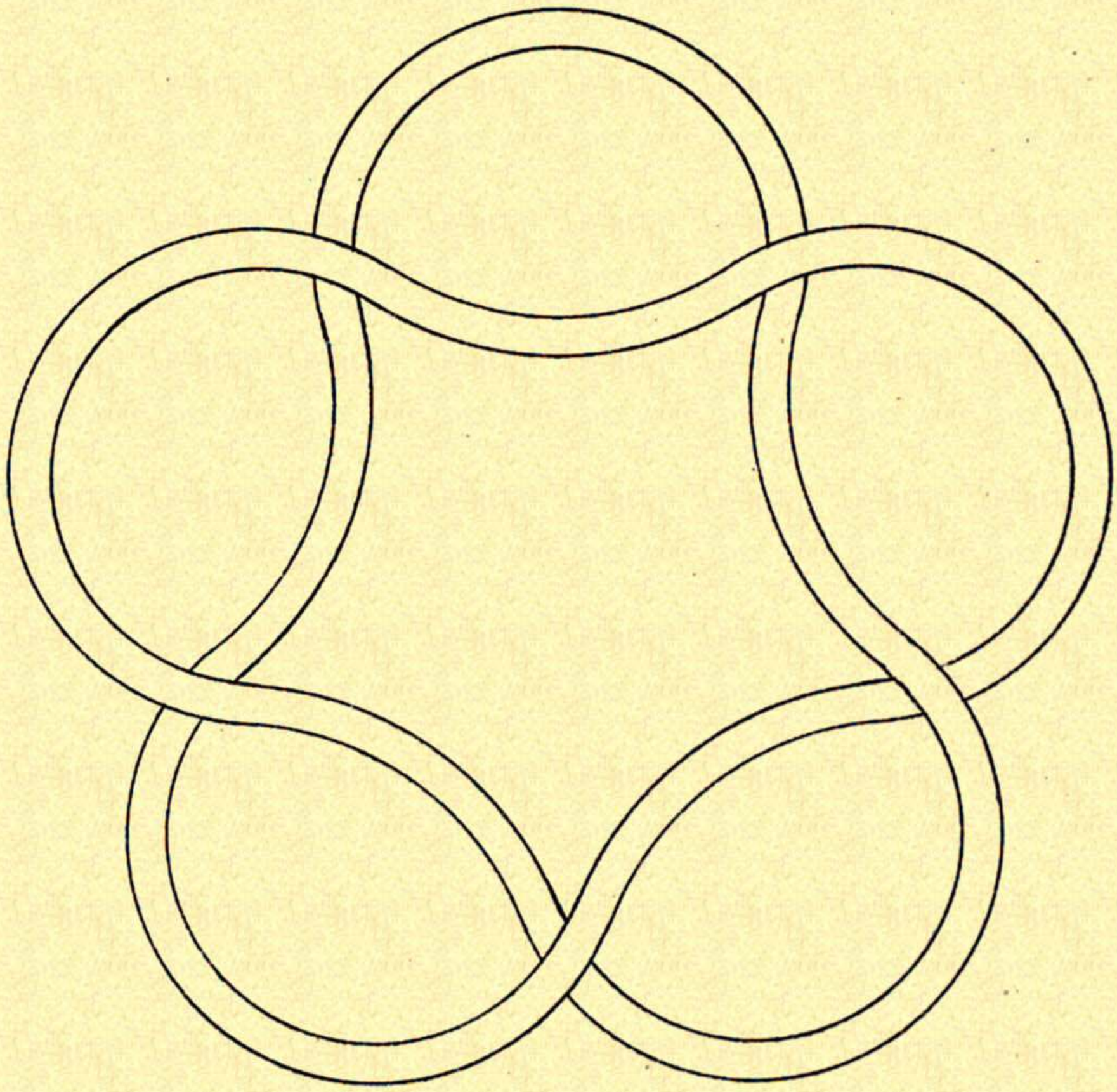
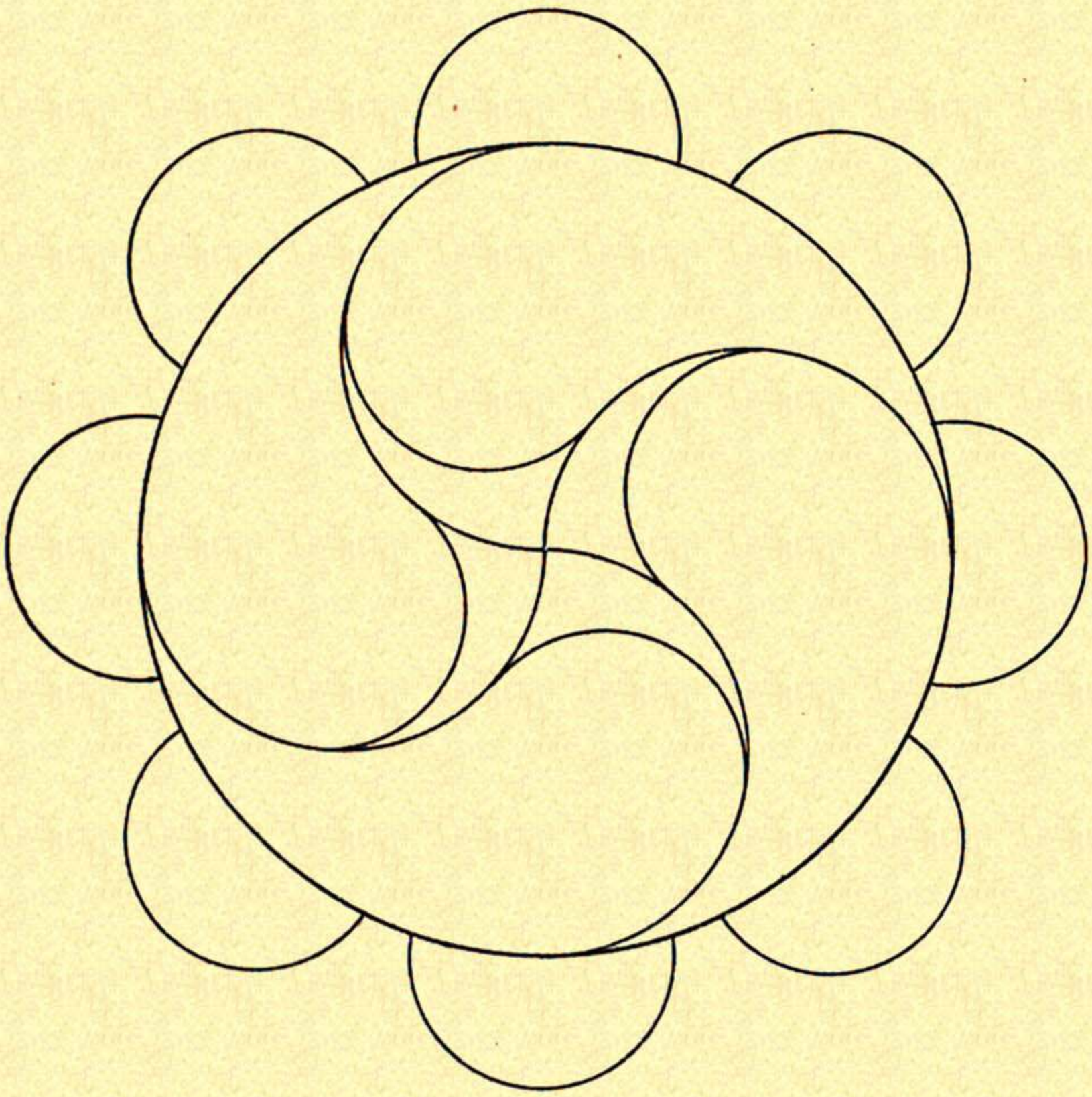
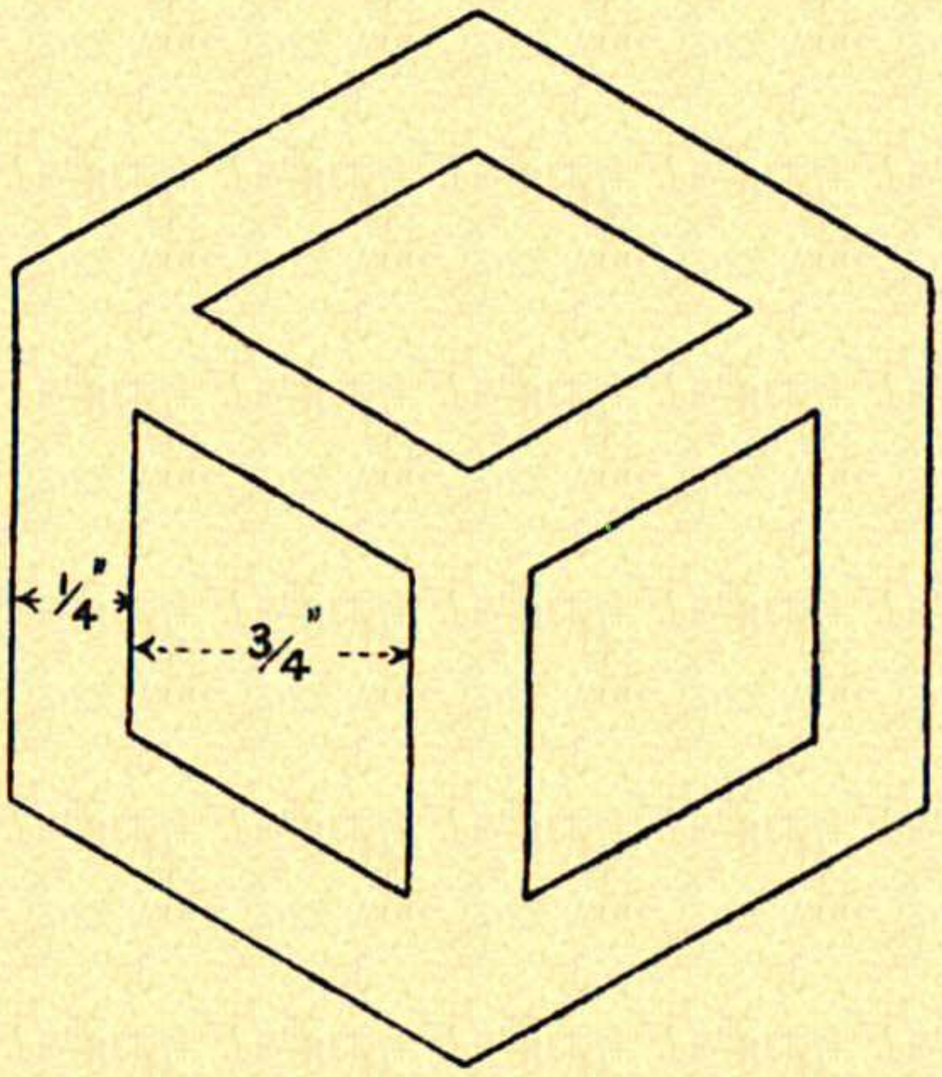
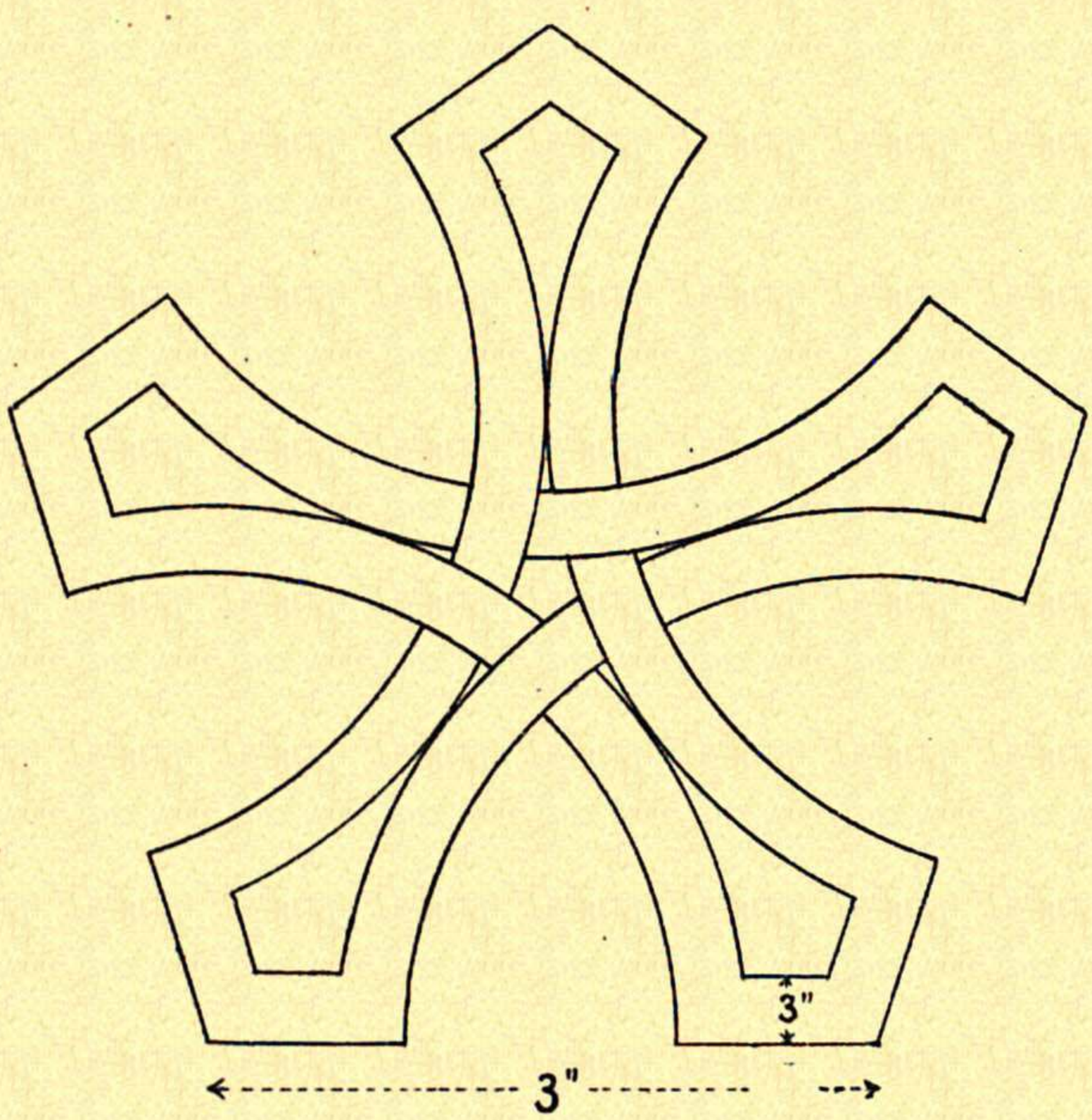


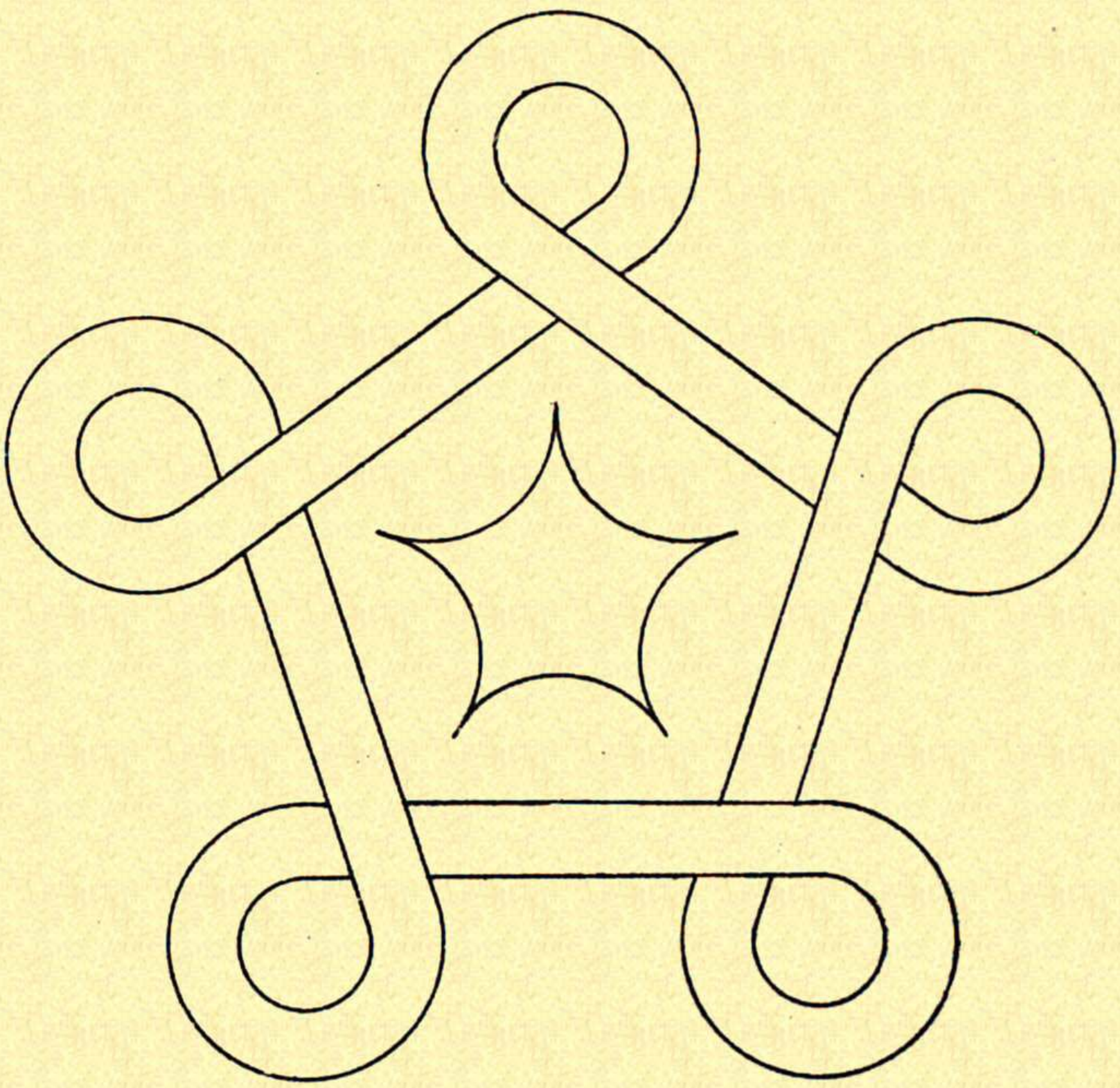
radius ·6"
1·2"
·6"
1·2"
·6"

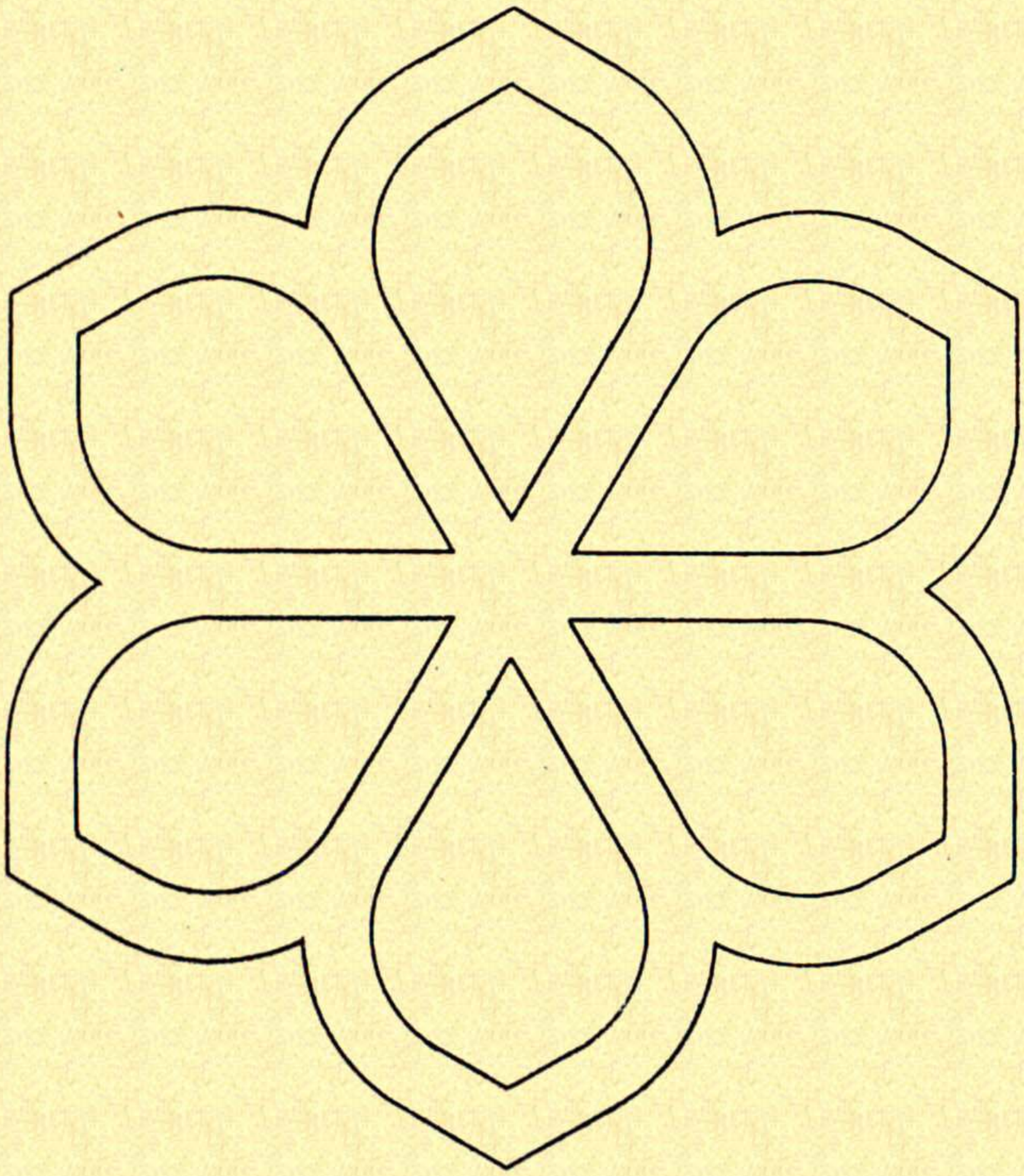





1/4"
3/4"

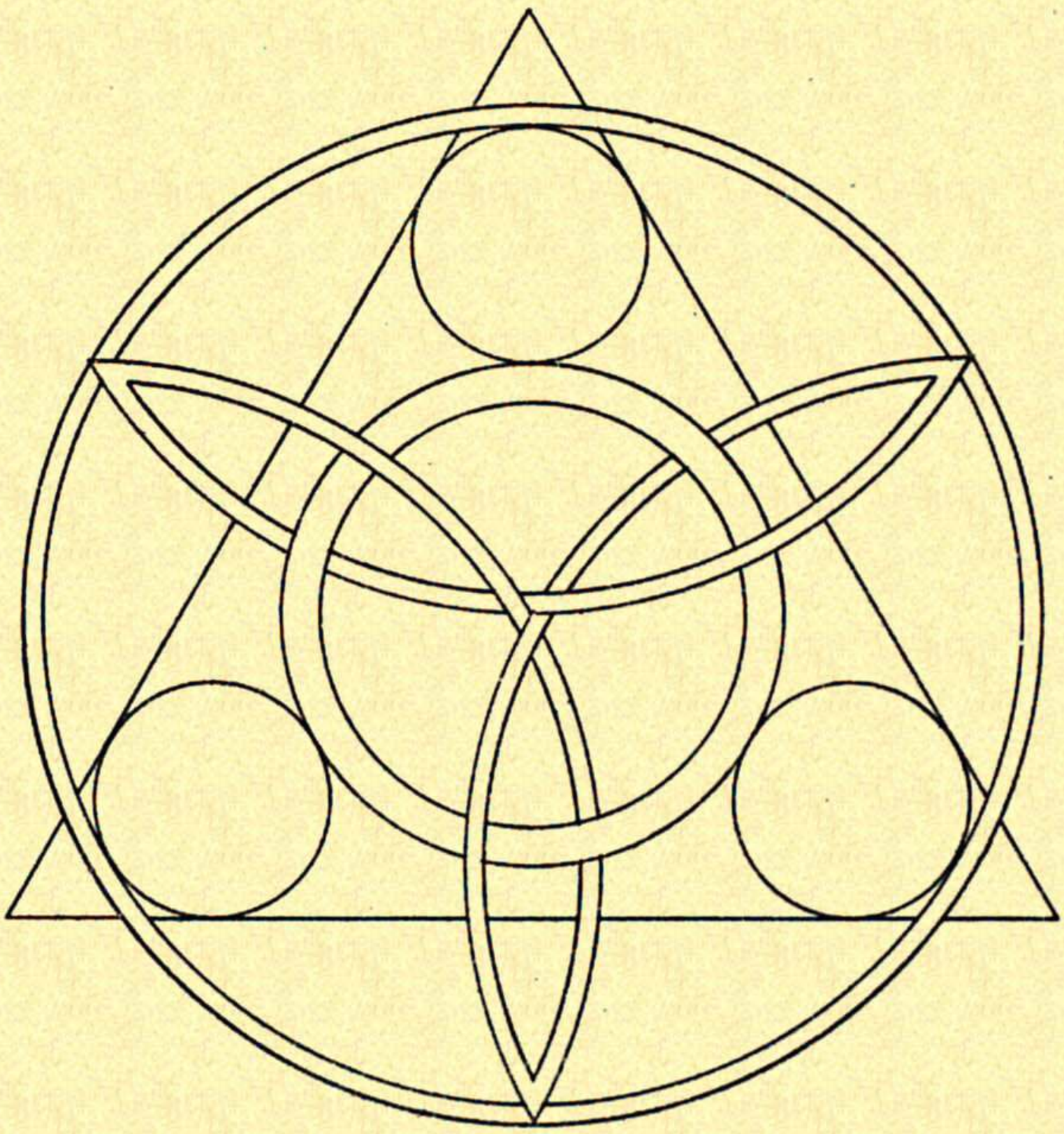

3"
3"

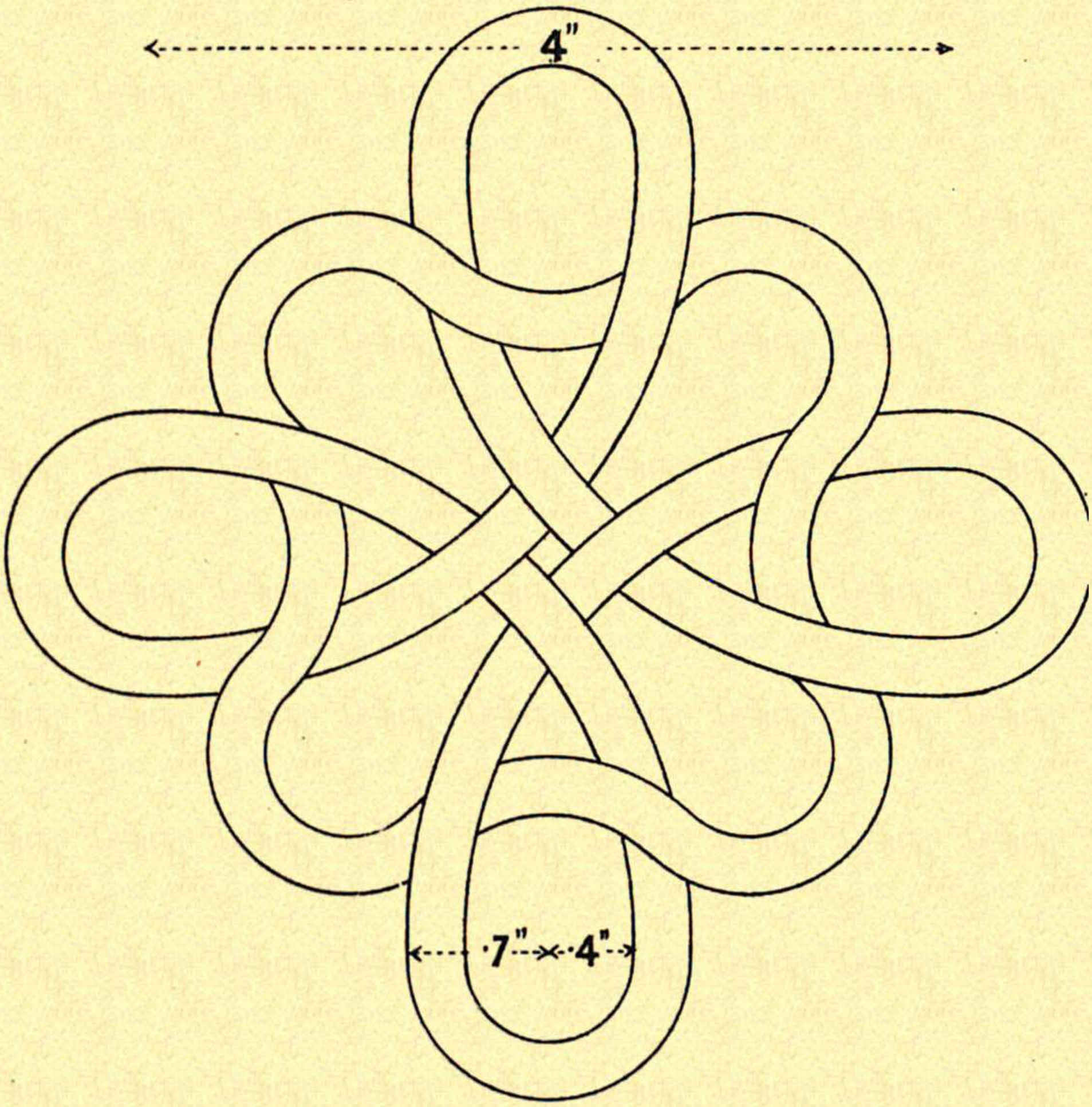


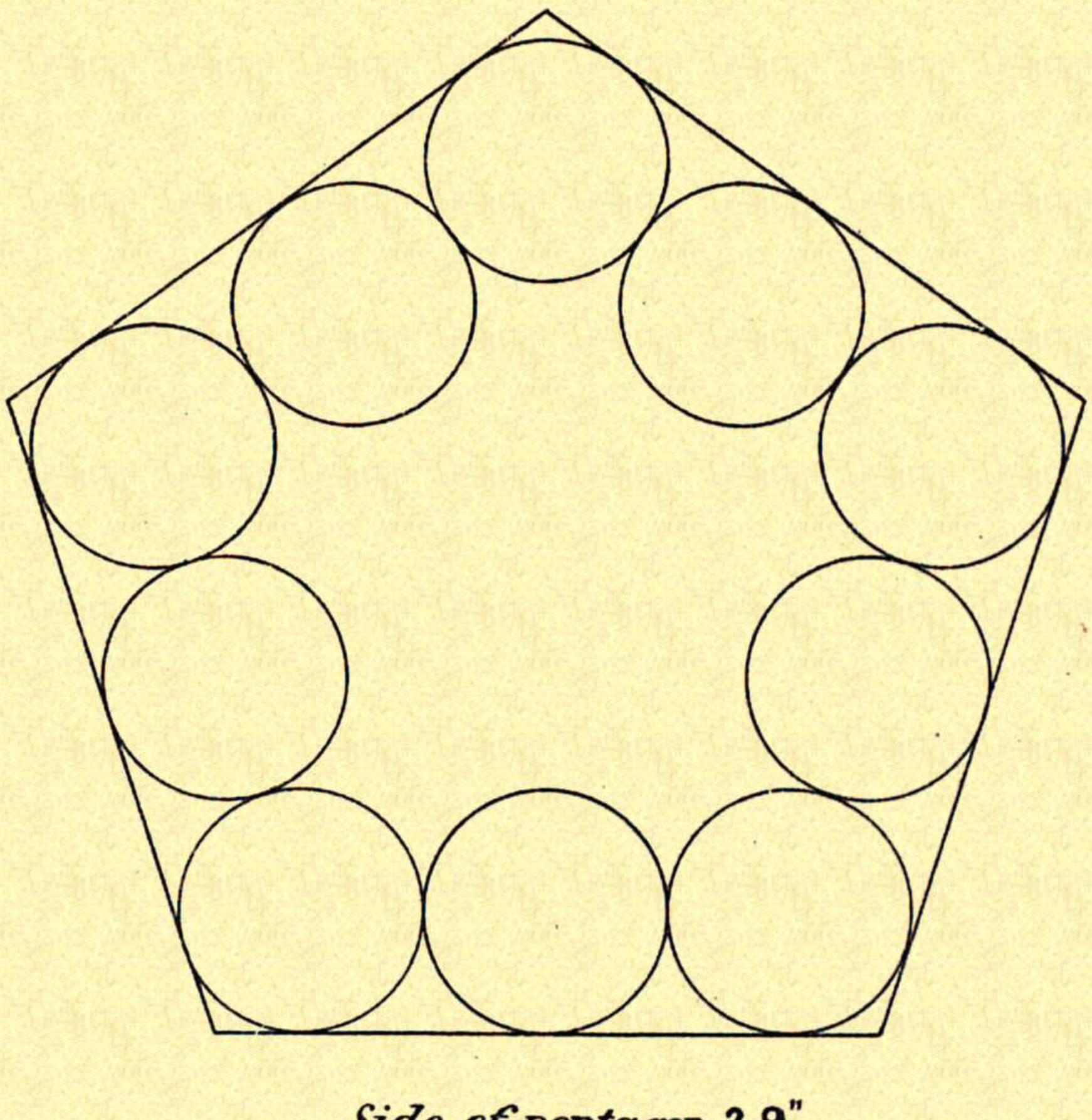


Side of outer hex 2"

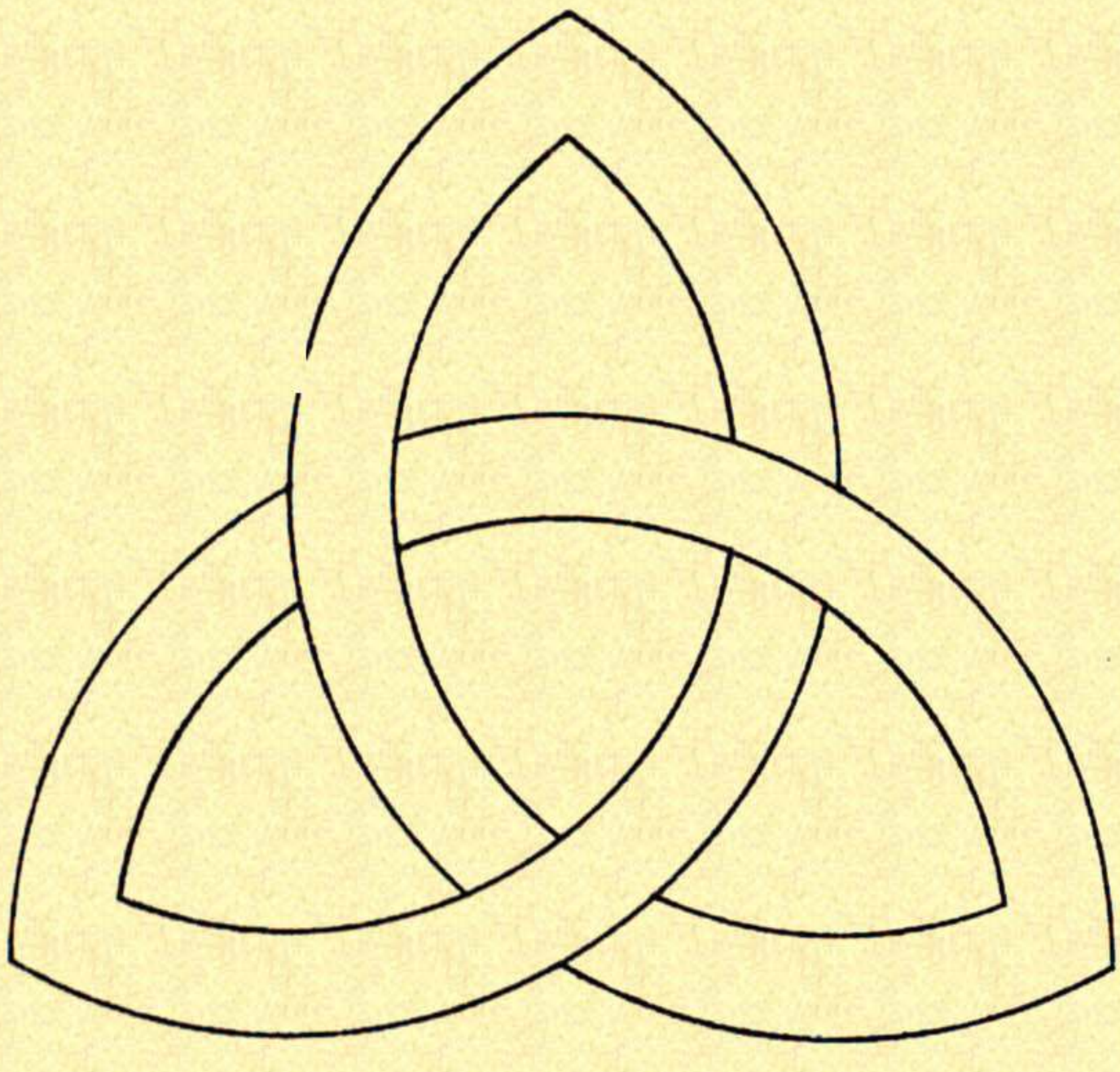
band ⅕"



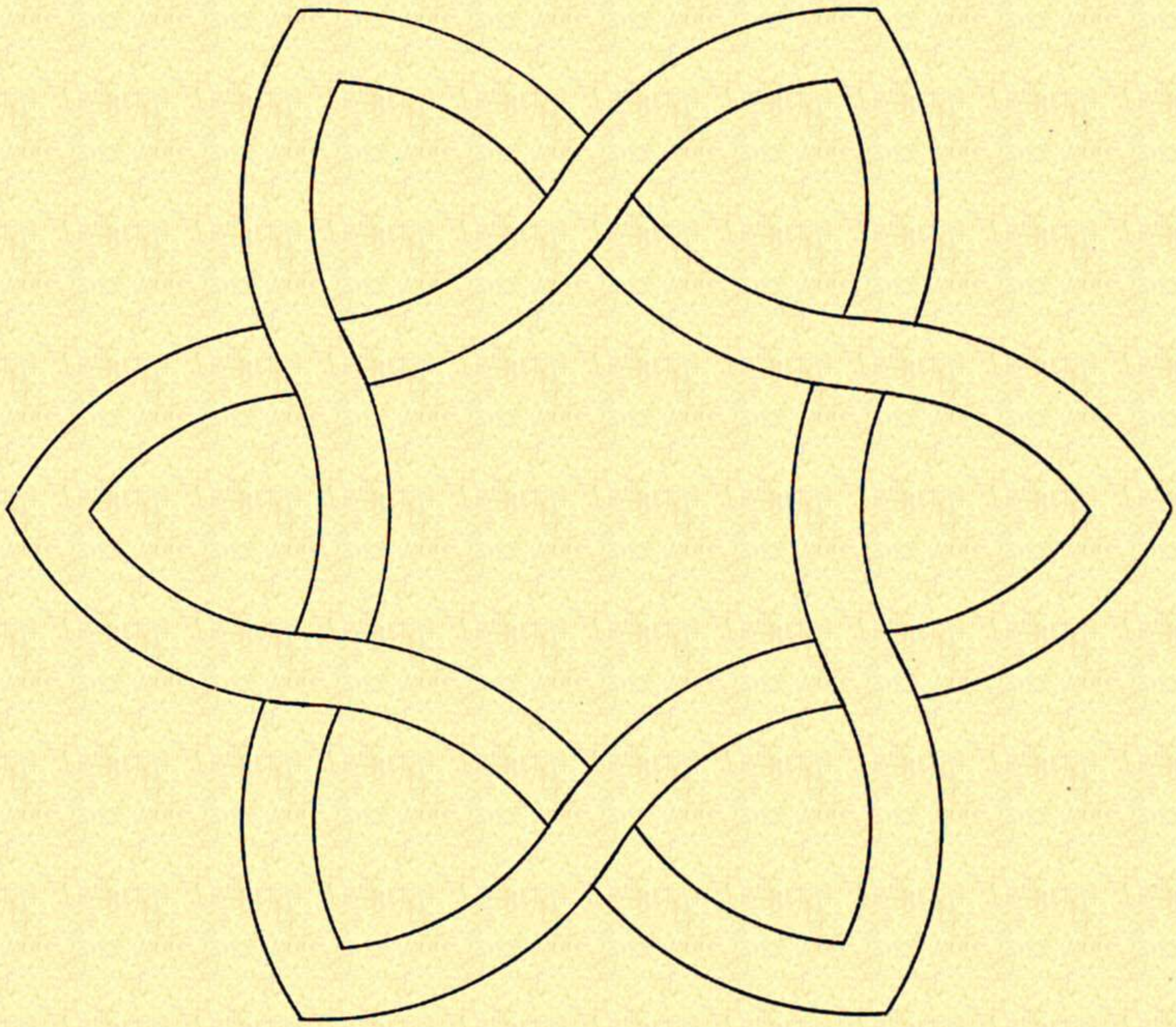

4"
7"
4"



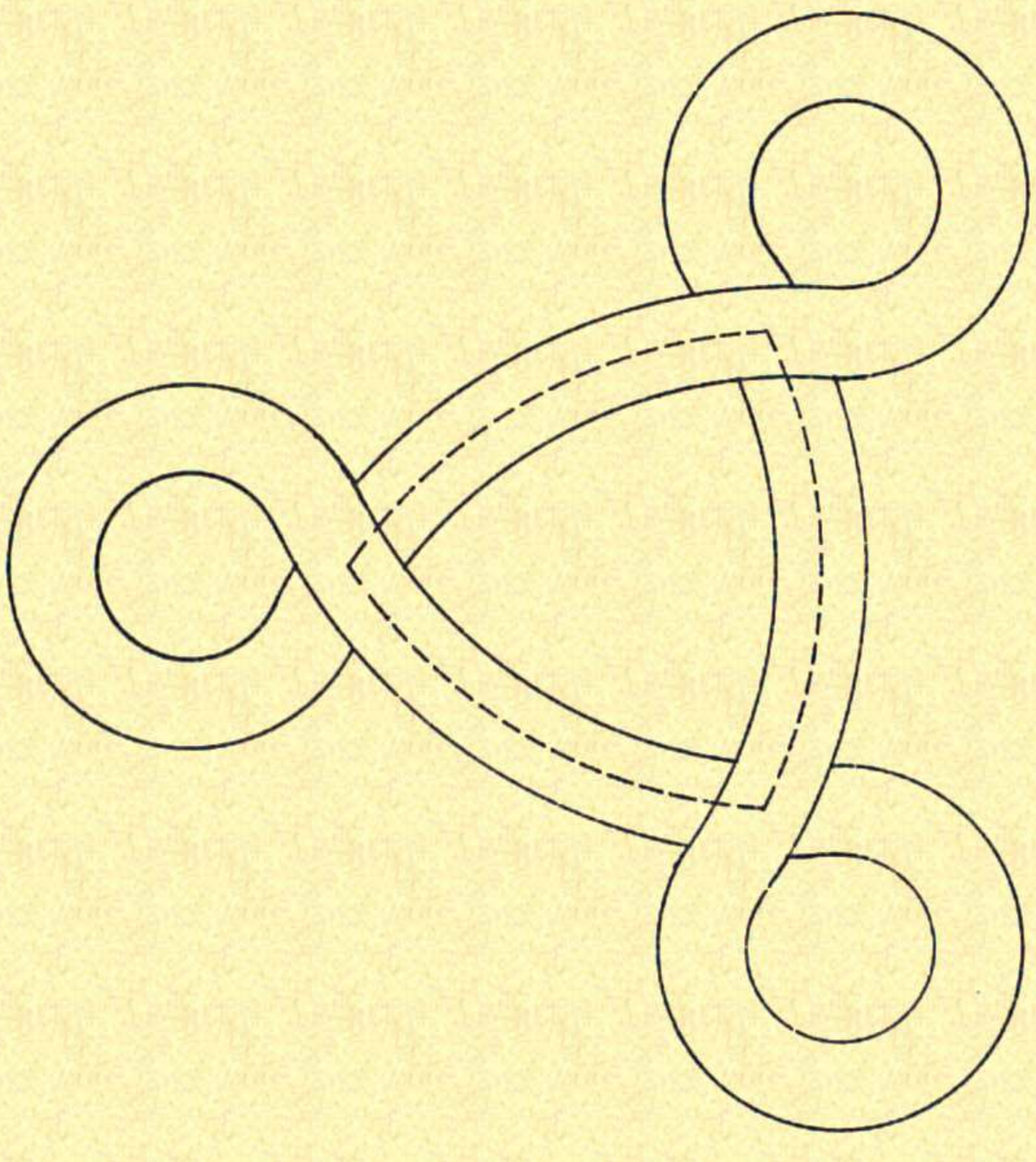
Side of pentagon 2·9"



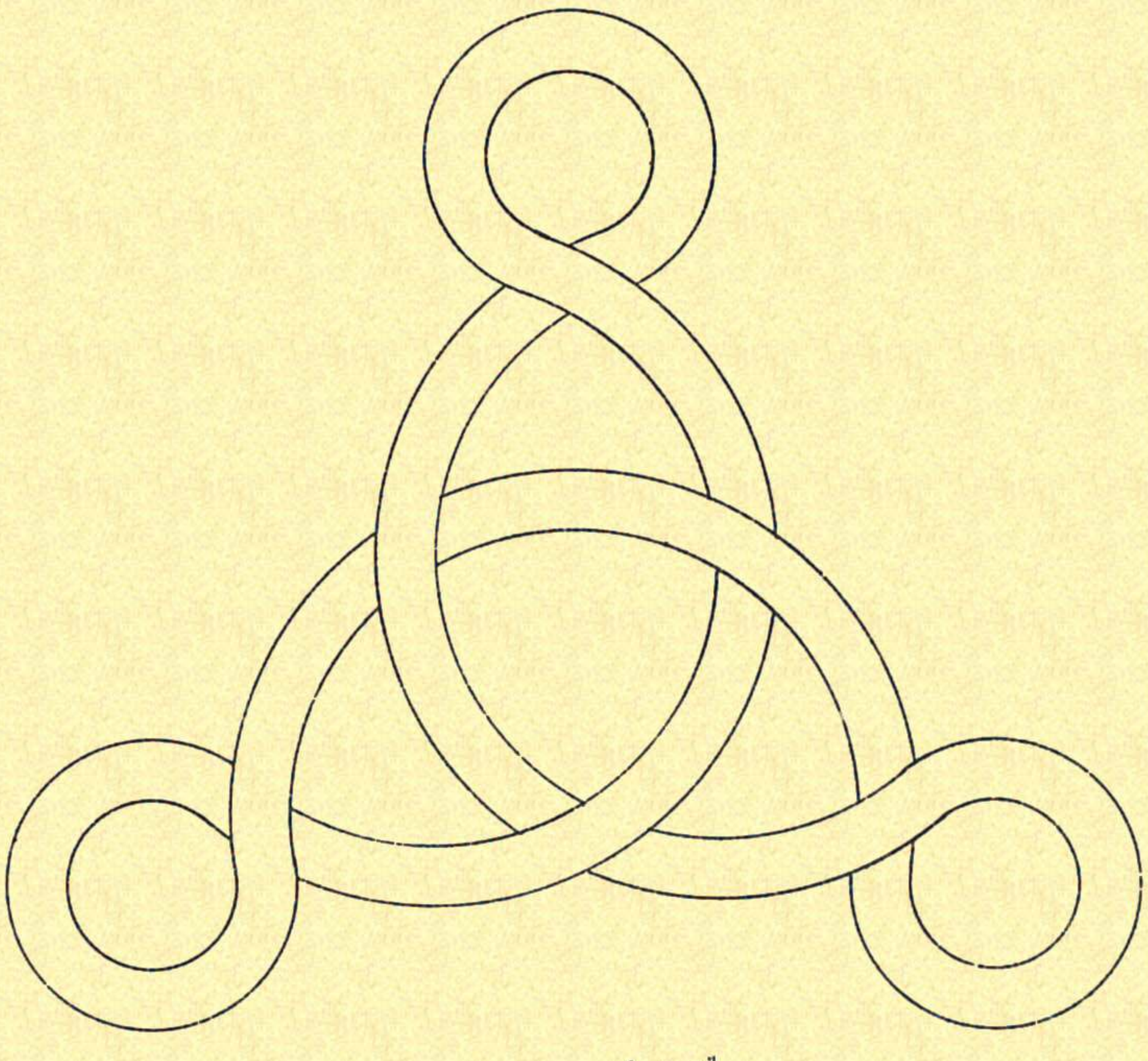
Radii 1·3"
1·7"



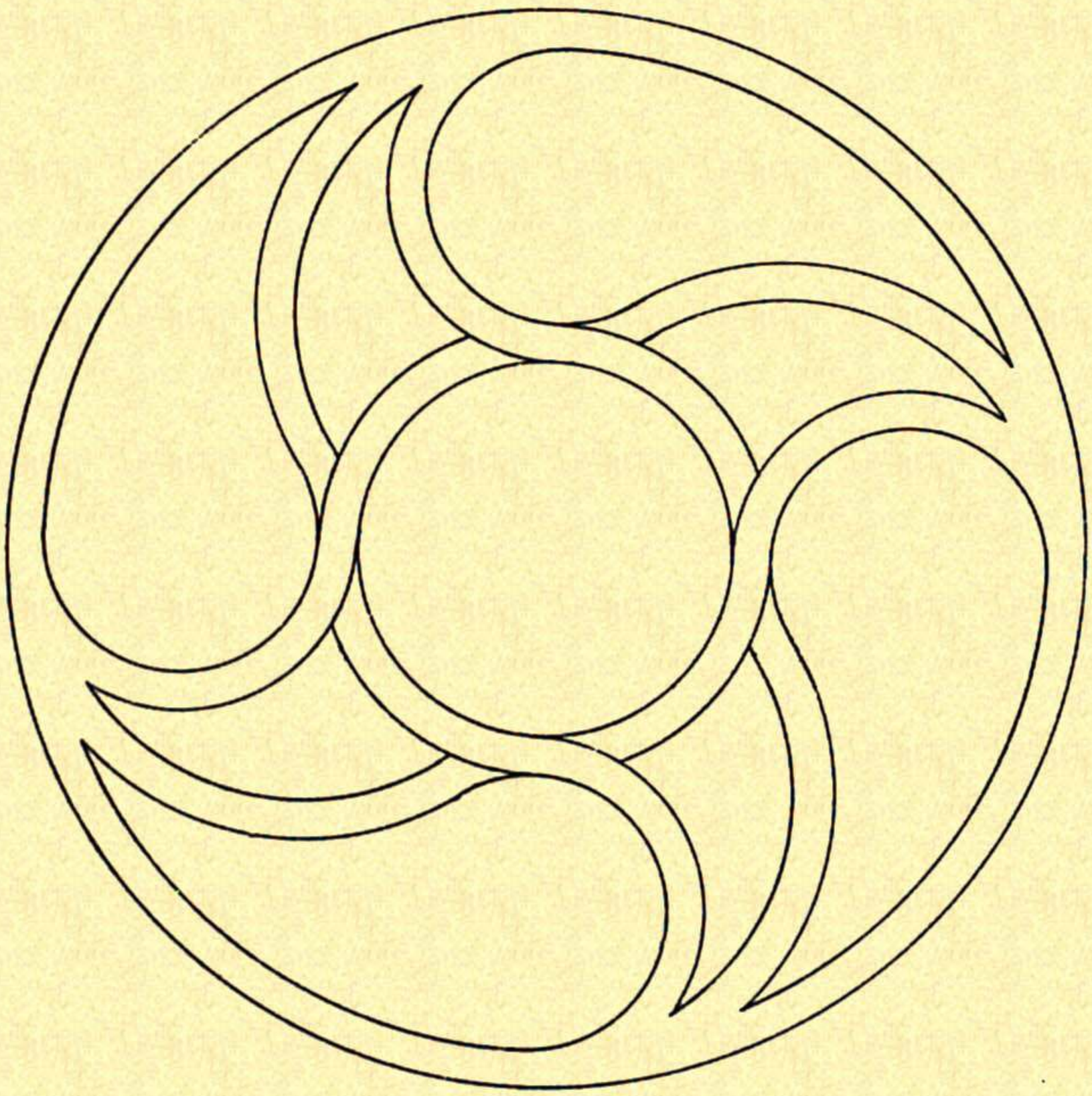
Radii 1·2"
1·4"

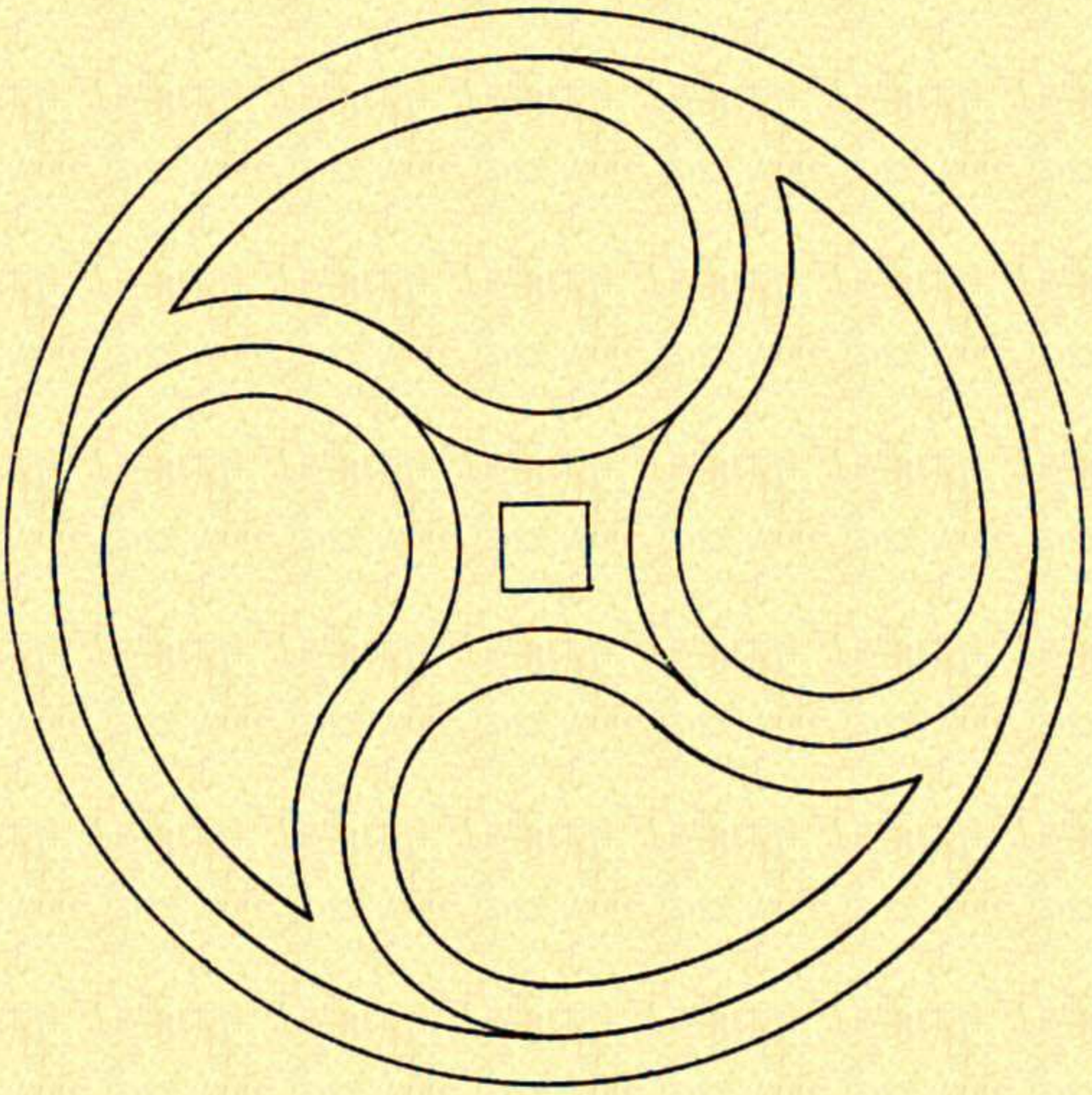


Radii 1½", 1¾"
¼", ½"

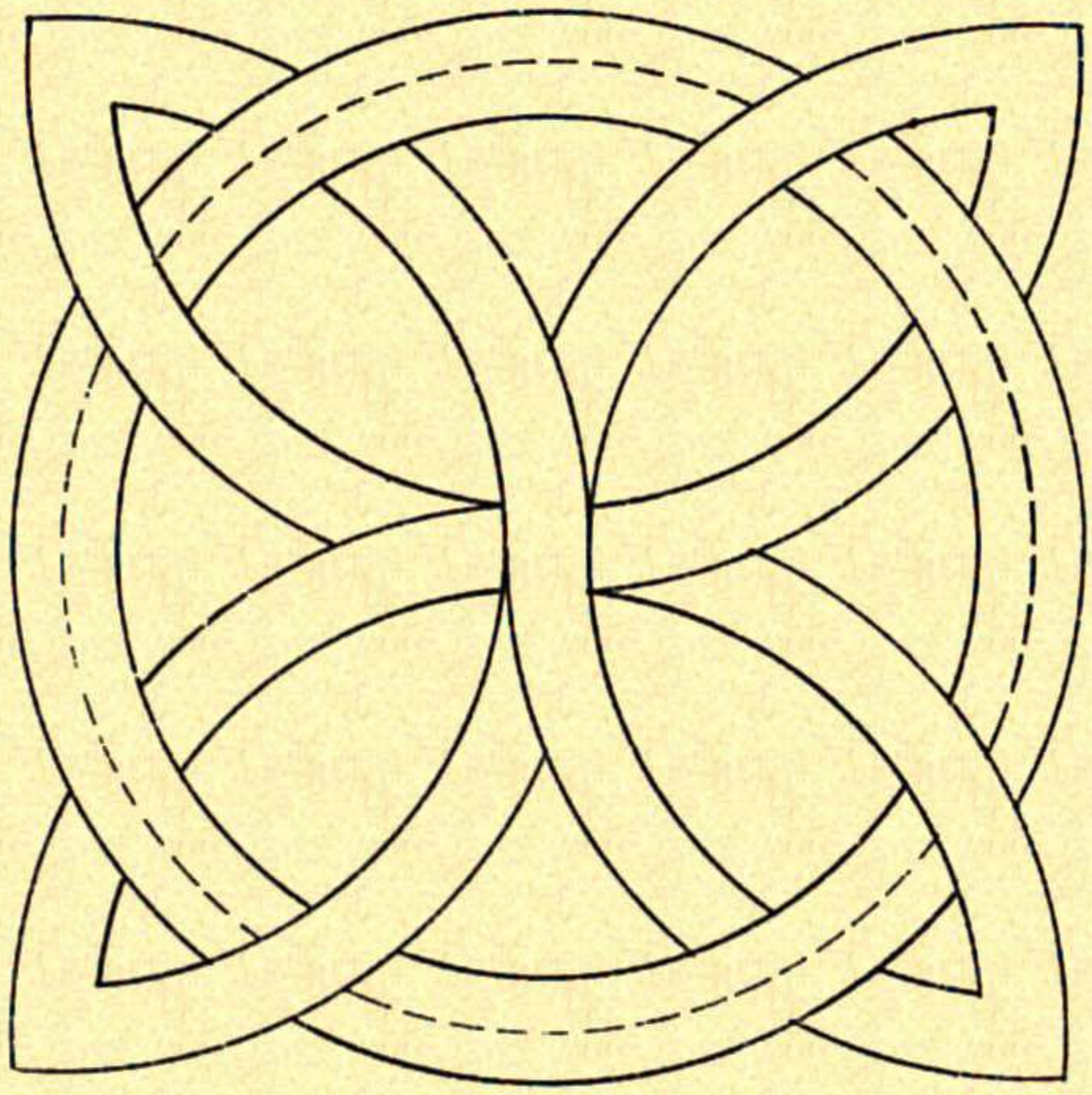


Radii ·9", 1·1"
·4, ·6"

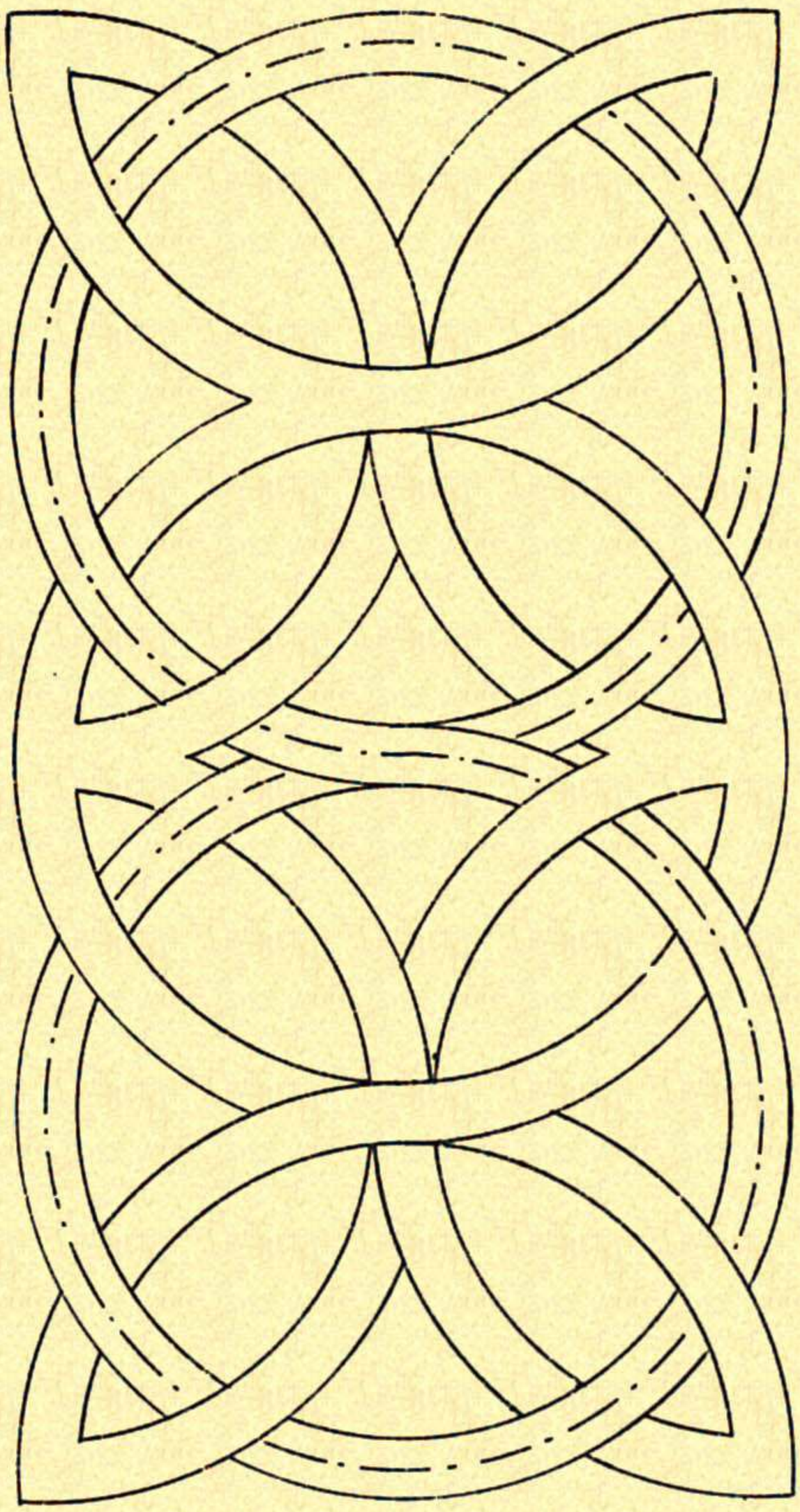




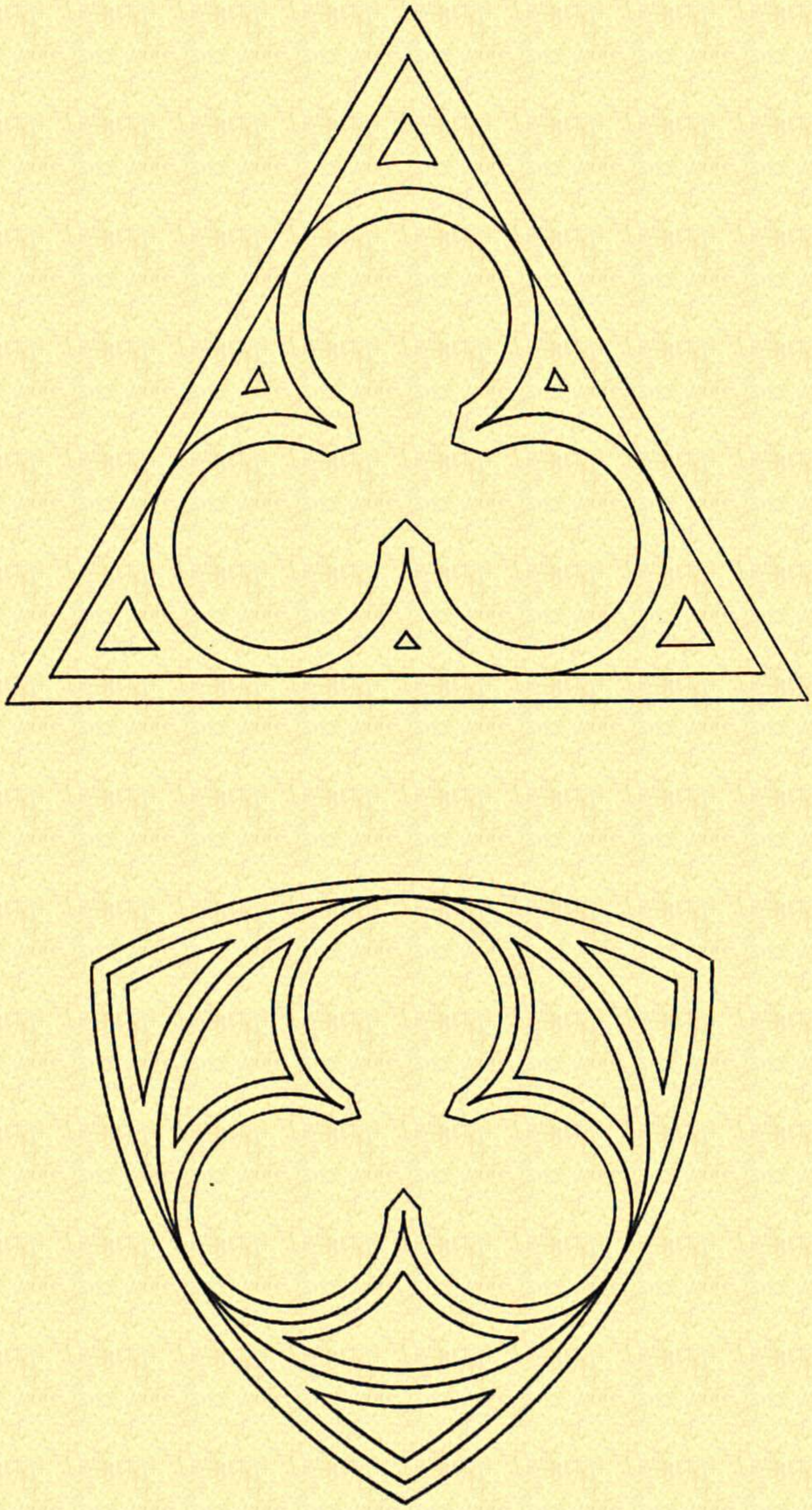
Outer circle 2½" radius
Band ⅕"

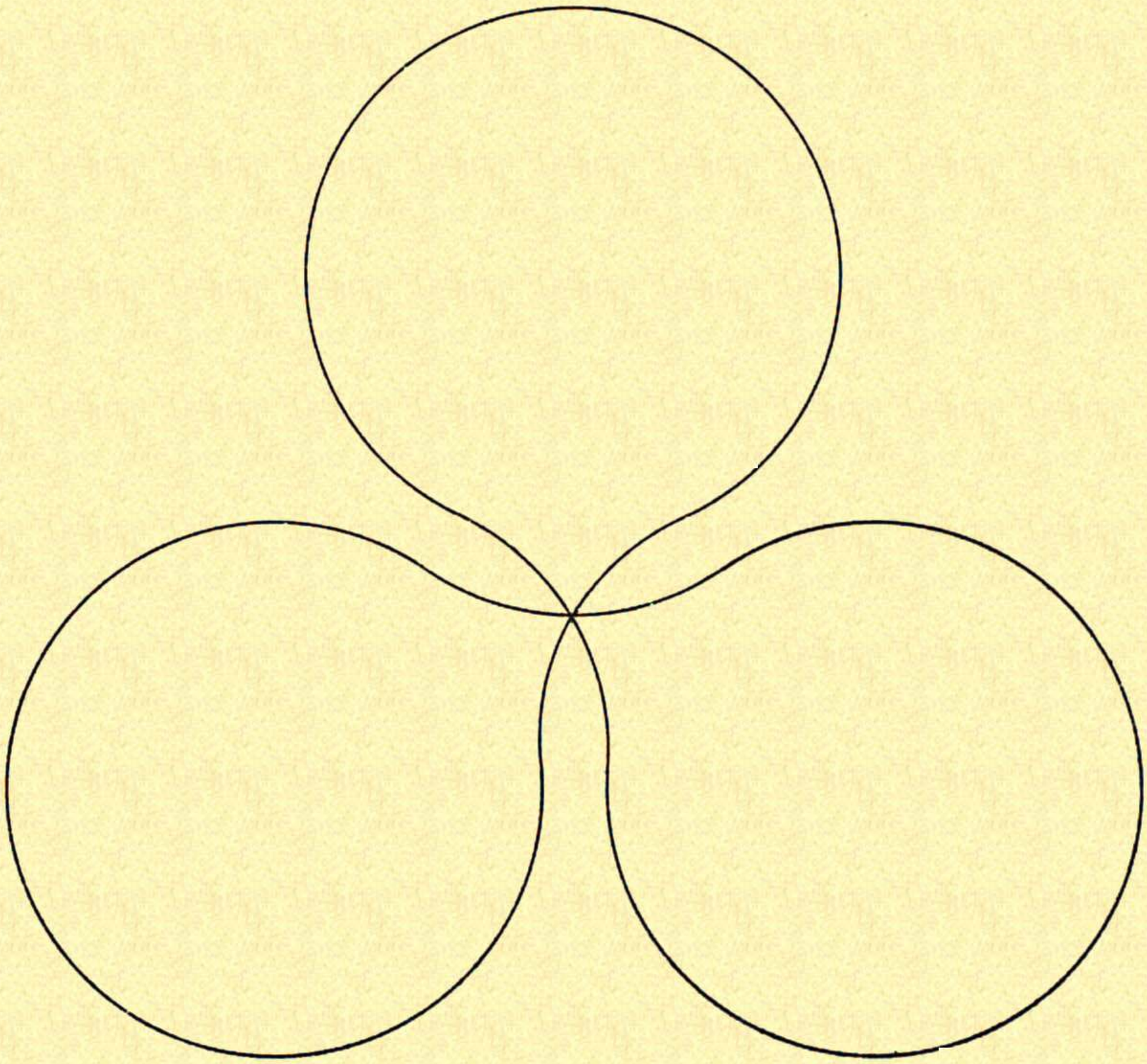


Radii 1", 1·2"

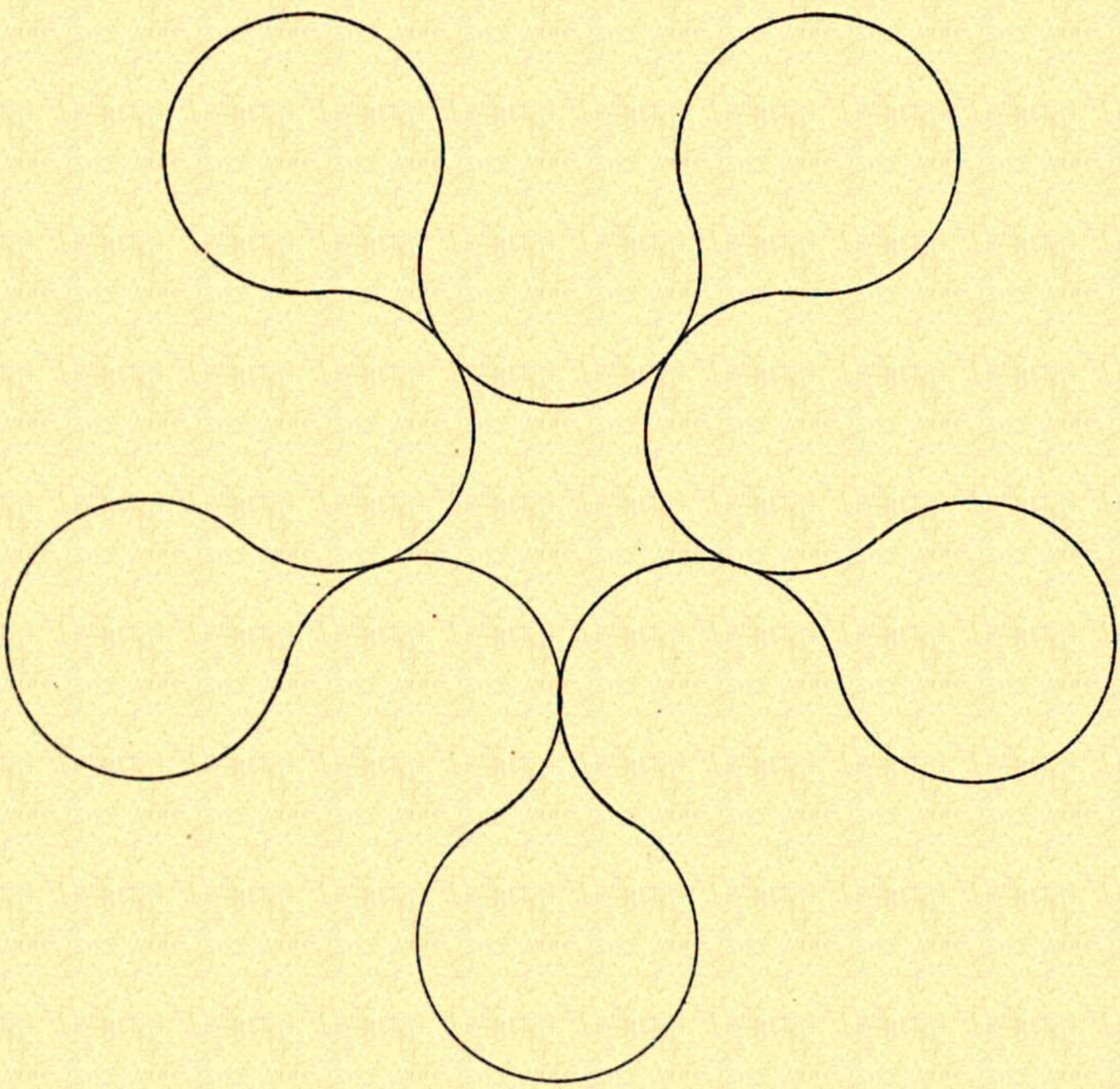


Radii 1," 1·2"

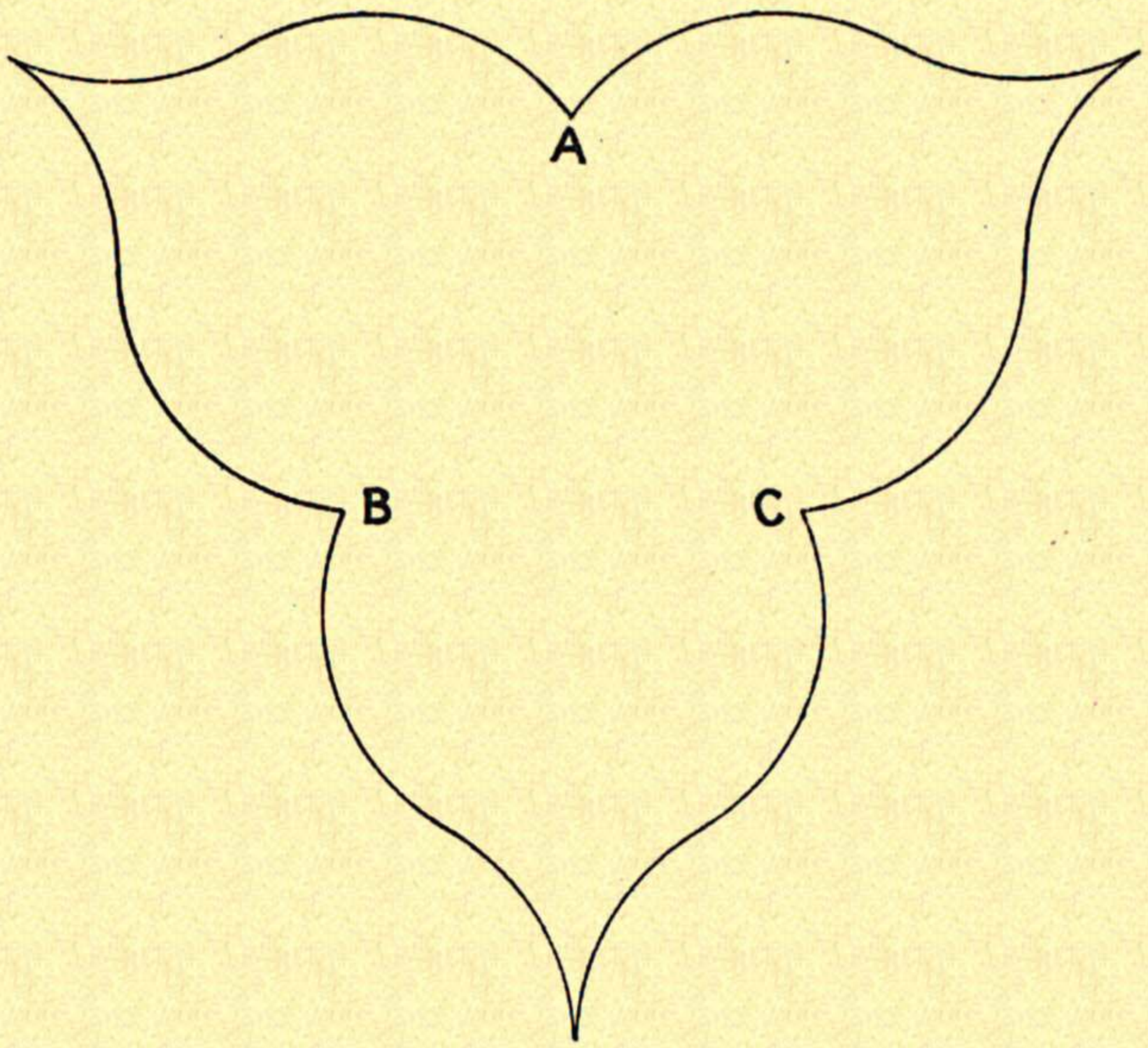




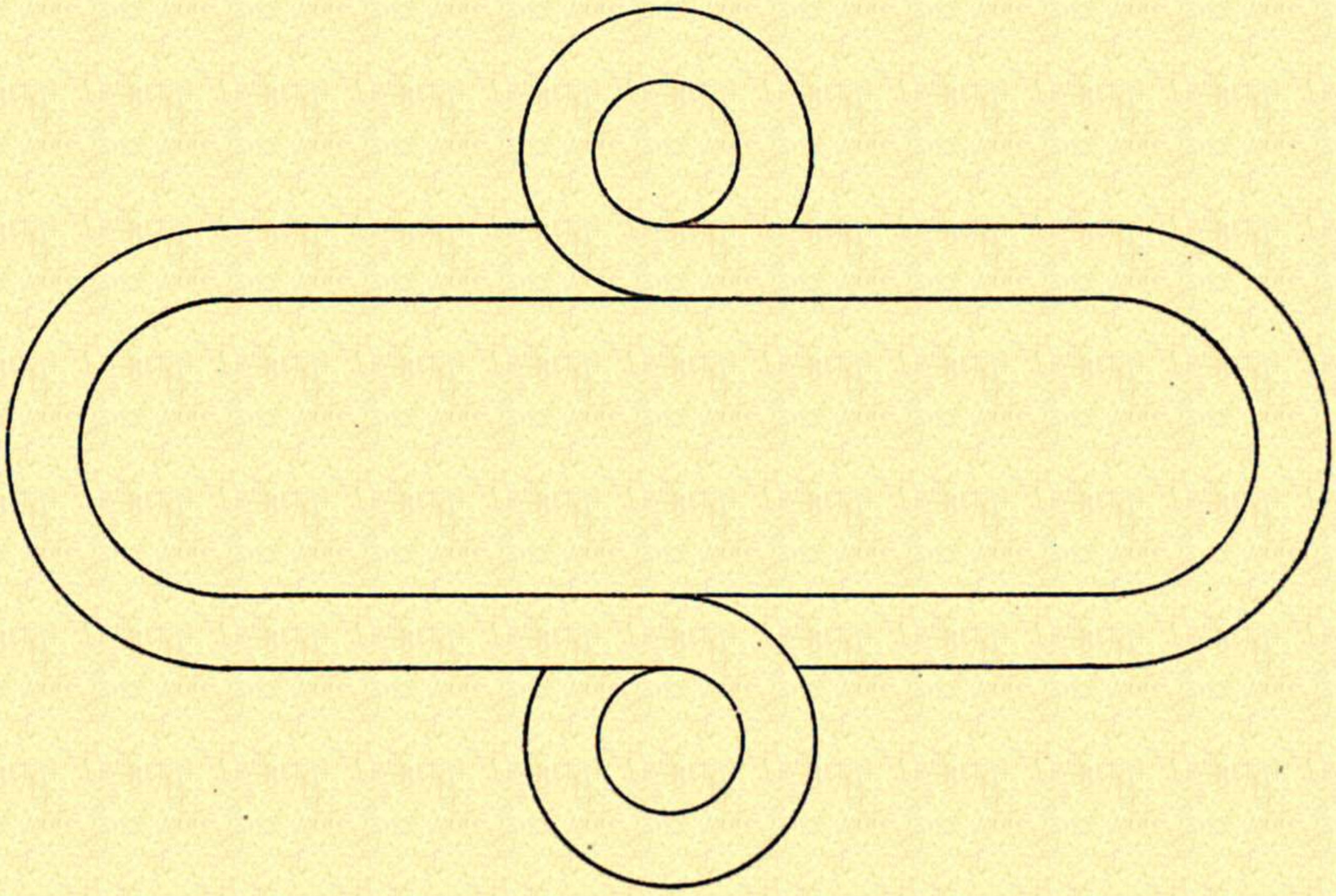
Radii of all arcs 1"

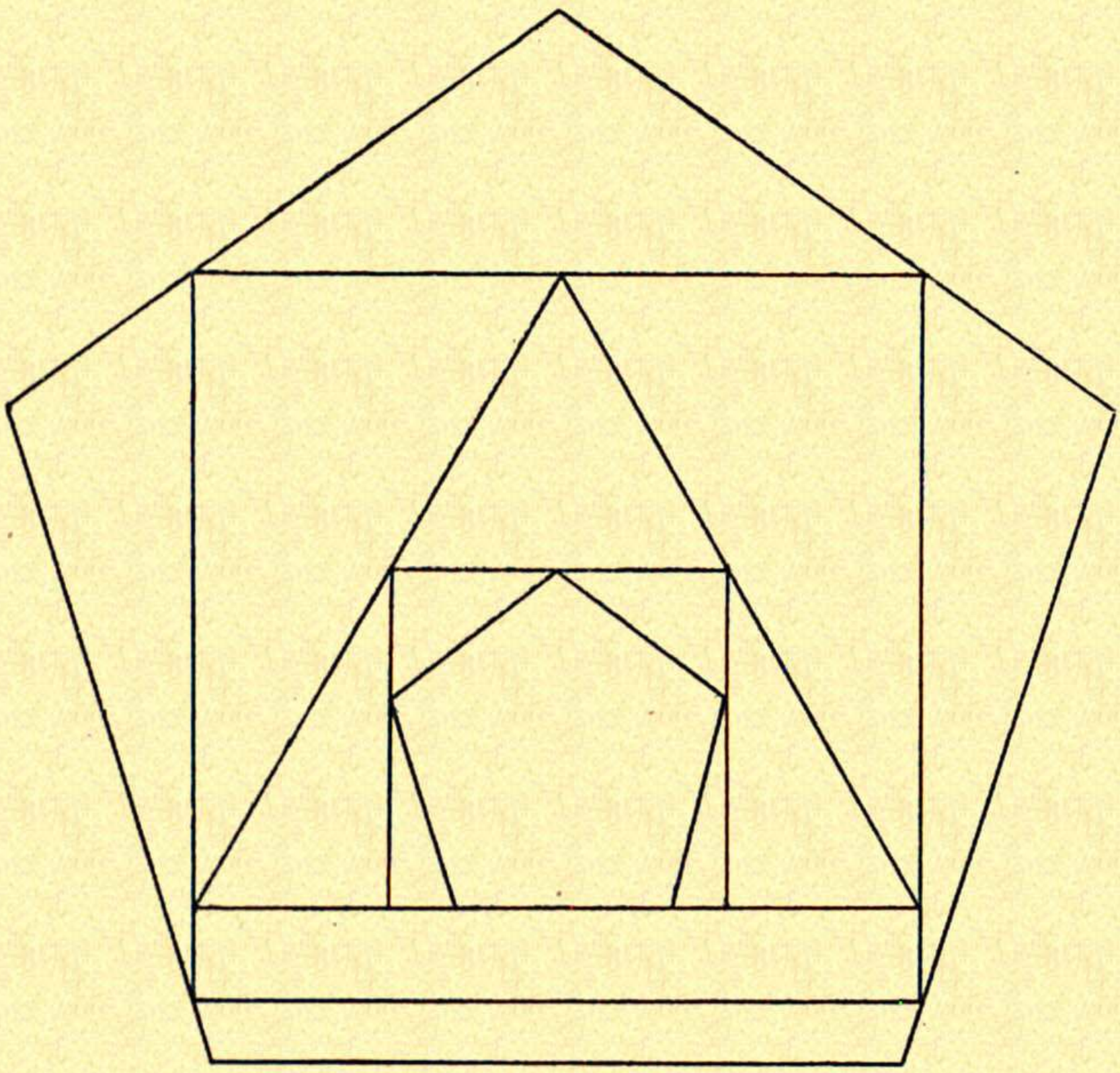


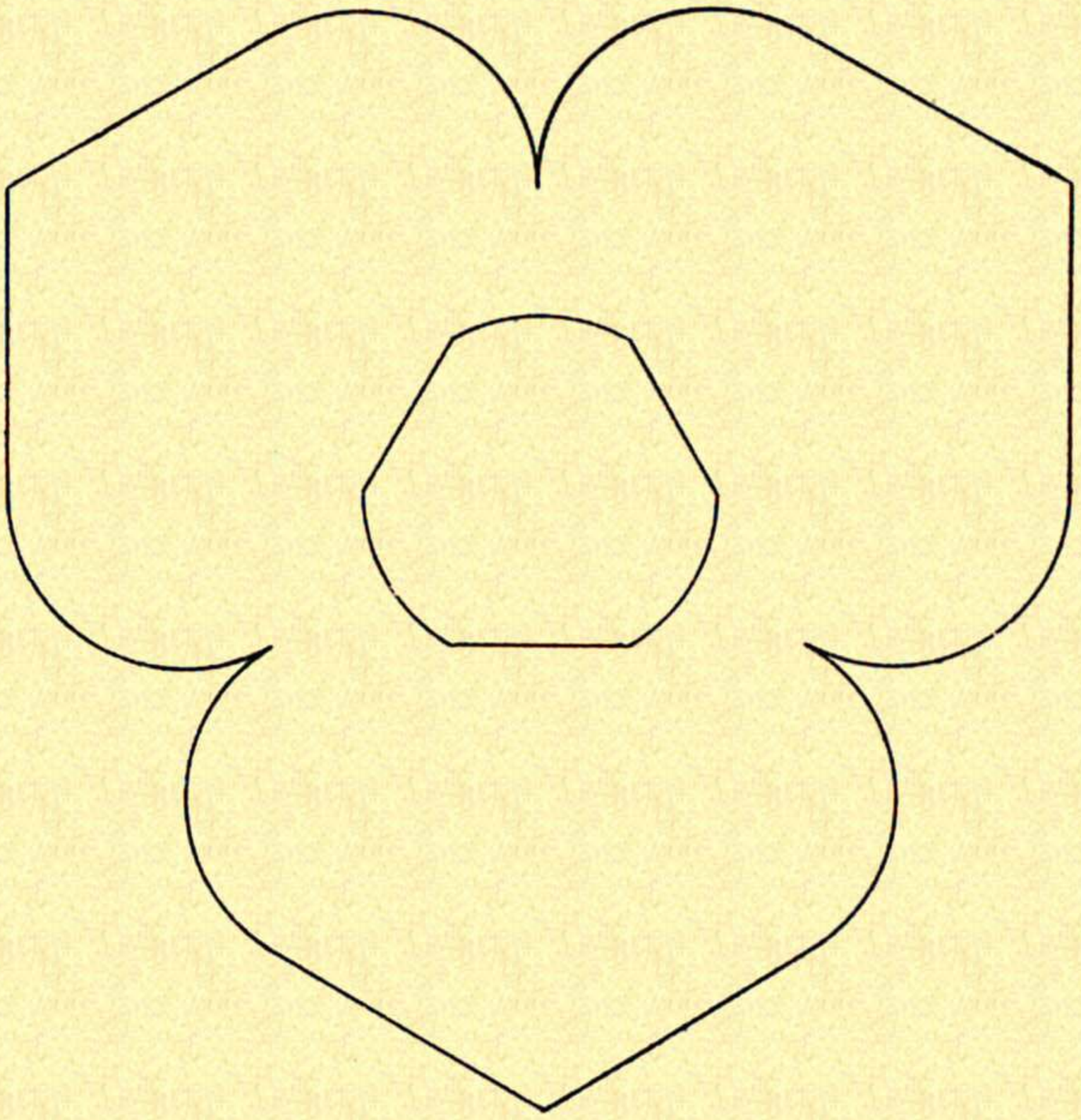
Radii of all arcs ½"



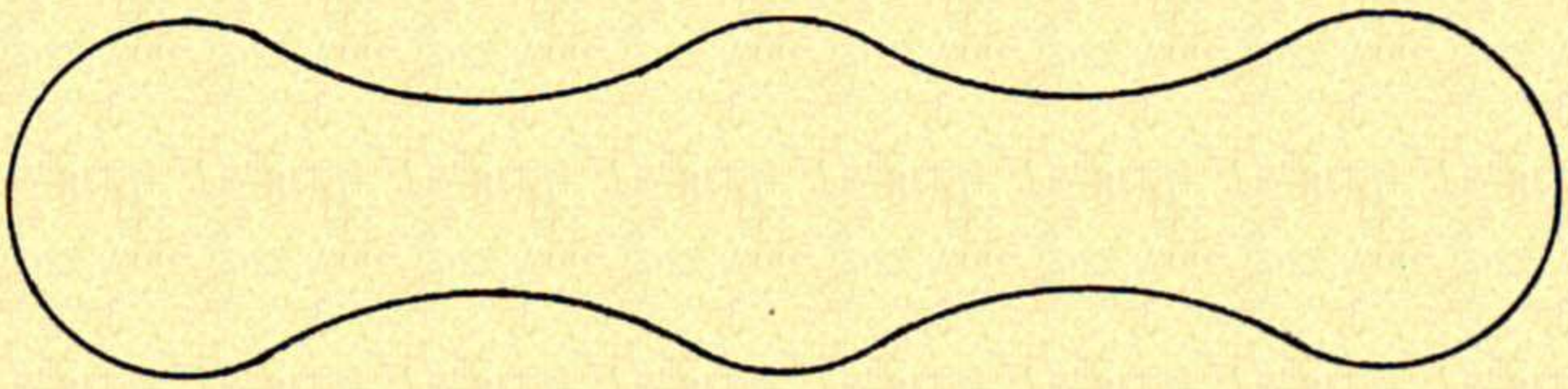
A B 1 3/8"
Radii of all arcs 3/4"

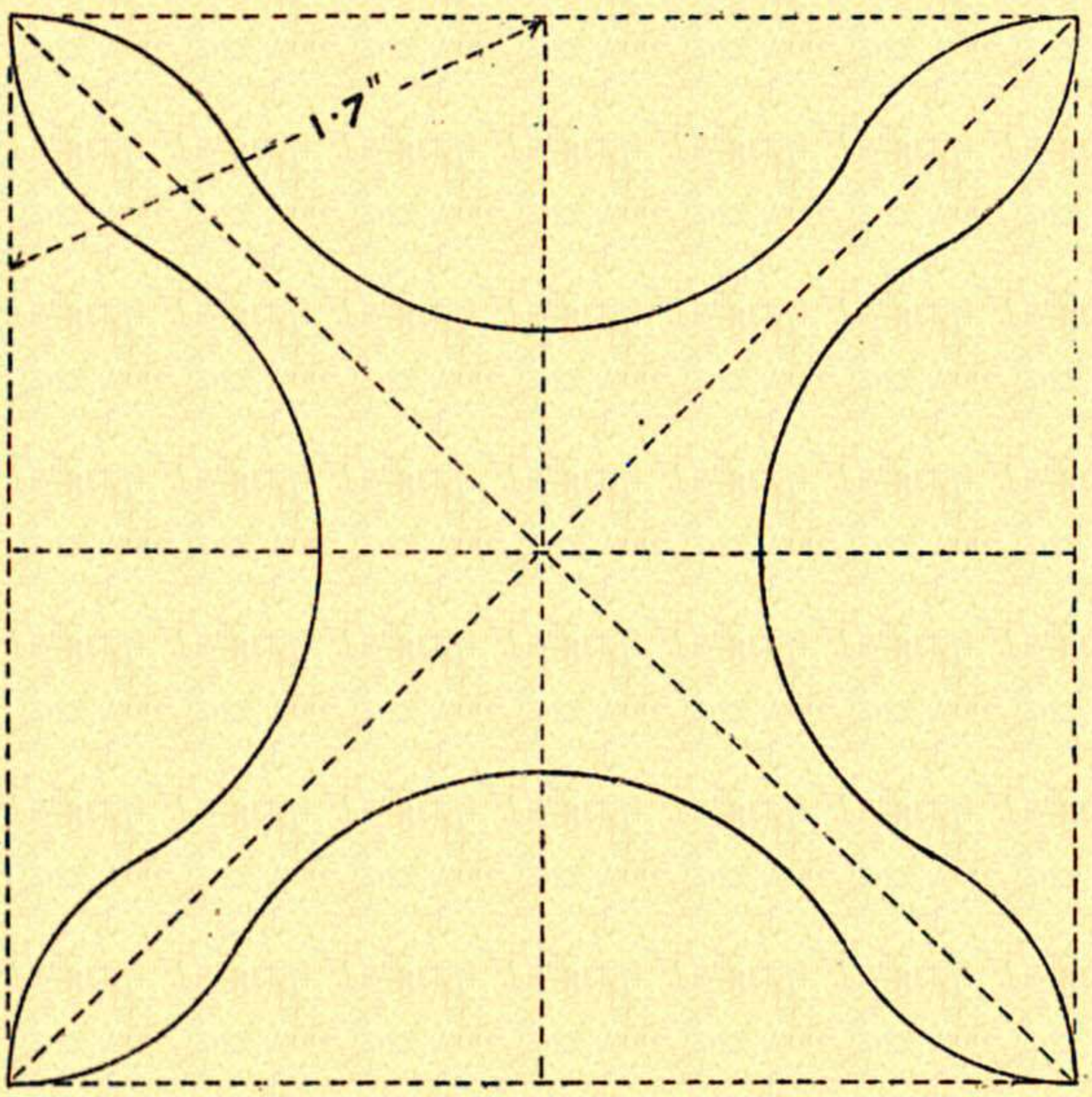




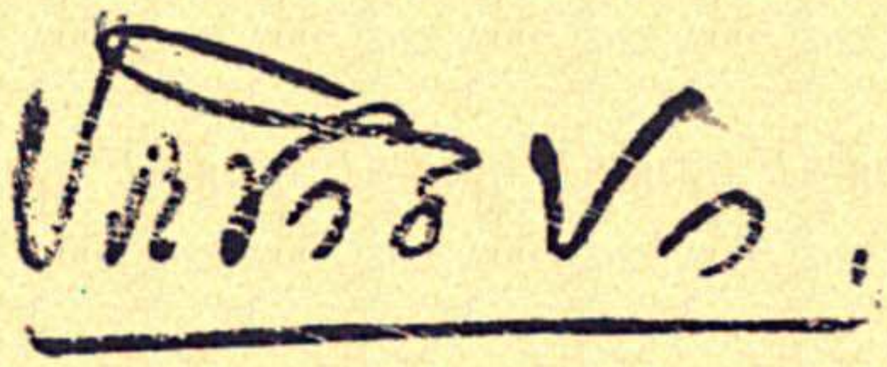


All radii · 6"



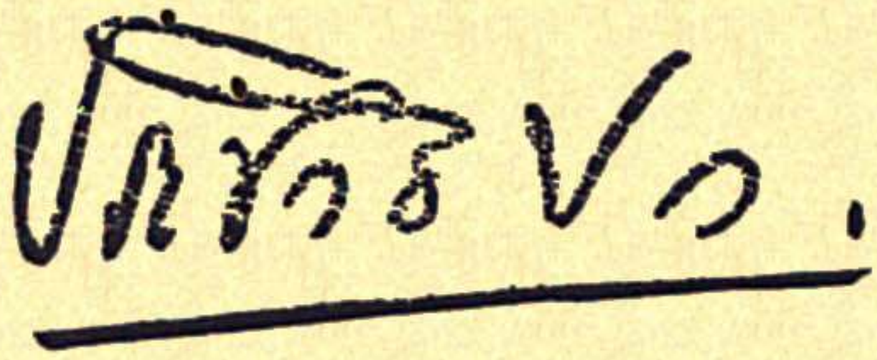


Smaller arcs intersect at right angles. Radii ·7" & 1".



PART I.

PLANE GEOMETRY.



TABLES.

LINEAR MEASURE.

English.

12 inches (″) =1 foot.
36″=3 feet (′)=1 yard.
63360″=1760 yards=1 mile.

$5\frac{1}{2}$ yards =1 rod, pole or perch.
40 poles =1 furlong.
8 furlongs=1 mile.

100 links =1 chain=22 yards.
10 chains=1 furlong.

Metric System.

1 Myriametre =10000 metres (m.).
1 Kilometre (Km.) =1000 m.
1 Hectometre (Hm.)=100 m.
1 Decametre (Dm.) =10 m.

1 decimetre (dm.) $=\frac{1}{10}$ m. =·1 m.
1 centimetre (cm.) $=\frac{1}{100}$ m. =·01 m.
1 millimetre (mm.) $=\frac{1}{1000}$ m.=·001 m.

Rough Equivalents.

8 Km. =5 miles.
1 m. =39·37″.
20 m. =1 chain.

SQUARE MEASURE.

English.

144 sq. inches = 1 sq. foot.
9 sq. feet = 1 sq. yard.
4840 sq. yards = 1 acre.
640 acres = 1 sq. mile.

$30\frac{1}{4}$ sq. yards = 1 sq. pole.
40 sq. poles = 1 rood.
4 roods = 1 acre.

---

10 sq. chains = 1 acre.

Metric System.

For land (*i.e.* fields, etc.) the unit is the are.

1 are (a.) = 1 sq. Dm. = 100 sq. m.
1 Hectare (Ha.) = 100 ares = 10000 sq. m.
1 centiare (ca.) = $\frac{1}{100}$ are = ·01 are = 1 sq. m.

Rough Equivalent.

1 Hectare = $2\frac{1}{2}$ acres.

RULES TO BE OBSERVED IN DRAWING SCALES.

1. Find the R.F. if it is not given. Simplify as much as possible. Reduce the numerator to unity.

2. Calculate the total length of the scale (in inches or centimetres *correct to hundredths*). If a distance is not specified, take the nearest convenient length to 6″.

3. Find the primary and secondary divisions by construction *below* the line; join only the end lines. Mark the divisions (in plain scales) by tiny uprights. The divisions should be multiples or submultiples of 10 if possible. For parallel lines in diagonal scales take $\frac{2}{25}$ or $\frac{2}{20}$ on the marquois scales.

4. Figure plain scales *above* the line, but diagonal scales *below*.

5. Ink in a single line only in plain scales, using Indian ink.

6. Write above the scale a description of it and its R.F.

Typical Scales.

1. On an Austrian map a distance of $2\frac{1}{2}$ Austrian miles is represented by 1·15″. Draw a scale of English miles for the map, showing 30 miles. Give R.F.

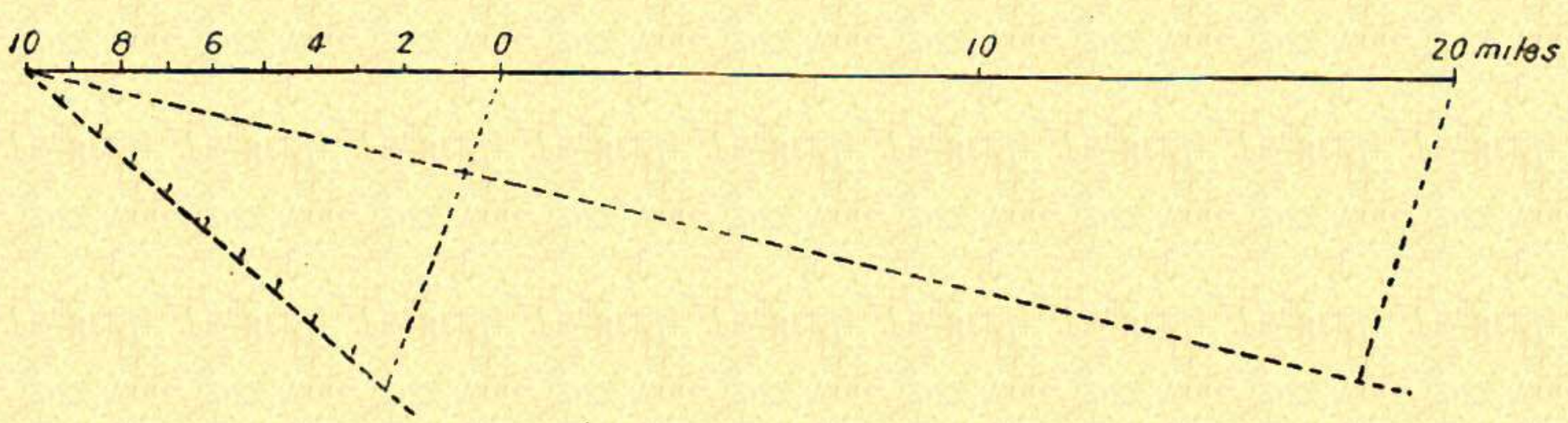
(1 Austrian mile=3·3312 English miles.)

$$\text{R.F.} = \frac{\overset{23}{\cancel{115}}}{\underset{50}{\cancel{250}} \times 3{\cdot}3312 \times 63360} = \frac{23}{10553241{\cdot}6} = \frac{1}{458837}.$$

$$\text{30 miles occupy } \frac{23 \times 30 \times \cancel{63360}}{50 \times 3{\cdot}3312 \times \cancel{63360}} = \frac{69}{16{\cdot}656} = 4{\cdot}14''.$$

Scale of miles.

$$\text{R.F.} = \frac{1}{458837}.$$



(Dotted lines should be in pencil.)

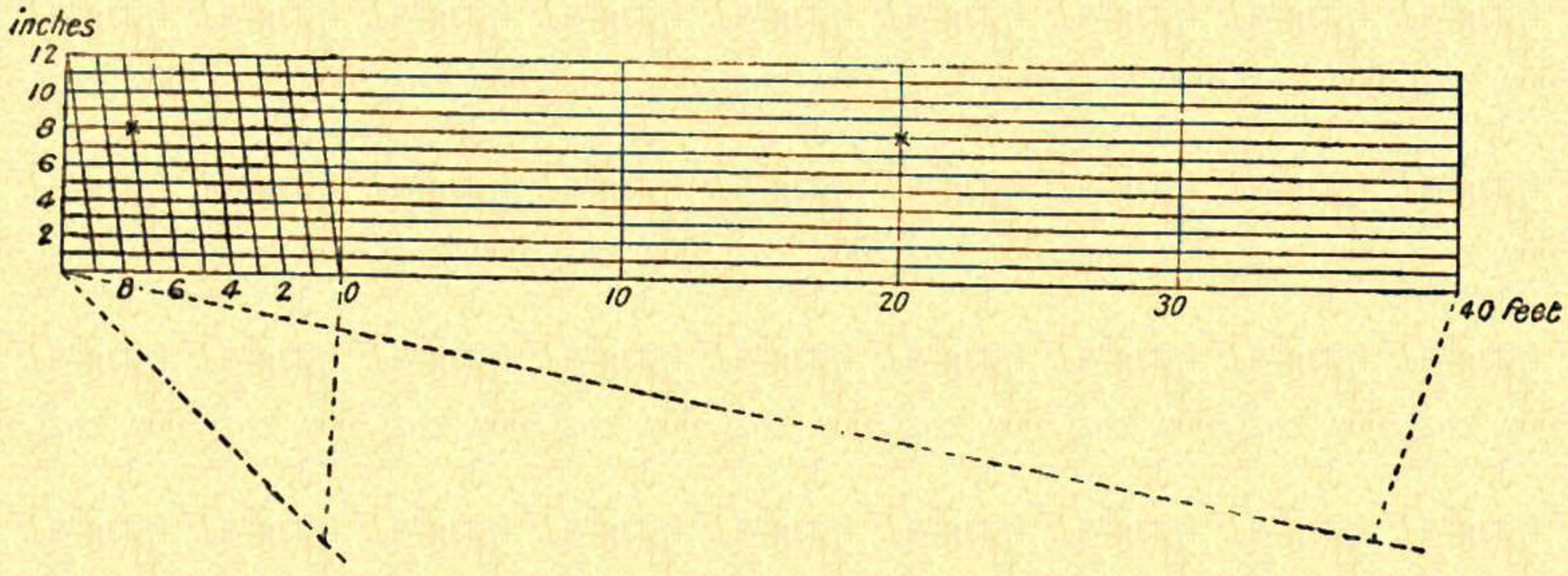
2. On a plan a distance of 37′ is shown by $4\frac{3}{4}''$; construct a diagonal scale representing 50′, by which single inches may be measured. Figure your scale properly, and write above it the R.F. Indicate, by two small marks on the scale, the points you would take in order to measure off a distance of 27′ 8″. Show all your calculations.

$$\text{R.F.} = \frac{19}{4 \times 37 \times 12} = \frac{19}{1776} = \frac{1}{93}.$$

$$50' \text{ occupy } \frac{19 \times \overset{25}{\cancel{50}} \times \overset{1}{\cancel{12}}}{\underset{2}{\cancel{4}} \times 37 \times \cancel{12}} = \frac{475}{74} = 6{\cdot}42''.$$

Scale of feet and inches.

$$\text{R.F.} = \frac{1}{93}.$$



(*Dotted lines should be in pencil.*)

GEOMETRICAL DRAWING
EXAMPLES.

N.B.—*The Patterns marked with an asterisk* (*) *are the easier.*

PLAIN SCALES.

Exercise I.

1. Draw a straight line 4·7″ long and divide it into seven equal parts. Divide the first part into three equal parts.

2. Draw a straight line 10·3 cm. long and divide it into six equal parts. Divide the first part again into five equal parts.

3. Construct a plain scale of yards with R.F. $\frac{1}{320}$. Show 50 yards.

4. Draw a plain scale of yards, R.F. $\frac{1}{225}$, to show 40 yards.

5. Draw a plain scale of yards, R.F. $\frac{1}{700}$, to show 100 yards and to read to 2 yards. With the same R.F. draw a scale of metres to read to 2 m.

6. Draw a plain scale of miles and furlongs with R.F. $\frac{1}{73400}$.

7. Draw a scale of miles and furlongs whose R.F. is $\frac{1}{47520}$.

8. Construct a scale of feet, 1 : 900, to show 400′, and divided to show 10′.

9. Construct a plain scale of miles and furlongs, 1 : 90000, and show 8 miles. Show by two marks 4 miles 5 furlongs.

10. Draw a scale of kilometres and hectometres, 1 : 40000. Show 5 Km.

11. Draw a scale of kilometres and hectometres, R.F. $\frac{1}{80000}$. Show 8 Km.

12. On a map 20 Km. occupy 1 cm. Find the R.F. and draw a scale of kilometres, showing 200 Km. and reading to distances of 5 Km.

13. Draw a scale of metres with R.F. $\frac{1}{900}$.

14. A plan is drawn on the scale $\frac{1}{8}''$ to a foot: draw a scale to suit the plan, showing 50′.

15. Make a scale of chains, $2\frac{3}{4}''$ to the mile, show 150 chains and divide it to show spaces of 5 chains.

16. On a map a distance of 3 miles 3 furlongs is represented by 2·97″. Construct a scale for the map showing 6 miles and divide it to show furlongs.

17. Construct a scale to read feet and inches, 7·5″ representing $6\frac{3}{4}'$.

18. Find the R.F. for a plain scale in which 319 yards is measured by 3·7″. Draw a scale to show 500 yards and to read to distances of 10 yards.

19. A distance of 11 miles 5 furlongs is represented by 3·72″. Draw a scale of miles.

20. Draw a scale of yards for a map, on which 39 yards occupy 4·29″.

21. On a French map 37 Km. occupy 9 cm. Calculate the R.F. and draw a scale of miles for the map. Show 40 miles.

22. On a plan 7 yards 2 feet occupies 3″. Find the R.F. and draw a scale of metres and decimetres for the plan.

23. On a French map a distance of 54 Km. was denoted by 12·87 cm. Construct a scale of miles for the map.

24. On a French map it is found that 3 Km. occupy 2 cm. Draw a scale of kilometres, showing 20.

25. On a certain map 13·2 Km. occupy 2·7 cm. Draw a scale of kilometres for the map.

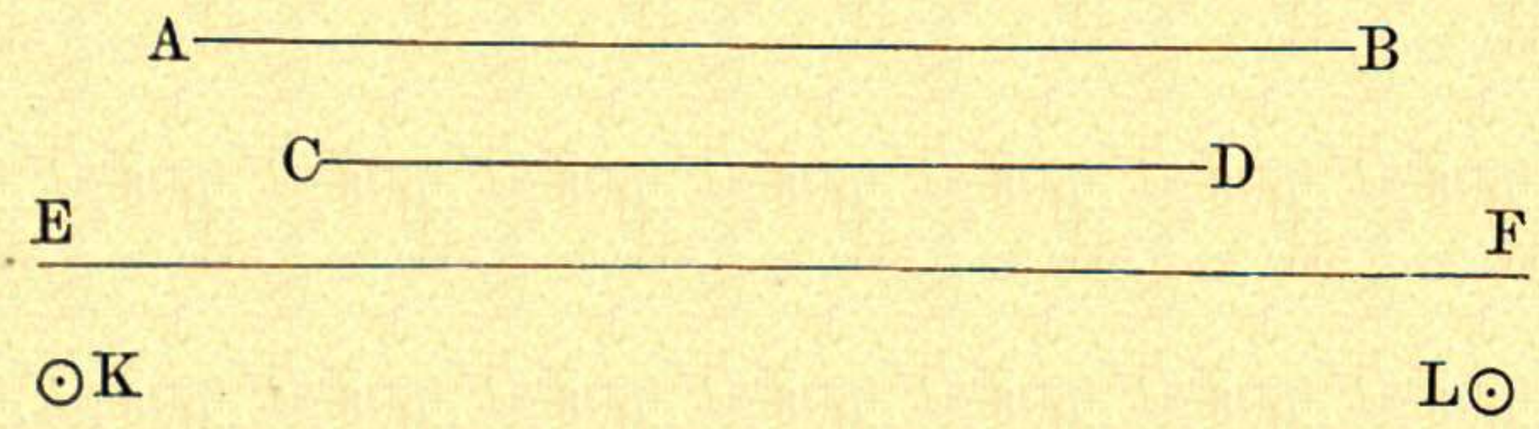
26. Draw a plain scale for a map on which 33 Km. are represented by 9·24 cm. Show 50 Km., and divide your scale so as to show single kilometres. Give the R.F.

27. On a map AB represents 24 miles. Calculate the R.F. and draw a scale of miles.

28. On a plan CD represents 18 yards 1 foot. Calculate the R.F. and draw a scale of yards.

29. On a map EF represents 3·2 Km. Draw a scale of Km. and Hm.

30. On a map K and L represent two towns 80 miles apart. Draw a scale of miles for the map to read to 5 miles.



31. Draw a plain scale of metres for a plan on which 57·9 m occupy 13·72 cm.

32. On a plan 5′ $3\frac{1}{2}$″ is denoted by 5·94″. Draw a plain scale of feet and inches for the plan.

33. Draw a scale of furlongs and chains for a map on which 5 furlongs 3 chains is represented by 4·9″.

34. Draw a scale in which a length of 8 chains is represented by 1″, to measure as a maximum 5 furlongs and as a minimum 20 yards.

35. Draw a scale of feet and inches for a map on which 1″ represents $\frac{3}{8}$ of a yard.

36. Draw a scale of yards, 10 yards to $1\frac{2}{3}$″, to show 30 yards.

37. A military sketch has been drawn to a scale of 1000 paces to the inch. Draw a plain scale for the sketch to measure distances between 100 and 5000 yards. Give the calculation and R.F., assuming the pace to measure 30″.

38. A distance of 3750 paces is represented on a map by 9·2″. Draw a scale of yards for the map showing 2000 yards, and divided to show distances of 50 yards. Show all your calculations, figure your scale properly, and write above it its R.F. (A pace= 32″.)

39. For a map, on which 145 yards is represented by an inch, make a scale of furlongs and chains.

40. A distance of $1\frac{3}{4}$ miles is represented on a map by $2\frac{1}{4}$″. Draw a scale of yards for the map showing 8000 yards, and divided to show distances of 200 yards. Show your calculations, figure the scale properly, and write above it its R.F.

41. The distance between two forts is known to be 840 yards, and measures on a map $3\frac{1}{2}$″. Draw a scale of paces for the map, showing 1500 paces, and divided to show distances of 50 paces. Show clearly all necessary calculations, figure your scale properly, and write above it its R.F. (A pace=32″.)

42. The distance from London to Portsmouth is known to be 74 miles, and measures on a large map 3′ 4″. Construct a scale to read miles and furlongs, showing 10 miles; calculate and write down the R.F.

43. The distance between two forts is known to be 1350 yards, and measures on a map 2·5″; draw a plain scale of yards for the map showing 3000 yards, and divide it to show distances of 50 yards. Show all your calculations, figure the scale properly, and write above it its R.F.

44. A sketch has been made on a scale of 1000 paces to an inch. Draw a plain scale to measure distances from 100 to 3000 yards, corresponding to scale of the sketch. (A pace=30″.)

SQUARE MEASURE DATA.

Exercise II.

1. A portion of a survey measures 28″ by 7″ and represents 100 sq. miles of country. Calculate the R.F. and construct a scale (to show miles and furlongs) on which the survey was made.

2. On a garden plan 1 acre occupies 360 sq. inches. Obtain the R.F. Draw the scale, showing 50′ and single feet.

3. An area of 1 sq. mile is represented on a map by a portion 9″ long and 4″ wide. Draw a scale of yards to show 1500 and to read to 50.

4. On a plan of a field 10 acres occupy 9 sq. inches. Find the R.F. Draw a scale of chains for the plan. Show 20 chains.

5. On a plan of some country a rectangular field, whose area is known to be 2 acres, measures 5″ by 4″. Draw a scale of yards for the plan (to read to distances of 2 yards).

6. Draw a scale of yards for a map on which 160 acres occupy 9 sq. inches.

7. Draw a suitable scale for a map 10″ long and 4·5″ wide on which 5 sq. miles of country are shown.

8. On a plan of an estate 5 acres occupy $40\frac{1}{2}$ sq. inches: determine the R.F. of the plan. Draw a suitable scale.

9. Draw a scale of chains for a map on which a rectangular field of 12 acres measures 4″ by 3·5″.

10. A map 12″ long by 14″ wide, and representing an area of 40 acres has no scale. Draw a scale to suit the map and give its R.F.

11. The size of the plan of a field containing 20 acres is 9″ by 3·7″. Draw a plain scale of feet for the plan.

12. On a map a large field of 110 acres measures 3″ by 2·5″. Find the R.F. of the map and draw a suitable scale.

13. A map of a French estate of 250 hectares is 40 cm. long and 16 cm. wide. What is the R.F.? Draw a scale of metres, showing 1000 m., and reading to distances of 10 m., for the map.

14. On a map of an estate 500 hectares covers an area of 50 cm. by 10 cm. What is the R.F.? Draw a scale of metres showing 1500, and reading to distances of 30 m.

15. The area of a certain French farm is 40 hectares and on a plan is represented by 1000 sq. cm. What is the R.F.? Draw a scale of Decametres for the plan showing 30.

16. A map of a French estate of 1406 hectares 25 ares measures 1·62 m. by 32 cm. Find the R.F. and draw a suitable scale.

COMPARATIVE SCALES.

Exercise III.

1. Draw a plain scale with R.F. $\frac{1}{135710}$ to measure Spanish leagues up to 40. (1 Spanish league=2·63 English miles.)

2. Two points are 13 Km. apart and on the drawing the distance is 3·25″. Find the R.F. of the scale if 1 metre=1·093 yards—and draw a scale of miles for the map, up to 15.

3. On a French map a distance of 30 Km. is represented by 2″, draw a scale showing 100 Km.: also draw a comparative scale of English miles. (8 Km.=5 miles).

4. Draw a scale of versts (1 verst=693 yards), the R.F. being $\frac{1}{100000}$. Show 25.

5. On a map the distance between two points (known to be 21 Km. apart) measures 6·5″. Draw a scale of English miles suitable for the map. (1 Km.=1093 yards).

6. Infantry marches at the rate of $2\frac{3}{4}$ miles an hour. Draw a scale of time for a map whose R.F. is $\frac{1}{31680}$, to measure intervals of ten minutes.

7. A horse canters 300 yards a minute. Construct a scale of time for a map whose R.F. is $\frac{1}{20000}$. Show ten minutes and divide it to show intervals of ten seconds.

8. Draw a time scale for a cyclist who rides 16 miles an hour, showing intervals of ten minutes. His map is on the scale 1 : 800000.

9. The distance between two places on a Russian map is $7\frac{1}{2}$ versts. The actual distance on the map is 5·73″. Construct a scale of English yards for the map (to read to distances of 100 yards). (1 verst=·663 mile).

10. Draw a scale of paces for a map whose R.F. is $\frac{1}{4000}$. A pace=32″. The scale is to read to 10 paces.

11. Draw a plain scale of Spanish leagues comparative to an English scale 20 miles to the inch. (1 Spanish league=2·63 English miles).

12. Draw a scale of "Hours of March" when rate is 14 miles in 5 hours for a map of which the R.F. is $\frac{1}{2000000}$.

13. The distance between two places on a Russian map scales $3\frac{3}{4}$ versts. The actual distance on the map is 2·85″. Construct a scale of miles and furlongs for the map. (1 verst=3500′.)

14. Assuming that the length of your pace is 33″ and that you have to make a sketch map of a position on the scale 12″ to the mile: construct a comparative scale of paces (to read to 10 paces) for your sketch.

15. A map has to be verified. Several distances on it are measured and calculated on the assumption that the scale is 6″ to the mile, but it is found that the true distance is in each case $\frac{1}{10}$ less than the calculation makes it. Construct a correct scale of yards (to read to distances of 25 yards) for the map.

16. In Cape Colony

12 Cape feet=1 Cape rood

and 600 sq. roods=1 morgen=2·116 English acres.

With R.F. $\frac{1}{120}$ draw a scale of Cape roods to show Cape feet. (N.B.—A Cape rood is a length and not an area.)

17. A distance of 20 Flemish miles is represented on a Flemish map by $8\frac{1}{4}$ English inches. Draw a plain scale of English miles for the map showing single miles up to 50. Show all necessary calculations, figure your scale properly and write above it its R.F. (1 Flemish mile=6869 English yards.)

18. On a weighing machine the distance between zero point and the 1 cwt. division was 5·6″. Draw a scale for the machine by which weights, increasing by 2 lbs., can be weighed up to 1 cwt.

19. If 96 Prussian feet are equal to 33 English yards, draw a scale of Prussian feet to correspond to a plan, on which 50 English yards are represented by 10″. Give the R.F.

20. An English map of the South Eastern Counties has a scale of miles on it with R.F. $\frac{1}{200000}$. Make for it a corresponding scale of versts for use in the Russian War Office. (1 verst= 1160 yards.)

21. On a map a distance of 27 French leagues is represented by 3·75″. Construct a scale of English miles for the map. Give the R.F. (1 French league=4262·84 English yards)

22. Draw a scale of versts (1 verst=3500′), the R.F. being $\frac{1}{100000}$.

23. Define a "comparative scale." Draw a scale of Milan miles comparative to a scale of 8 miles to an inch. (1 Milan mile=1808·81 English yards.)

24. A distance of 37 English miles is represented on a map by 4·3″. Draw a scale of Irish miles for the map showing 40 Irish miles. (1 Irish mile=2200 yards.)

25. A length of 41 yards is represented on an English plan by 1·6″. Draw a scale of Russian sachines for the plan, showing single sachines up to 50. Show all necessary calculations, figure your scale properly, and write above it its R.F. (1 sachine=7 English feet.)

26. Draw a scale of Milan miles comparative to a scale of 10 English miles to an inch. (1 Milan mile=1808·81 English yards.) Show 40 Milan miles. Give your calculations neatly

and clearly and the R.F. of the scale. Draw a line measuring 27 Milan miles.

27. A distance of 154 geographical miles is represented on a map by 3·37″. Construct a scale of English statute miles for the map showing 250 statute miles, and by which distances of 10 statute miles may be measured. (60 geographical miles=69½ statute miles.)

28. On a Spanish map a distance of three Spanish leagues is represented by 1″. Construct a scale of English miles for the map, showing 50 miles. Show clearly all necessary calculations, figure your scale properly, and write above it its R.F. (A Spanish league=4565½ English yards.)

29. On a French plan a length of 12 m. is represented by 3·25″. Draw comparative scales of metres (showing 20), and English feet (showing 60). Give the R.F. 1 m.=39″.

30. The R.F. of a scale is $\frac{1}{50000}$. Construct it so as to show 8000 yards. Make also a comparative scale of kilometres and hectometres.

PRACTICE IN THE USE OF MARQUOIS SCALES.

Exercise IV.

1. Draw a series of 11 parallel lines at equal distances apart so that the total width is 1″.

2. Draw a series of seven parallel lines, each at least 5″ long, at equal distances apart so that the total width may be 1″.

3. Draw a series of ten parallel lines, each about 5″ long, and at equal distances apart so that the total width is 1″.

4. Draw five parallel lines, each 5·3″ long, and ·2″ apart.

5. Draw a square with sides 3″, and divide it into 144 equal squares.

6. Draw an equilateral triangle with sides 2″, and divide it up into 25 equal equilateral triangles by lines parallel to its sides.

7. Draw a rhombus with sides 2·5″ and one angle 50°, and divide it into 100 equal rhombi by straight lines parallel to its sides.

8. Divide a rectangle whose sides are 4″ and 3″ into 100 equal and similar rectangles.

9. Draw a rectangle 5″ by 2¼″ and divide it up into ¼″ squares.

10. Draw a parallelogram with sides 4″ and 3″, and included angle 55°. Divide it up into 100 similar and equal parallelograms.

11. Draw a series of parallel lines $\frac{3}{7}$″ apart, and cross them at right angles by another series of parallel lines $\frac{4}{9}$″ apart. Make the whole figure 3″ by 4″.

12. Draw 12 parallel lines $\frac{2}{9}$″ apart, and cross them at an angle of 60° by a series of 12 parallel lines $\frac{4}{2\,5}$″ apart.

13. Draw a square with sides 3·7″ and inside it describe five other squares, similarly situated, with their sides $\frac{7}{2\,0}$, $\frac{7}{3\,0}$, $\frac{7}{4\,0}$, $\frac{7}{5\,0}$, $\frac{7}{6\,0}$″ apart respectively. What is the length of the diagonal of the smallest square?

14. Describe an equilateral triangle with sides 3″ long. In it inscribe three others similarly situated with their sides $\frac{6}{2\,5}$″ apart, and outside three more with their sides $\frac{6}{3\,5}$″ apart.

[Patterns on pp. 1*, 2*, 3, 4*, 5, 7*, 35.]

DIAGONAL SCALES.

Exercise V.

1. Draw a scale of inches to show hundredths of an inch.

2. Draw a scale of miles and furlongs, to show chains diagonally. R.F. $\frac{1}{6\,3\,3\,6\,0}$.

3. Construct a scale to measure yards, feet and inches. R.F. $\frac{1}{4\,0}$. Show 5 yards.

4. Draw a diagonal scale of metres. R.F. $\frac{1}{3\,2\,0\,0}$.

5. Draw a diagonal scale of metres (to read to 20 m.). R.F. $\frac{1}{6\,3\,3\,6\,0}$.

6. Draw a diagonal scale of feet and inches, R.F. $\frac{1}{5\,0}$, to show 40′. Mark on the scale a distance 23′ 7″.

7. Construct a scale of furlongs, chains and poles. R.F. $\frac{1}{5\,5\,0\,0}$.

8. Draw a scale of yards. R.F. $\frac{1}{3\,0\,0\,0}$.

9. For a Norwegian map draw a diagonal scale of English miles. R.F. $\frac{1}{5\,0\,0\,0\,0\,0\,0}$. Also a corresponding scale of Norwegian miles. (1 Norwegian mile=7 English miles.)

10. Make a scale of French metres, $\frac{1}{2\,5\,0}$, showing decimetres diagonally.

11. A Russian map has on it a scale of versts, $\frac{1}{3\,0\,0\,0\,0}$. Affix a corresponding diagonal scale of Kilometres, divided to show decametres.

12. Given the R.F. $\frac{1}{40000}$, construct a diagonal scale of miles, furlongs and chains. Mark with two dots on the scale a distance of 3 miles 6 furlongs 8 chains.

13. Draw a diagonal scale, 1 : 11000, to show furlongs and poles. Show 1 mile.

14. The R.F. of a map is $\frac{1}{3500}$. Draw a scale of single yards.

15. Draw a scale of yards, feet and inches. R.F. $\frac{1}{120}$. Show a distance of 7 yards 1 foot 3 inches on the scale by two small crosses.

16. Construct a diagonal scale of cables and fathoms. R.F. $\frac{1}{12000}$. (1 nautical mile=10 cables=1000 fathoms=6086 feet.)

17. A plan of a house is drawn in such a way that the side of a room 19′ 8″ long is represented by 3·54″. Draw a scale for the plan, showing 30′, and divide it to show inches diagonally.

18. Draw a scale showing 40 miles, and furlongs diagonally, to suit a map on which a distance of 16 miles is represented by 2·4″. Give the R.F. and show all calculations.

19. Construct a diagonal scale to measure yards, feet and inches; $3\frac{1}{2}$ yards being represented by 2·4″. Show 8 yards.

20. Draw a scale showing 5 miles, furlongs, and chains diagonally to suit a map on which a distance of 2 miles 5 furlongs is represented by 2·8″.

21. On a certain map 8 Km. occupy 4·8 cm. Construct a scale of miles for the map showing furlongs diagonally.

22. A distance of 3·7 miles measures on a map 2·22″. Draw a scale of miles and furlongs. Mark off 3 miles 7 furlongs on it.

23. The distance between two places is $27\frac{1}{2}$ miles and measures 2·75″ on the map. Construct a diagonal scale of miles and furlongs showing 50 miles. Mark off a line 23 miles 5 furlongs.

24. Suppose a line, 11″ long, denotes on a map the distance between two towns, which are 85 miles apart, lay down a scale of miles up to 40, showing furlongs diagonally. Set off a distance 22 miles 5 furlongs.

25. Suppose a line 11″ long on a map denotes a distance 73 miles. Lay down a scale of miles, up to 40, showing furlongs diagonally. Mark, with small crosses, a distance 19 miles 3 furlongs.

26. On a plan a length of 3·4″ represents 76·5 yards. Make a diagonal scale of chains and links, showing 6 chains. By two small marks on the scale show a distance 3 chains 74 links.

27. On a map a distance of 9 miles 3 furlongs is represented by $3\frac{3}{4}''$. Draw a scale of miles for the map showing furlongs diagonally. Show 15 miles. Mark a distance 7 miles 5 furlongs on the scale.

28. On the scale of a certain French map 1 dm. represents 735 m. Construct a scale of metres for the map and indicate by two crosses a length of 593 m.

29. Draw a diagonal scale of miles, furlongs and chains (to read 5 miles) for a map in which 7 furlongs occupy 1·1". From it draw a line to represent 3 miles 2 furlongs 4 chains.

30. On a map 5 miles 5 furlongs occupy 1·5". Draw a diagonal scale of miles and furlongs to suit the map. Show 20 miles.

31. Draw a diagonal scale, showing yards, feet and inches, for a map on which 2 yards 1 foot is represented by 1·75". Show 8 yards.

32. Construct a diagonal scale of feet and inches having a R.F. of $\frac{1}{84}$. Show 50′. Show all necessary calculations, figure your scale properly, and show by two small marks on it the points you would take in order to mark off a length of 17′ 8".

33. Draw a scale $\frac{1}{47520}$ to measure miles, furlongs, and (diagonally) chains, 4 miles being the longest distance shown. From the scale draw a line 3 miles 5 furlongs 7 chains long. Give the calculations.

34. Draw a scale to show 5 furlongs, and poles diagonally, for a map whose R.F. is $\frac{1}{7200}$.

35. Draw a diagonal scale of $\frac{1}{1050}$ to measure feet.

36. Construct a diagonal scale of $\frac{1}{792}$ to show single yards. Let your scale represent 150 yards, figure it properly, and show by two small marks on the scale the points you would choose in order to take off a length of 67 yards.

37. Construct a diagonal scale of $\frac{1}{660}$ to show chains and single feet.

38. Draw a scale $\frac{1}{47250}$ to measure miles, furlongs, and chains (diagonally). From the scale draw a line 3 miles 5 furlongs 7 chains long.

39. Construct a diagonal scale of $\frac{1}{88}$ to measure feet and inches. Show 40′. Figure the scale properly and show your calculations. Show, by two small marks on the scale, the points you would choose in order to take off a distance of 17′ 8".

40. Given the R.F.$=\frac{1}{4000}$. Draw a diagonal scale of chains to read to 10 links and showing 30 chains; show, by two small marks on the scale, the points you would take in order to measure off 23 chains 70 links.

41. Construct a scale of miles having a R.F. of $\frac{1}{633600}$, and showing furlongs by the diagonal method. The scale is to be properly figured, and all necessary calculations shown. Show, by two small marks on the scale, the points you would take in order to measure off a length of 27 miles 5 furlongs. Make your scale long enough to measure 50 miles.

42. Construct a scale of geographical miles to measure distances from 2 to 800 miles, if 115 statute miles are represented by ·8″. (60 geographical miles = 69½ statute miles.)

43. The distance between two points is known to be 440 yards, and measures on an English plan exactly 4″. Draw a diagonal scale of metres for the plan. Show all the calculations, and write above it its R.F.

44. On a certain map 1′ represents 1000 miles. Draw a diagonal scale for the map showing single miles. By two crosses on your scale show a distance of 234 miles.

45. On a scale of a certain Italian map 1 cm. represents 78·5 m. Construct a scale of metres for the map. From it draw a straight line which would represent 739 m.

46. A map is constructed on the scale 4″ to 3 miles. Draw a diagonal scale to show miles, furlongs and chains.

47. Draw a plain scale to show metres and decimetres; 5·8 m. being represented by 7 cm. Draw also a diagonal scale comparative to the above to show yards, feet and inches.

48. The distance from London to Oxford (57 miles) occupies on a map 3·23″. Draw a scale to measure single miles up to 100 and place on it a diagonal scale to read furlongs. Set off a length 38 miles 5 furlongs.

49. On a map 2 miles 5 furlongs occupy ·91″. Draw a diagonal scale for the map, showing miles and furlongs.

50. On a map 10 miles 5 furlongs is represented by 11″. Draw a scale of miles, furlongs and chains. Using it, rule a straight line to measure 3 miles 2 furlongs 1 chain.

51. On a map 26 miles 3 furlongs are represented by 6·33″. Draw a scale of miles and furlongs for the map.

52. Draw a scale of yards, feet and inches, $\frac{7}{8}$″ representing 1 yard. Show a line 4 yards 1 foot 7 inches long.

53. On a plan 3 chains 62½ links are represented by 4″. Draw a scale of chains and links.

54. Construct a scale of 1 in 600 to show chains and single feet.

55. Make a diagonal scale of 1 in 540 to show 300′.

56. Draw a scale of metres (to read to centimetres) suitable for the plan of a house in which 6·42 m. are represented by 9·3 cm.

57. On a map AB represents 25 miles. What is the R.F.? Draw a scale of miles and furlongs for the map.

58. On a map C and D represent two towns 32 Km. apart. Draw a scale of Km. and Hm. for the map.

59. On a plan E to F represents a distance of 16 metres. Draw a scale showing metres and decimetres.

60. A distance of 47 chains is represented on a plan by $7\frac{1}{2}''$. Construct a diagonal scale of yards for the plan, showing distances of 2 yards.

61. What is a scale, and in what different ways can it be shown on a plan? When would you use a diagonal scale? Draw a scale showing yards, feet and inches. R.F. $\frac{1}{36}$.

62. The distance between two points is known to be 970 yards, and measures on a military plan 11·64". Draw a scale of yards for the plan showing 500 yards. Show single yards by the diagonal method. Show, by two small marks on the scale, the points you would take in order to measure off a length of 237 yards.

63. On a plan a distance of a furlong is represented by 2·25". Draw a diagonal scale of yards to suit the plan, showing 500 yards. Find the R.F.

64. On a map a distance of 5500 paces measures $8\frac{3}{4}''$. Draw a diagonal scale to show miles and paces, 10 paces being the minimum length shown. Give the R.F. Assume 1 mile=2000 paces.

65. On a map 67·5 miles are represented by 9·3". Draw a scale of miles for the map, showing 30 miles. Show furlongs by the diagonal method. Show, by two small marks on the scale, the points you would take in order to measure off a distance of 17 miles 3 furlongs.

66. (*a*) Construct a plain scale of yards having a R.F. $\frac{1}{432}$. Show 60 yards.

(*b*) Apply the diagonal method to the above scale to show feet. Show, by two small marks on the scale, the points you would take in order to measure off 22 yards 2 feet.

67. If a length of 3·3″ represents 2 miles 7 furlongs, draw a diagonal scale of miles, furlongs and chains to show 5 miles. Give the R.F. Mark off on the scale the line whose length is 3 miles 5 furlongs 6 chains.

68. On a plan a distance of 37′ is shown by $4\frac{3}{4}$″; construct a diagonal scale representing 50′ by which single inches may be measured. Indicate, by two small marks on the scale, the points you would take in order to measure a distance of 27′ 8″.

69. The distance between two places is $27\frac{1}{2}$ miles, and measures 3·25″ on the map. Construct a diagonal scale of miles and furlongs showing 50 miles. Mark off a line 23 miles 5 furlongs long.

70. A distance of 134 yards is represented on a plan by $2\frac{1}{4}$″. Construct a diagonal scale for the plan by which single yards may be measured. Show 300 yards. Show by two small marks on the scale the points you would take in order to measure off a distance of 183 yards.

71. Draw a scale of yards, 10,000 yards long, for a map when $3\frac{1}{4}$ miles are represented by $4\frac{1}{4}$″. Give the R.F.

72. A fort, whose greatest measurement is 560 yards, is reduced on a plan so that 25′ occupy ·075″: make a scale of yards for the plan long enough to take in all the lines of the figure.

73. Make a scale of yards, feet and inches (diagonal); 1″ to a yard. Give the R.F., and indicate upon it a line 3 yards 2 feet 6 inches long.

74. A French map is drawn to a scale of 1 metre to an inch. Construct a diagonal scale of feet and inches for the map showing 20′. Show all necessary calculations, figure the scale properly and write above it its R.F. Show, by two small marks on the scale, the points you would take in order to measure off a length of 11′ 8″. (A metre=39·37″.)

75. 1250 yards are represented by 15·5″. Make a comparative scale of French metres. (1 m.=1·0936 yards.)

76. Draw a diagonal scale of English miles and furlongs to suit a map, whose R.F. is $\frac{1}{460800}$; also a comparative plain scale of French kilometres. (8 kilometres=5 miles.)

77. Construct an English scale for a French map, on which 75 metres occupy 3″ and mark single yards diagonally. (1 m.=·39″.) Mark off a distance of 135 yards on it.

78. Make a scale of metres corresponding to an English scale of chains, 10 chains=3″. (1 Km.=·6213 English miles.)

79. A length of 3 m. 4 dm. is represented on a French plan by two English inches. Construct a diagonal scale for the plan showing 10 m., and reading to 2 cm. Show all your calculations,

and figure your scale properly. Show, by two small marks on the scale, the points you would take in order to measure off a length of 7 m. 8 dm. 4 cm.

80. In a map drawn on the metric system it is found that a distance of 8 Km. measures 1·3″. Construct a scale of miles for the map showing furlongs diagonally. (1 yard=·914 m.)

81. Draw a scale of $\frac{1}{2\frac{1}{5}}$ to measure metres (up to 4), tenths of metres, and (diagonally) hundredths of metres. From the scale draw a line 2·63 m. long.

82. On a French map, a distance of 4·48″ represents, according to the scale, a distance of 8000 m. Contruct a scale showing 10,000 English yards to suit the plan. (A metre= 1·0936 yards.)

83. The distances on a sketch are found to be inaccurate owing to an error in pacing. On comparing the sketch with an accurate survey it is found that a length of 685 yards on the survey is only 650 yards on the sketch. The sketch was intended to be drawn on a scale of 6″ to 1 mile. Draw a scale of yards to suit the sketch, showing 1000 yards.

84. On a Greek map a distance of 25 Greek miles is represented by 2·3″, draw a comparative scale of English miles, showing furlongs diagonally. Give the R.F. (One degree=86 Greek miles=69 English miles.)

85. A district measuring 20 miles by 13 miles is to be mapped on a sheet of paper measuring 11″ by $7\frac{1}{2}$″. The map is to be the largest possible size.

Draw a scale by which miles, furlongs and chains may be measured.

86. A plan is ·875 miles to 1″. Lay down a scale of miles, furlongs and chains. A portion of this is to be enlarged, so that any given distance shall be three times as long. Draw a scale showing 3000 yards for it.

87. On a map 1250 yards are represented by 13·5″. Make a scale of metres for it.

88. On a certain German map 3·75 cm. represented 257 m. Draw a scale of yards for the map, to read to 2 yards.

89. On an English plan 37′ 4″ occupy 3·27″. Draw a suitable scale of metres and decimetres.

90. Construct a scale R.F. $\frac{1}{20000}$ for your horse at a trot. You find that whilst trotting half a mile you rise 250 times in the saddle.

91. You have to make a sketch on horseback on a scale 3″ to 1 mile, and you ascertain that your horse when walking covers half a mile in 800 steps, and when trotting in 520. Construct suitable scales to read to ten steps.

92. In China,

100 chih=10 chang=1 yin=117½ English feet.

Draw a scale of yin, chang, chih. R.F. $\frac{1}{1440}$.

93. In Russia,

500 sajen=1 verst=·663 English miles.

Draw a diagonal scale of versts and sajen to read to 5 sajen comparative to an English scale of 1″ to the mile.

94. In China,

100 fun=10 tsun=1 chih=1·175 English feet.

Draw a scale (comparative to an English scale of 1″ to 1′) showing 5 chih and reading to fun.

CONSTRUCTION OF ANGLES.

Exercise VI.

Construct the following angles :—

1. 60° and 120°.

2. 30°, 90° and 150°.

3. 15°, 45°, 75°, 105°, 135° and 165°.

4. 22½°, 67½°, 112½° and 157½°.

5. At a given point construct all the angles you can (without using the dividers), each being measured by a whole number of degrees.

6. 7½°, 37½°, 52½ and 82½°.

7. 97½°, 127½°, 142½° and 172½°.

8. Construct a triangle on a base 3″ with base angles 45° and 75°.

9. Construct a right-angled isosceles triangle whose hypotenuse is 7·3 c.m.

10. Draw the polygon ABCDE in which AB=BC=2″, CD=DE=2·7″, ∠ABC=105°, ∠BCD=112½°, ∠CDE=82½°. Measure and write down the length of AE.

11. Construct angles of 30°, 60°, 90° at a point, and with your dividers obtain angles of 10°, 20°, 30°, 40° . . . etc.

12. By the method of question 11 divide the circumference of a circle (radius 2″) into 36 equal parts.

13. Draw a circle with radius 3″. Make an angle of 15° at its centre : also by trial obtain an angle of 10° and so an angle of 5°. Divide this by trial into single degrees.

DIAGRAMS OF COURSES, AND THE DETERMINATION OF DISTANCES BETWEEN INACCESSIBLE POINTS.

Exercise VII.

In this Exercise all angles must be constructed, if it is possible to do so without using trial methods.

1. A man walked 57 yards N.E. and then 47 yards N.W. How far was he from his starting point? [Scale $\frac{1}{640}$.]

2. A man cycles 11 Km. S., then 12 Km. S.W., and finally 15 Km. N. Draw a diagram of his journey and measure the direct distance of his finishing point from his starting point, and the bearing of the former place from the latter place. [Scale, which need not be drawn, $\frac{1}{200000}$.]

3. Two ports (A and B) at sea are E. and W. of each other and $7\frac{1}{2}$ miles apart. A vessel starts from the Eastern port A, and sails N.W. at the rate of 10 miles an hour. Another vessel starts at the same time from B, and sails S.E. at the rate of 8 miles an hour. Draw a plan of the position of the ships at the end of 45 minutes, and find their distance apart. [Scale, which need not be drawn, 2 miles to an inch.]

4. A ship sails N. 5 Km., then N.W. for 4·2 Km., and finally S.S.W. for 7·8 Km. Draw a plan of her course. How far is she from her starting point? [Scale $\frac{1}{100000}$, which need not be drawn.]

5. A man walks 200 yards due N., then 100 yards due E., and 80 yards N.E. At this point he turns to the right through an angle of 75° and walks 55 yards. How far is he from his starting point? [Scale, which need not be drawn, 100 yards to the inch.]

6. A man walks from A to B $\frac{1}{2}$ mile, turns to the right half a right angle, from B to C 1 mile, turns to the left 30°, from C to D $\frac{3}{4}$ mile, turns to right 75°, from D to E 2 miles. How far is A from E in a straight line? [Scale, $1\frac{1}{2}$" to the mile.]

7. A ship's course is as follows:—

E.N.E. for 4 miles,
then W for 5 ,,
,, N.E. for $3\frac{1}{2}$,,
,, S. for 6 ,,

Draw a diagram of her course. How far is she finally from her starting point, and what is her bearing from the same point? [Scale, $\frac{1}{66000}$ to be drawn.]

8. A man walks from A to B 2 miles, turns to right 45°, proceeds to C $2\frac{1}{4}$ miles, turns to right 60° and reaches D in $1\frac{1}{4}$ miles. How far is it from A to D direct? [Scale, 1″ to the mile.]

9. A person wishing to get from A to B (which is due N. of him, but separated from him by an impassable marsh), walks N.W. 2·75 Km. to C, then N. 15°E. for 1·75 Km. to D, and finds himself 2 Km. from B in a straight line. How far is it from A to B direct? [Scale, which need not be drawn, 2 cm. to 1 Km.]

10. Draw a diagram of the following walk on a suitable scale: A man walks 150 yards in a straight line, then turns to the right through an angle of 15°, 100 yards straight on and then he turns to the right 75° and walks 80 yards. Finally he turns to the left 105° and pursues a direct course for 250 yards. How far is he now from his starting point?

11. Draw a plan of the following walk on a suitable scale:—

A to B	..	5 miles	N. 15° E.
B to C	..	4 „	S. 30° E.
C to D	..	3·7 „	W. 30° N.
D to E	..	4·5 „	E. 15° N.

How far is it from A to E direct?

12. Draw a scale ·57″ to the mile; and on this scale construct a diagram of the course of a ship which sails

from A to B	..	3 miles	E.N.E.
B to C	..	$2\frac{3}{4}$ „	W.N.W.
C to D	..	3 „	E.N.E.
D to E	.	$3\frac{1}{2}$ „	W.N.W.
E to F	..	7 „	S.

How far is it from A to F?

13. Construct a diagram of the course of a ship which sailed

from A to B	..	10 miles	N.N.E.
B to C	..	3 „	S.W.
C to D	..	$5\frac{1}{2}$ „	S.S.W.

and from D back to A.

In what direction was she finally sailing?

14. A man walks from P to Q 500 yards and then turns to the right 45°, walks 400 yards to R and there turns to the left 30° and goes 300 yards to S. and again turns to the left 60° and proceeds for 450 yards. How far is he from his starting point? [Scale $\frac{1}{12300}$.]

15. Draw a diagram of the following course KLMNOP :—

K to L 750 yards, then a turn 45° to right
L to M 675 „ „ 75° „
M to N 650 „ „ $37\frac{1}{2}°$ left
N to O 525 „ „ $22\frac{1}{2}°$ „
O to P 325 yards.

How far is it from K to P ? . [Scale (500 yards=1·14″) to be drawn.]

16. Draw a diagram of the following walk :—

A man walks in all 7 miles. At the completion of each mile he turns to the right through the following angles respectively : 15°, $22\frac{1}{2}°$, $37\frac{1}{2}°$, 75°, 90°, 105°.

How far from his starting point does he finish his walk? [Scale 2″ to 1 mile.]

17. Draw the polygon ABCDE in which

AB=BC=CD=DE=5 cm.

$\angle$ABC=150° $\angle$BCD=105° and $\angle$CDE=$127\frac{1}{2}°$.

Measure AE.

18. It is required to find the range of two of an enemy's forts P and Q and their distance from each other. To do this a base line AB of 1500 yards is taken and the following angular measurements are made :—

$\angle$BAP=$97\frac{1}{2}°$ $\angle$ABP=$37\frac{1}{2}°$
$\angle$ABQ=$112\frac{1}{2}°$ $\angle$QAB=$22\frac{1}{2}°$

Draw a plan to the scale 3″ to a mile and measure and write down the lengths of AP, AQ, BP, BQ and PQ.

19. Starting from A, a man walks 3 miles due E., then $1\frac{1}{2}$ miles due N , then in a N.W. direction till he is $3\frac{1}{2}$ miles from A direct. Afterwards he walks 2 miles due S. How far is he now from A ?

20. A man walks from a point A, $3\frac{1}{2}$ miles in a straight line due N. to B, when he turns to the right through an angle of 45° and walks in the new direction $1\frac{1}{2}$ miles to C, where he again turns to the right through an angle of 75° and walks $2\frac{3}{4}$ miles to D. From D he walks due S. for $2\frac{1}{2}$ miles to E. Draw a scale of $\frac{3}{4}$ mile to an inch, and draw a plan of his walk to that scale. Write down the distance in miles from E to A. A protractor is not to be used.

21. Draw a figure ABCD, the given dimensions being AB=750 yards, $\angle$ABC=105°, $\angle$BAD=50°, $\angle$ABD=80°, $\angle$BAC=30°. Measure BC and AD, and write down their lengths in yards. [Scale, 200 yards to 1″.]

22. Along one bank of a river a line AB is measured 700 yards long. At A and B observations are taken to C and D on the opposite bank, the angles BAC, BAD being 100° and 45° respectively, and ABC, ABD 30° and 75° respectively. Draw a plan showing the points A, B, C, D ; measure the distances

AC, CD, BD, and write down their lengths in yards. [Scale (which should be drawn) 200 yards=1″.]

23. Draw to a scale of 10′ to an inch, the irregular figure of which the dimensions, &c., are as follows :—CB=37′, AB=20′, FA=25′, EF=22′, ED=22′, ∠B=105°, ∠A=135°, ∠F=120°, ∠E=75°. The angles are to be constructed. Measure and write down the length in feet of the side CD, and the magnitude of the angles EDC and DCB.

CONSTRUCTION OF IRREGULAR POLYGONS AND THE DETERMINATION OF THEIR AREAS BY REDUCTION TO TRIANGLES.

Exercise VIII.

"Triangles on the same base and between the same parallels are equal to one another."

1. Draw a plain scale of 9′ to the inch, and on that scale draw the quadrilateral ABCD ; given AB=29′, BC=24′, CD=23′, DA=26′, AC=33′ ; calculate its area.

2. Draw a plain scale of feet R.F. $\frac{1}{54}$, and from it draw the figure ABCDE in which AB=11′, BC=12′, CD=14′, DE=13′, EA=10′, AC=18′, CE=19′, and determine its area.

3. ABCD is a field, in which BC=240 yards,

∠ABC=80°, ∠BCD=105°,
∠ACB=70°, ∠CBD=47°.

If 1″ represents 100 yards, draw a plan and find the area of the field.

4. Draw the quadrilateral whose sides are 1·8″, 2·1″, 2·7″, 3·2″, the angle between the first two sides being 105°. Reduce it to a triangle and find its area. [N.B.—Construct the angle 105° if possible.]

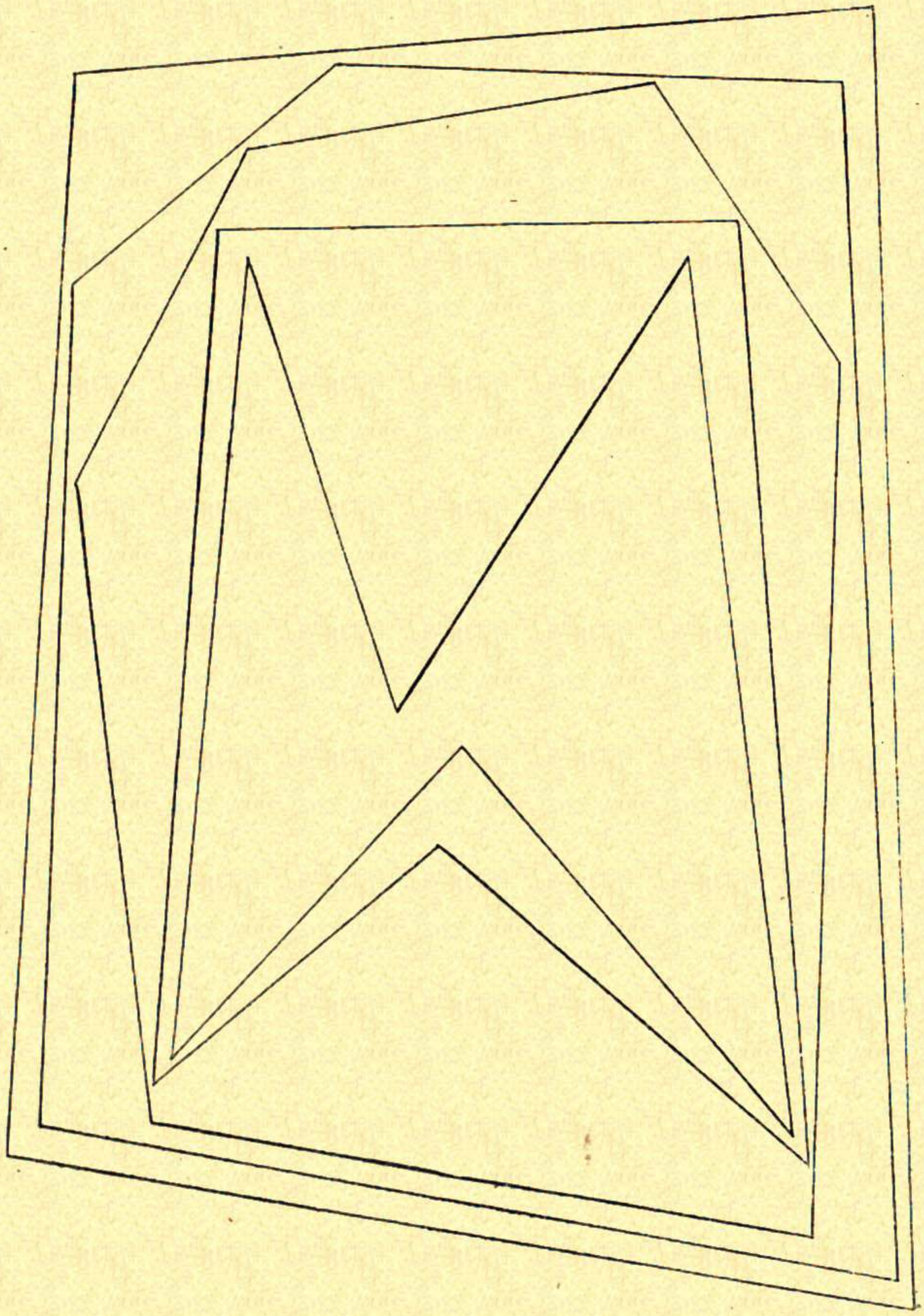
5. Draw the quadrilateral ABCD, having given AB=3·2″, BC=2·1″, CD=3″, DA=4·1″, and ABC=30° (to be made by construction). Find its area in sq. inches by reducing to an equivalent triangle.

6. A plot of ground is marked off by posts A, B, C and D ; AB is measured 150 yards, and the angles ABC, ABD, BAC, BAD are observed to be 105°, 80°, 40° and 70° respectively. Draw a plan of the ground and find its area. [Scale, which should be drawn, $\frac{1}{1800}$.]

7. Make a scale of $17\frac{3}{5}$″ to the mile, and with it construct a quadrilateral, having two opposite sides parallel and of lengths 425 and 225 yards, and the other two sides each 200 yards. Obtain its area.

8. Draw a figure ABCD. Given AB=2″, CD=4·7″, ∠ACB=30° and ∠ABC=∠DAB=121°. Find its area.

9. The figures below are numbered consecutively (the largest being No. 1). Copy them exactly. Reduce them to triangles and find the area of each :—



(α) No. 1. Full size. Area in sq. in.
(β) No. 2. Full size. Area in sq. cm.
(γ) No. 3. Scale $\frac{1}{3600}$. Area in sq. yards.
(δ) No. 4. Scale 1 : 10000. Area in hectares.
(ε) No. 5. Scale 1 : 20000. Area in acres.

10. Draw a plan of a five-sided field ABCDE, from the following measurements:—AB=80 yards, BC=75 yards, CD=95 yards, DE=50 yards, EA=105 yards. Diagonals AC=AD=120 yards. [Scale, which should be drawn, $\frac{1}{1200}$.] Calculate its area.

11. In order to measure the area of an irregular pentagonal field ABCDE a man walks from A to B, a distance of 85 yards, then turns through an angle of 75°, to the right, and walks 70 yards to C, then turns at right angles, to the right, and walks 110 yards to D, and then walks 90 yards to E, which is equidistant from A and D. Make a scale (100 yards being represented by 3″), to suit, and give the area in acres. [N.B.—There are no re-entrant angles.]

12. Construct a figure ABCDE having given AB=100 yards, ∠ABC=135°, ∠BAE=120°, AE=75 yards, AC=160 yards, ED=90 yards, and DC=150 yards. [Scale, to be drawn, 40 yards to 1″.] *Construct* the angles. Find the area.

13. Construct the polygon ABCDEF in which AB=4, AF=$3\frac{1}{2}$, AC=$6\frac{1}{2}$, AD=8, AE=$6\frac{1}{2}$, ∠BAC=35°, ∠CAD=25°, ∠DAE=40°, ∠EAF=25°. Lengths given are in yards. [Scale $\frac{1}{72}$.] Find its area.

14. With scale 13 yards to the inch, draw the figure ABCDE in which

AB=BC=40 yards,
AE=ED=31 yards,
∠BAE=51°, ∠AED=134°, ∠ABC=90°.

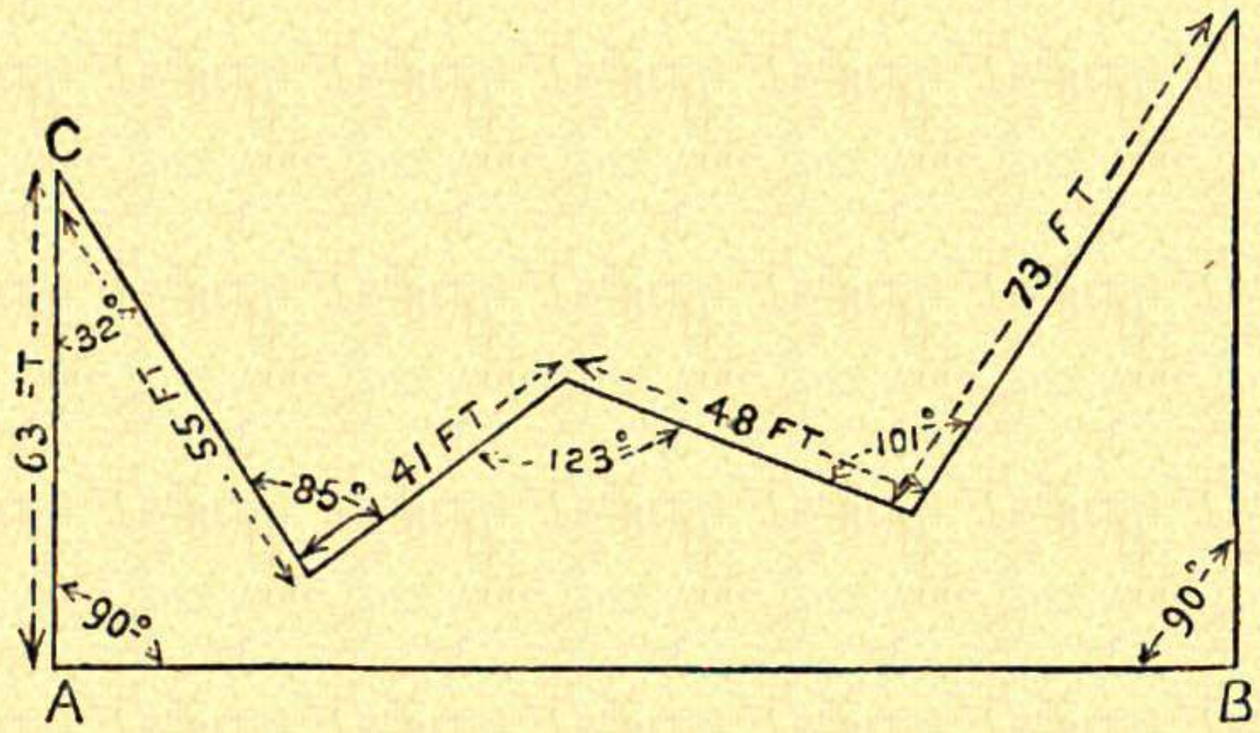
[The angle at E is to be re-entrant.]
Reduce it to a triangle and find its area.

15. ABCDEF is a field bounded by six straight lines, and having a re-entrant angle at E. The lines BC, BD, BE, BF, FE are equal to 170, 245, 135, 185, 130 yards respectively, and the angles CBD, DBE, FBA, AFB to 27°, 29°, 32° and 62° respectively. Draw a scale of yards $\frac{1}{2500}$ and a plan of the field. Reduce the figure to a triangle and find its area.

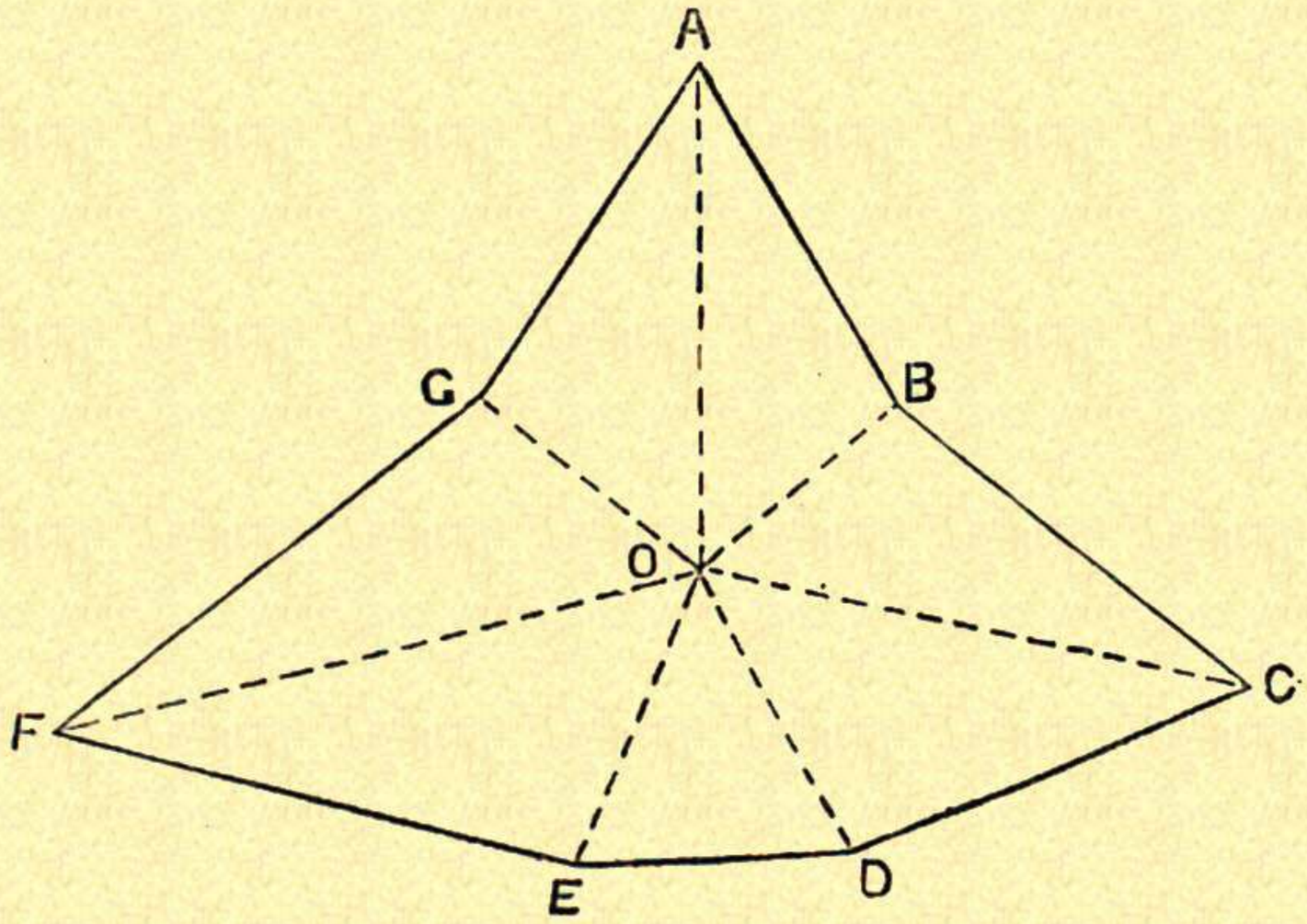
16. Draw a scale of yards having a R.F. of $\frac{1}{3960}$. Draw a straight line AB representing 240 yards to this scale, and bisect it in C. On the same side of AB *construct* the angles ACD=45°, ACE=75°, and ACF=120°. Make CD=250 yards, CE=165 yards, CF=285 yards, and join ADEFB. Reduce the figure to a triangle of equal area having its apex at E, and its base in AB produced. Find its area.

17. Draw a square on a side of 3″. Inscribe in it an equilateral triangle. Make a triangle equal to the excess of the area of the square over that of the equilateral triangle.

18. Construct from the given dimensions the irregular figure shown in the diagram. Reduce the figure to a right-angled triangle of equal area, having AB for one of its sides, and its other side in AC (produced if necessary). What is its area? [Scale 30′=1″.]



19. Make a scale of 9′ to an inch, and draw the figure shown below to that scale. The angles AOB, BOC, etc., are all equal.



OA=25′, OB=10′, OC=24′,
OD=12′, OE=12′, OF=30′,
OG=12′.

Write down the length of the sides of the figure AB, BC, etc. Reduce the figure to a triangle of equal area, having its apex at A and its base in ED produced. Determine its area.

20. Construct the figure ABCDEF, having given AB=2″, BC=$1\frac{1}{2}$″, AF=$1\frac{1}{4}$″, FE=$2\frac{1}{2}$″, AD=$4\frac{1}{2}$″, CD=2″, AC=3″, BF=$2\frac{1}{2}$″, BE=$3\frac{1}{2}$″, and determine its area in square inches by reducing it to an equivalent triangle.

21. The sides of a quadrilateral figure ABCD are as follows: AB=45 yards, BC=38 yards, CD=33 yards, and AD=39 yards, and the diagonal AC=66 yards. Make a scale of 15 yards to an inch, and draw the figure to that scale and bisect it by a straight line drawn from the point B. Determine the area of the whole figure in sq. yards.

22. The four sides of a field ABCD measure as follows:— AB=280 yards, BC=320 yards, CD=330 yards, DA=360 yards, diagonal DB=300 yards. Draw a scale of 120 yards to an inch, draw a plan of the field to that scale, and calculate its area in yards.

23. Construct the figure ABCD having given AB=3″, ABC=108°, ABD=42°, BAC=30° and BAD=82°. Measure and write down the lengths of AD, DC and CB. Reduce it to a triangle. Find the length of a side of a square of equal area.

24. A man walks from A to B, a distance of 12 yards, then turns to the left through an angle of 75°, and walks 11 yards to C, when he again turns to the left through an angle of 45°, and walks to D, a distance of 14 yards. He then walks straight back to A. Find from your scale the distance DA, and by reducing the figure ABCD to a triangle, find its area in sq. yards. [Scale, 5 yards to an inch.]

25. A plot of ground is marked off by posts at A, B, C, D; AB is measured 750 metres, and the angles ABC, ABD, BAC, BAD are observed to be 112°, 80°, 37° and 70° respectively. Draw a plan of the ground and find its area in hectares.

26. In order to measure the area of a five-sided cricket ground ABCDE, a boy walks round the edge of it, keeping it always on his right. He first walks 90 yards from A to B, then turns through an angle of 60° and walks 70 yards to C, then turns at right angles and walks 120 yards to D, then turns through an angle of 30° and walks 90 yards to E, and then walks back to A. Construct a plan of his walk on the scale of $\frac{1}{1500}$ (which should be drawn), and obtain the area of the ground, the distances from B to D, and E to A, and the magnitudes of the angles DEA and EAB.

27. ABCDE is a pentagonal figure. The dimensions are as follows:—AB=5″, BD=4″, AD=DC=DE=2″. A, D and C are in one straight line, EDC is a right angle. Construct the figure, and determine its area.

28. In order to measure the area of an irregular pentagonal field ABCDE a man walks from A to B a distance of 90 metres, then turns through an angle of 60° and walks 70 metres to C, then turns at right angles and walks 170 metres to D, and then walks 100 metres to E, which is equidistant from A and D. Make a scale to suit, 200 yards being represented by 4·5″, and give the area in hectares. [The field has no re-entrant angles.]

AREAS FROM FIELD NOTES.

Exercise IX.

Draw plans (on suitable scales) of the fields whose dimensions are determined by the following notes

In each case calculate the area of the field *without drawing any new lines.*

1.

	to C		
	950		
to B 100	700		
	500	150 to D	Measurements in links.
	from A		

2.

	to G		
	150		
	90	35 to H	
to F 25	50		Measurements in metres.
	from E		

3.

	to N		
	1500		
	1000	200 to P	
to M 150	750		
to L 100	350		Measurements in links.
	from K		

4.

	to D		
	130		
to C 25	75	35 to E	
to B 45	55		Measurements in feet.
	from A		

5.

	to D		
	130		
to C 35	100		
	80	45 to E	
to B 25	20		Measurements in metres.
	from A		

6.

to B		
9		
7·7	2·1 to C	
4·4	1·2 to D	
1·4	3·2 to E	Measurements in chains.
from A		

7.

	to D		
	300		
to C 40	270		
	210	30 to E	
to B 50	120		Measurements in metres.
	from A		

8.

	to T	
	1000	
to S 200	850	
to R 140	600	
to Q 180	500	Measurements in links.
	from P	

9.

	to V		
	50		
	40	10 to U	
to W 35	35		
	30	20 to T	Measurements in paces.
	10	15 to S	1 pace=33″.
	from R		

10.

	to E		
	100		
to D 15	95		
to C 30	75	20 to F	
to B 25	45		Measurements in paces.
	35	15 to G	1 pace=33″.
	from A		

SCALES OF CHORDS.

Exercise X.

1. Construct a scale of chords, radius 3″, to measure angles from 15° to 180° (rising by increments of 15°), and determine the lengths of chords of 45° and 135°.

2. Draw a scale of chords, radius 3·5″, to show angles of 5°. Use it to construct angles of 50° and 65° at the base of a triangle 2·4″ long. With it also measure the third angle of the triangle.

3. Use your scale of chords to construct a regular pentagon in a circle whose radius is 3″.

4. Make a scale of chords with radius 2″. Use it to inscribe a regular nonagon in a circle, radius 2″.

5. Make a scale of chords with radius 5 cm. and use it to circumscribe a circle of radius 5 cm. with a regular nonagon.

6. Use the scale of chords on your protractor to draw a regular quindecagon in a circle whose radius is 3″.

REGULAR FIGURES IN A CIRCLE.

Exercise XI.

1. In a circle of radius 4 cm. inscribe a regular hexagon.

2. Inscribe a square in a circle of radius 5·2 cm.

3. Inscribe an equilateral triangle in a circle, radius 1·3″.

4. In a circle, whose radius is 1·92″, inscribe an octagon.

5. In a circle, whose radius is 5 cm., inscribe a dodecagon.

6. In a circle (radius 3·7 cm.) inscribe two squares whose sides cut each other at 45°.

7. In a circle (radius 2·03″) inscribe two equilateral triangles cutting each other symmetrically.

8. In a circle, whose radius is 1·75″, inscribe a regular pentagon.

9. Inscribe a regular heptagon in a circle, whose radius is 6 cm.

10. In a circle, whose diameter is 12·8 cm., inscribe a regular decagon.

11. Inscribe two regular pentagons in a circle (radius 2·33″) cutting each other symmetrically.

12. Find A, B, C, D and E, consecutive points of a regular pentagon in a circle (radius 1·85″) and join them in the following order :—AC, CE, EB, BD, DA, to make a star.

13. Describe a circle of 1″ radius and divide it into three equal sectors.

14. In a circle (diameter 4·7″) inscribe a regular heptagon.

15. Draw a circle, radius 1·95″. Inscribe in it a regular octagon. Draw inside it a concentric octagon having each of its sides $\frac{4}{5}$ of a side of the former.

16. In a circle, whose diameter is 9·3 cm., inscribe a regular hexagon : by joining the middle points of its sides describe another hexagon. Circumscribe the latter with a circle. What is its radius ?

17. In a circle, whose radius is 6 cm., inscribe a regular pentagon. By reducing this to an isosceles triangle find its area.

18. Inscribe a square in a circle, whose radius is $1\frac{3}{4}''$: and in this square place another symmetrically, so that the sides of the latter are $\frac{1}{4}''$ from those of the former.

19. In a circle, whose radius is 3·7 cm., inscribe an equilateral triangle. What is the length of its side? In this triangle describe another symmetrically with sides 2·7 cm.

20. In a circle of diameter 7·9 cm. inscribe a regular octagon: and on its sides describe semicircles alternately inside and outside the octagon.

21. In a circle (radius 2·47″) inscribe a regular decagon: symmetrically place inside it another so that the perimeter of the latter may be $\frac{2}{3}$ the perimeter of the former.

22. Inscribe a regular heptagon in a circle, whose radius is 2·3″, and in it draw another heptagon, each whose sides is ·3″ from a side of the former.

23. Inscribe a regular pentagon in a circle whose radius is 2·5″, and in it inscribe five successive pentagons with their sides ·15″ apart.

24. Inscribe a square in a circle whose radius is 2·32″, and in it inscribe four successive squares with sides $\frac{1}{7}''$ apart.

25. In a circle, whose radius is 2·04″, inscribe a regular octagon, and in it inscribe four successive octagons with sides $\frac{2}{9}''$ apart.

26. In a circle, whose radius is 1·98″, inscribe a regular heptagon, and in it inscribe four successive heptagons with sides $\frac{2}{7}''$ apart.

27. Construct a regular pentagon in circle 2″ radius, and determine its area by two different methods.

28. Describe a circle of 4″ diameter and in it inscribe a regular octagon. Within this octagon inscribe a second having each of its angles in the centre of a side of the first. Within this second octagon inscribe a third having each of its angles in the centre of a side of the second. Continue this process till six regular octagons have been drawn.

29. In a circle, radius 2·5″, inscribe a regular hexagon; within it draw three other hexagons with the same centre whose sides are parallel to its sides and ·5″ apart.

30. Draw a circle with radius 2·32″ and in it inscribe a regular pentagon. In the pentagon place five similarly situated pentagons with their sides $\frac{4}{25}''$ apart.

31. Draw a circle with radius 2·17″ and in it inscribe a regular hexagon. In the hexagon place five similarly situated hexagons with their sides $\frac{6}{35}$″ apart.

32. In a circle, whose radius is 2·43″, inscribe a regular heptagon. Draw four more similarly situated heptagons, two inside and two outside, such that their sides are $\frac{11}{50}$″ apart.

CONSTRUCTION OF REGULAR FIGURES ON A GIVEN BASE.

Exercise XII.

1. On a base 3″ describe an equilateral triangle. Determine its area.

2. On a base 3″ describe a square. Draw another square by joining the middle points of its sides.

3. Draw a regular hexagon on a base 2″ long and in it place symmetrically another hexagon with sides 1·65″.

4. Draw a regular octagon on a base 1·52″. Determine its area by two different methods.

5. Describe a regular pentagon on a line 1·7″ long, and place in it four other pentagons, with the same centre, whose sides are ·25″ apart and parallel to those of the former pentagon.

6. Draw a regular pentagon with sides 1·75″. Find its area by two different methods.

7. Draw two octagons, symmetrically situated, one inside the other, having sides 1″ and 1·5″ respectively.

8. On a straight line 5 cm. long describe a regular hexagon and outside it place a similarly situated hexagon, with the same centre, having sides 6 cm.

9. Draw a pentagon whose side is 2·2″. In it place three other pentagons, whose sides are parallel to, and equidistant from, the sides of the first pentagon, the distance between parallel sides of each successive pentagon being 0·14″.

10. Describe a nonagon on a base 1″. Reduce it to an isosceles triangle and find its area.

11. Draw two heptagons, symmetrically, one inside the other, the larger having sides 1·25″, and the sides of the latter being ·4″ distant from those of the former.

12. Draw a regular pentagon on a base 2·3″ and two others inside it, having their sides ·3″ apart.

13. A square and a pentagon are described on the same side of a line 6·3 cm. long. Find, in sq. cm., the area contained between these two figures.

14. On a straight line 1·11″ long describe a regular decagon. Reduce one-half of it to an isosceles triangle of equal area, and so calculate the area of the whole decagon.

15. On a straight line 2·3″ describe a regular heptagon and inside it place a similarly situated heptagon (with the same centre) whose sides are 1·9″.

16. On a base of ·75″ long describe a regular octagon. Find its area.

17. Describe a square of $2\frac{1}{4}$″ side, and on each side describe an isosceles triangle having each of its equal sides $1\frac{3}{4}$″. Circumscribe the whole figure with a circle.

18. Describe a regular pentagon on a line 2·4″, and place in it four other pentagons with the same centre, whose sides are ·25″ apart and parallel to those of the first pentagon.

19. Draw three octagons, symmetrically situated, one inside the other, and having sides of 1″, 1·25″, and 1·5″ respectively.

20. Describe a square on a base of $2\frac{1}{2}$″, and describe a circle to pass through its angular points.

21. Describe a regular hexagon of $1\frac{1}{2}$″ side, and a second of 1″ side inside the former, and having its sides equidistant from and parallel to those of the former.

22. On a straight line 3·1″ long describe an equilateral triangle, and in it draw seven others similarly situated, their sides being $\frac{1}{6\,0}$″ apart.

23. Describe a square with sides 3·2″ and in it describe six other squares, similarly situated, with their sides $\frac{7}{4\,5}$″ apart.

24. Describe a regular pentagon on a base 2·05″ long, and in it inscribe two others, similarly situated, with their sides $\frac{4}{3\,5}$″ apart, and outside it also two others with their sides the same distance apart.

25. Describe five regular octagons, similarly situated, one inside the other, with sides alternately $\frac{1}{1\,2}$″ and $\frac{1}{6}$″ apart. The side of the middle octagon is to be $1\frac{1}{4}$″.

REGULAR FIGURES CIRCUMSCRIBED ABOUT CIRCLES.

Exercise XIII.

"A straight line, which touches a circle, is at right angles to the radius at the point of contact."

1. Describe an equilateral triangle about a circle whose radius is 1″.

2. Describe a square about a circle whose radius is 3 cm.

3. Describe a regular pentagon about a circle (radius 1·2″).

4. Circumscribe a circle (radius 1·1″) with a regular hexagon.

5. Circumscribe, with a regular heptagon, a circle, radius 3·4 cm.

6. Circumscribe a circle whose radius is 1·43″ with a regular octagon.

7. About a given circle whose radius is 3·7 cm. describe a regular nonagon.

8. About a given circle whose radius is 1·39″ describe a square and circumscribe the square with another circle.

9. Describe a regular pentagon about a circle whose radius is 3·4 cm. and circumscribe the pentagon with a circle.

10. Describe a regular heptagon about a circle whose radius is 1·7″ and find its area by reducing it to a triangle.

11. About a circle of 1·84″ radius describe a regular octagon.

12. Draw a square of $1\frac{3}{4}$″ side, and about it describe a circle. About this circle describe a second square having its sides parallel to the *diagonals* of the first. About the second square describe a circle, and about this circle describe a third square having its sides parallel to the *sides* of the first.

13. About a circle, whose radius is 1·37″, describe a regular pentagon, and outside it draw four more similarly situated pentagons with their sides $\frac{7}{60}$″ apart.

14. Circumscribe a circle, whose radius is 1·2″, with a regular heptagon, and outside it draw four more similarly situated heptagons with their sides $\frac{13}{45}$″ apart.

15. Draw a series of six similarly situated concentric octagons with sides $\frac{6}{35}$″ apart, so that the circle which is inscribed in the third smallest has a radius 1″.

16. Draw a circle with a radius of 1·2″, and in it inscribe a regular pentagon. About the circle describe a square.

17. Describe two concentric circles with radii of 2″ and ½″. In the larger circle inscribe a regular pentagon, and about the smaller one describe an equilateral triangle, making one side of the pentagon parallel to one side of the triangle. Reduce the figure contained between the triangle and pentagon to a triangle of equal area.

[Patterns, pp. 6*—10*, 19*, 24*, 26, 28*, 30, 31, 36, 41, 44, 45.]

CIRCUMCIRCLE OF A TRIANGLE AND CIRCLE PASSING THROUGH THREE GIVEN POINTS.

Exercise XIV.

"The centre of a circle passing through two given points is on the perpendicular bisector of the line that joins them."

1. Draw a triangle with base 7·9 cm. and base angles 75° and 45° (to be constructed). Circumscribe it with a circle and measure its radius.

2. Describe a triangle with two sides 2″ and 2·5″ and included angle 120° (to be constructed). Circumscribe it with a circle and measure its radius.

3. Draw a triangle having sides 2″, 4·8″ and 5·2″. Draw a circle passing through the middle points of its sides. Measure the radius.

4. Draw an isosceles triangle with base 4·7 cm. and vertical angle 30°. Circumscribe it with a circle, and measure the radius. [The protractor is not to be used for drawing the angles.]

5. Draw a triangle, sides 4·2 cm. and 3·5 cm., and included angle 105° (to be constructed). Circumscribe a circle about it and measure the radius.

6. Draw a triangle with sides 3·7″, 2·5″ and 2·93″. Draw perpendiculars from the angular points to the opposite sides. Through the feet of perpendiculars describe a circle. Measure the radius.

7. Draw a straight line AB 4″ long and find a point C 3″ from A and 2¼″ perpendicularly from AB. Describe a circle to pass through the points A, B and C. Measure its radius.

8. Draw a square ABCD, side 2″; on AB describe an equilateral triangle ABE outside the square. Describe a circle to pass through the points B, D, E and measure its radius.

9. Draw a triangle with sides 10·4 cm., 9·2 cm., 7·9 cm. Circumscribe it with a circle. Also draw a circle to pass through the middle points of the sides of the triangle. Measure the radii of the circles.

10. ABCD is a square, side 2″; on the diagonal BD (remote from A) is described an equilateral triangle BDE. Draw a circle passing through E, F and G (the middle points of AB and AD respectively). Measure its radius.

IN- AND E- CIRCLES OF TRIANGLES, SECTORS, AND KITE-SHAPED QUADRILATERALS.*

Exercise XV.

"The centre of a circle touching two straight lines is on the bisector of the angle between them."

1. Draw a triangle with sides 6, 7 and 8 cm., and inscribe a circle.

2. Draw a triangle with sides 1·3″, 1·4″ and ·9″. Produce all the sides. Draw the three escribed circles.

3. Draw a sector, radius 3″ and angle 37°. Inscribe a circle in it.

4. Draw a quadrant, radius 7 cm., and inscribe a circle in it.

5. Draw a triangle with two sides 3″ and 2·4″, and included angle 57°. Inscribe a circle in it.

6. Draw a triangle with base 3·4 cm. and base angles 73° and 57°. Produce all the sides and draw the three escribed circles.

7. Draw a sector, radius 1·9″ and included angle 30°. Produce the bounding radii and draw a circle to touch these lines (produced) and the arc of the sector.

8. Draw a right-angled triangle with hypotenuse 3·3″ and one side 1·8″. Inscribe a circle in the triangle.

9. Draw the kite-shaped figure ABCD in which
$AB=AD=1\frac{1}{4}''$, $BD=1\frac{1}{2}''$, $CB=CD=2''$,
and in it inscribe a circle.

10. Draw the "kite" PQRS in which
PQ=PS=4 cm., QS=5 cm., RQ=RS=4·8 cm.
Inscribe in it a circle.

*N.B.—A kite is a quadrilateral composed of two isosceles triangles on the same base and on opposite sides of it.

11. Show how to cut from a triangular sheet of metal (sides $1\frac{3}{4}''$, $2''$, $2\frac{1}{4}''$) the greatest possible circle.

12. Construct a triangle with sides of $3''$, $4''$, and $5''$. Draw the inscribed circle. Measure and write down the length of the radii of the circle and the size of the angles of the triangle.

13. Draw a triangle with sides $3\frac{1}{2}''$, $3''$ and $1''$, and draw a circle touching the two longer sides produced, and the shorter side.

14. Construct a triangle ABC, given the angle at A$=48°$, side AB$=3''$ and AC$=3\frac{3}{4}''$. Within this triangle inscribe another whose sides are parallel to those of ABC, and $\cdot2''$ from them. Inscribe a circle in the smaller triangle.

15. Draw a quadrant of a circle, radius $3''$, and inscribe a circle.

16. On a base of $2\frac{1}{2}''$ describe an isosceles triangle whose vertical angle is $40°$, and on the opposite side of the base describe an equilateral triangle. In the figure so formed inscribe a circle.

17. On opposite sides of a straight line $2\frac{1}{2}''$ long, as base, describe isosceles triangles with vertical angles $100°$ and $70°$. Inscribe, in the quadrilateral so formed, a circle.

EQUAL CIRCLES TOUCHING EACH OTHER AND THE SIDES OF REGULAR FIGURES.

Exercise XVI.

1. Describe an equilateral triangle on a base 9 cm., and in it inscribe three equal circles, each touching the other two circles and one side of the triangle.

2. Describe an equilateral triangle with sides $4''$, and in it inscribe three equal circles, each touching two others and two sides of the triangle.

3. Inscribe an equilateral triangle in a circle, whose radius is $2''$, and in the triangle inscribe three equal circles, each touching two others and one side of the triangle.

4. Describe an equilateral triangle, whose altitude is $2{\cdot}3''$, and in it place three equal circles, each touching the other circles and two sides of the triangle.

5. Draw a square with sides 7·3 cm. and in it inscribe four equal circles, each touching two other circles and one side of the square.

6. Draw a square with sides 3·7″ and in it inscribe four equal circles, each touching two other circles and two sides of the square.

7. Inscribe four circles in a square (side 2″) each touching one side of the square and two other circles.

8. In a square whose diagonal measures 10·2 cm. inscribe four equal circles, each touching two others and one side of the square.

9. In a circle (radius 2·3″) inscribe four equal circles, each touching two others and the circle.

10. Make an equilateral triangle whose altitude is 3·5″. In it place three circles, each touching two others and two of the sides of the triangle.

11. Inscribe three equal circles in a circle, of radius 2″, so that each may touch two others and the original circle.

12. Draw a circle having a radius 1·25″. Trisect this circle (by construction) by straight lines drawn from the centre to the circumference. In each of the three sectors thus formed draw a circle touching the two radii and the circumference of the first circle.

13. About a circle of 1″ radius describe six equal circles, each touching two others and the given circle.

14. Describe a circle of 2″ radius, divide it into quadrants, and in each quadrant inscribe a circle.

15. Describe a circle of 1·54″ diameter. Surround this circle by six equal circles touching the given circle and each other successively. Circumscribe the six circles by a circle.

16. In a circle, radius ·9″, inscribe three equal circles, each touching two others and the first circle; also about the circle describe six equal circles, each touching two others and the first circle.

17. In a regular hexagon of 2″ side, inscribe three equal circles, each touching two others and two sides of the hexagon.

18. In a circle of 2″ radius inscribe five equal circles, each to touch the original circle and two others.

19. Describe a circle (radius ·75″) and about it describe five equal circles, each touching two others and the original circle.

20. On a base of 2·8″ describe a regular pentagon, and in it inscribe five equal circles, each touching two others and two sides of the pentagon.

21. Draw a pentagon with sides 2″, and in it inscribe five equal circles, each touching two others and one side of the pentagon.

22. Draw a pentagon in a circle, radius 4·4 cm., and in it inscribe five equal circles, each touching two others and one side of the pentagon.

23. Draw a pentagon with sides 4·7 cm., and in it inscribe five equal circles, each touching two others and two sides of the pentagon.

24. Draw a circle with radius 1·73″. From it cut off a quadrant, and in the remainder inscribe four equal circles to touch consecutively.

25. Draw a circle with radius 4·8 cm., and divide it into four sectors, whose angles are proportional to 3, 4, 5 and 6; in each sector inscribe a circle.

26. Draw a regular heptagon in a circle whose radius is 1·78″, and in it inscribe seven equal circles, each touching two others and two sides of the heptagon.

27. In a circle, radius 2·72″, inscribe seven equal circles, each touching the original circle and two others.

28. Draw a circle with radius 1″ and surround it with seven equal circles touching each other consecutively.

29. Draw a regular heptagon with sides 1″. Draw seven equal circles, outside it, each touching two other circles and one side of the heptagon.

30. In a circle, radius 2·5″, inscribe a regular octagon. In the octagon inscribe four equal circles, each touching two others and two sides of the octagon.

31. Draw a circle with radius 2·5 cm.; in it inscribe a regular octagon; draw eight equal circles (outside the octagon) each touching two other circles and one side of the octagon.

32. In a square of side $2\frac{1}{2}$″ inscribe eight equal circles, such that each side is touched by two circles, and each circle touches the adjacent circles.

33. On a straight line BC, 4″ long, describe an isosceles triangle ABC, with sides 5″ long. (1) In the triangle inscribe two equal circles touching each other, and each touching the base BC, and one side of the triangle. (2) Inscribe a third circle touching each of the sides AB, AC, and each of the two equal circles.

[Patterns, pp. 14, 15*, 16*, 20*, 27*, 29*.]

FOILED FIGURES HAVING TANGENTIAL ARCS INSCRIBED IN REGULAR FIGURES.

Exercise XVII.

1. Describe a trefoil of tangential arcs in a circle (radius $1\frac{1}{2}''$).

2. Describe a quatrefoil of tangential arcs in a circle (radius 4·5 cm.).

3. Describe a cinquefoil of tangential arcs in a circle (radius 2″).

4. In an equilateral triangle (sides 3″) describe a trefoil of tangential arcs, each arc touching one side of the triangle.

5. In an equilateral triangle (sides 7·3 cm.) describe a cuspidate trefoil, each arc touching two sides of the triangle.

6. In a square (sides 3·2″) inscribe a cuspidate quatrefoil, each arc touching two sides of the square.

7. In a square (sides 8 cm.) inscribe a quatrefoil of tangential arcs, each arc touching one side of the square.

8. In a pentagon whose sides are 2·7″ inscribe a cinquefoil of tangential arcs; each arc is to touch one side of the pentagon.

9. In a circle, whose radius is 6·3 cm., inscribe a pentagon, and in the pentagon inscribe a cuspidate cinquefoil, each of whose arcs touches two sides of the pentagon.

10. In a circle (radius 2″) inscribe a foiled figure with seven tangential arcs.

11. In a circle, radius 2·5″, inscribe a cuspidate trefoil. The thickness throughout is $\frac{1}{5}''$.

12. Draw a regular pentagon with sides 6 cm., and in it inscribe a cuspidate cinquefoil, each of whose arcs touches two sides of the pentagon.

13. Draw a regular hexagon with sides 6·5 cm. and in it inscribe a foiled figure of six tangential arcs, each arc touching one side of the hexagon.

14. Inscribe a regular heptagon in a circle, whose radius is 6 cm., and in the heptagon inscribe a foiled figure of seven tangential arcs, each arc touching two sides of the heptagon.

CIRCLES *WITH GIVEN RADII* TO PASS THROUGH POINTS, OR TO TOUCH GIVEN LINES OR CIRCLES.

Exercise XVIII.

"If two circles touch externally, the distance between their centres equals the SUM of their radii; but if they touch internally it equals the DIFFERENCE of their radii."

1. Draw two straight lines inclined at an angle of 65°, and draw four equal circles (radii $\frac{1}{2}''$) to touch them.

2. Draw two straight lines meeting at an angle of 105° (to be constructed), and draw four equal circles (·7″ radii) to touch them.

3. Draw two straight lines meeting at an angle of $67\frac{1}{2}°$ (to be constructed), and draw four equal circles (1·7 cm. radii) to touch them.

4. Two straight lines AOB, COD cut one another at an angle of 25°. Draw four circles (radii 1·4 cm.) to touch both lines.

5. Obtain, by construction, an angle of $37\frac{1}{2}°$; and describe a circle (radius 3·2 cm.) touching each of the lines containing the angle.

6. Draw two parallel straight lines, 2·4″ apart, and take a point ·72″ from one of them. Draw a circle to pass through that point and to touch the two lines.

7. Draw a perpendicular at the end of a given line, and draw a circle touching each of the two lines at points 1″ from the point where they meet.

8. Draw two circles, radii 3·7 cm., to touch a given straight line and to pass through a given point 4·2 cm. from that line.

9. Draw two straight lines at an angle 75° (to be constructed). Divide this angle into four equal parts. In each of the four small angles inscribe a circle 1·7 cm. radius.

10. Draw two straight lines (unlimited in length), and inclined at an angle of 73°. Place four circles (having radii $\frac{3}{4}''$) so that each of the lines may lie at a distance $\frac{1}{4}''$ from the nearest points of the circles.

11. Draw a rectangle $3\frac{1}{2}''$ long and $2\frac{1}{2}''$ broad. Round off the corners with circular arcs having $\frac{3}{4}''$ radii.

12. Draw two straight lines meeting at an angle of 55°. Describe a circle, $1\frac{1}{2}''$ radius, cutting off from these lines chords of $1\frac{3}{4}''$ and $2\frac{1}{4}''$ respectively.

13. XY is a straight line and Z a point 2″ from it. With centre Z a circle $1\frac{1}{2}$″ radius is described. Draw two other circles, radii ·7″, to touch XY and the given circle.

14. AB is a given straight line and C a point 4 cm. from it. With centre C and radius 1·7 cm. a circle is described. Draw two circles (radii 4·5 cm.) to touch the given circle and AB. One of these circles is to touch the given circle externally and one internally.

15. PQ is a given straight line and R a point 2 cm. from it. A circle, centre R, radius 3 cm., is described. Draw two circles, whose radii are 1·5 cm., to touch PQ and the given circle, one externally and the other internally.

16. In a circle, whose radius is 3·7 cm., a chord 6·8 cm. is placed. In the larger segment draw a circle, radius 2 cm., to touch the given circle and the chord, and in the smaller draw a circle, radius 7 mm., fulfilling the same conditions.

17. Two parallel straight lines are 2″ apart, and symmetrically between them is drawn a circle whose radius is $\frac{1}{2}$″. Draw four circles, radii $\frac{3}{4}$″, to touch the given circle and one or other of the straight lines.

18. Through a circle, of radius 1·73″, is drawn a secant (·43″ from the centre). Draw all the circles possible, whose radii are ·4″, which touch the given circle and secant.

19. Draw two circles, radii 1″ and 1·25″ (centres 2·75″ apart), and draw circles ·75″ radii to touch both.

20. Draw three circles, radii ·7″, ·8″ and ·9″, to touch each other externally.

21. Draw three equal circles of 1″ radius, each touching the other two externally, and circumscribe them by another circle.

22. Draw two circles with radii $1\frac{1}{2}$″ and 1″ so that one may touch the other externally. Draw a third circle with radius 3″, which will touch and enclose both of them.

23. Draw two circles (radii 1·2″ and 1·6″) touching each other externally. Draw two other circles (radii ·75″) touching each of the first two.

24. Describe two circles of radii 1″ and ·7″ to touch each other, and each to touch internally a third circle of radius 2″.

25. Two circles have centres 4″ apart and radii $1\frac{1}{4}$″ and $1\frac{3}{4}$″. Describe a circle of radius $2\frac{1}{2}$″ which touches one of these internally and the other externally.

26. A circle, radius 2·8″, is drawn; in it place two other circles, radii 1·3″ and 1″ respectively, touching each other externally and the outer circle.

27. One circle, radius $\frac{3}{4}''$, touches internally another circle, radius $2\frac{1}{2}''$. Draw a third circle, radius $1''$, so that it may touch each of the former circles. (N.B.—The point of contact is not to be the same for all circles.)

28. A and B are points $3''$ apart. With A as centre draw a circle of $\cdot 75''$ radius, and with B as centre a circle of $1\cdot 5''$ radius. Describe a circle of $1\cdot 25''$ radius to touch them both.

29. Describe circles, radii 2 cm. and 3 cm., touching each other externally, and another circle, radius 7·5 cm., to touch and enclose the two smaller circles.

30. Draw a circle of radius $1\frac{1}{2}''$, and within it inscribe two other circles of diameters $1\frac{1}{4}''$ and $\frac{3}{4}''$ respectively which shall touch each other and the circumference of the first circle.

31. Draw an arc of a circle of $3''$ radius, and describe a circle of $\frac{1}{2}''$ radius touching it externally. Describe a second circle of $\frac{3}{4}''$ radius touching the first and the arc externally, and a third circle of $1''$ radius touching the arc externally and the second circle. Explain briefly your construction.

32. Three equal circles (radii $1''$) lie with their centres in a straight line, and each touches the adjoining one. Draw them, and place two more equal circles, each touching the other, and also touching two of the former circles.

TANGENTS.

Exercise XIX.

"The tangent of a circle is at right angles to the radius of the circle at the point of contact."

"The angle in a semicircle is a right angle."

1. Draw a pair of tangents to a circle of $1''$ radius from a point $1\cdot 75''$ from the centre of the circle.

2. From a point $2\cdot 3''$ from the centre of a circle of radius $1\cdot 4''$, draw two tangents to the circle.

3. A point lies $3\frac{1}{2}''$ distant from the centre of a given circle (radius $1''$). Draw a tangent from the point, and also draw the parallel tangent.

4. To a circle of radius $1\cdot 5''$ draw two tangents from a point $3\cdot 3''$ from the centre, and a third tangent equally inclined to the two others.

5. Given a circle, draw three tangents to it, so that two of them intersect at an angle of 30°, and the third cuts the other two at equal angles

6. On a chord AB $4\frac{1}{4}''$ long draw the segment of a circle to contain an angle of 55°. Produce AB to C, making BC$=2\frac{1}{2}''$. Through C draw a tangent to the segment.

7. Draw a segment of a circle containing an angle of 75° on a line 2″ long. Produce the line 1″ in each direction, and from the two extremities draw tangents to the circle.

8. From a point O outside a circle of 1″ radius, and 2″ from its centre, draw two lines tangential to the circle. Join the points where these lines touch the circle, and on the line so found construct a regular pentagon.

9. Two circles, radii 1″ and 2″, touch each other. Draw the common tangents to the circles.

10. Describe two circles of 1″ and $\frac{3}{4}''$ radii respectively, their centres being 3″ apart. Draw all possible common tangents to these two circles.

11. Two circles, radii 1·3″ and 2·17″, have their centres 2·5″ apart; draw the common tangents.

12. Draw four common tangents to two circles, radii 1·7 cm. and 2·9 cm., whose centres are 10 cm. apart.

13. Describe two circles whose radii are 1·3″ and ·8″ with their centres 1·75″ apart, and draw their common tangents.

14. Draw the four common tangents to two circles of radii $\frac{3}{4}''$ and $1\frac{1}{2}''$, whose centres are $4\frac{1}{2}''$ apart.

15. Draw two circles of radii 2·2″ and 1·5″ having their centres 3″ apart, and construct their common tangents.

16. Draw a straight line AB 3·5″ long, and from A and B as centres, with radii 1″ and 1·5″ respectively, describe circles. (1) Draw a common tangent to these two circles, touching them both on the same side of AB. (2) From the point A draw a tangent to the circle whose centre is B. In each case show clearly how the tangent point is determined.

17. Describe three circles of radii 1·2 cm., 1·8 cm. and 2·4 cm. to touch each other, and describe the triangle each of whose sides touches two of these circles.

18. Draw three circles, radii 2 cm., 2·3 cm. and 2·6 cm., touching each other, and describe a triangle each of whose sides touches two of these circles.

19. Describe three circles, radii ·5″, ·7″ and ·85″, each touching externally the other two. Draw a triangle each side of which touches two of these circles.

20. With radii of $\frac{3}{4}''$, $\frac{3}{4}''$ and $\frac{1}{2}''$, describe three circles touching each other, and about them describe a triangle, each side to touch two of the circles. The method of determining

the six points of contact must be clearly shown. Write down the length of each side of this triangle.

21. Describe three circles, with radii of ·70″, ·85″ and ·95″ respectively. to touch each other externally. Circumscribe the circles with a triangle.

22. A circle, radius $\frac{3}{4}$″, touches a given straight line. Draw a second circle, with radius 1″, which shall touch the first, and also the straight line. Draw afterwards a common tangent.

23. Show the construction for drawing a tangent to a circular arc of large radius, whose centre is out of reach, from a point on the circumference.

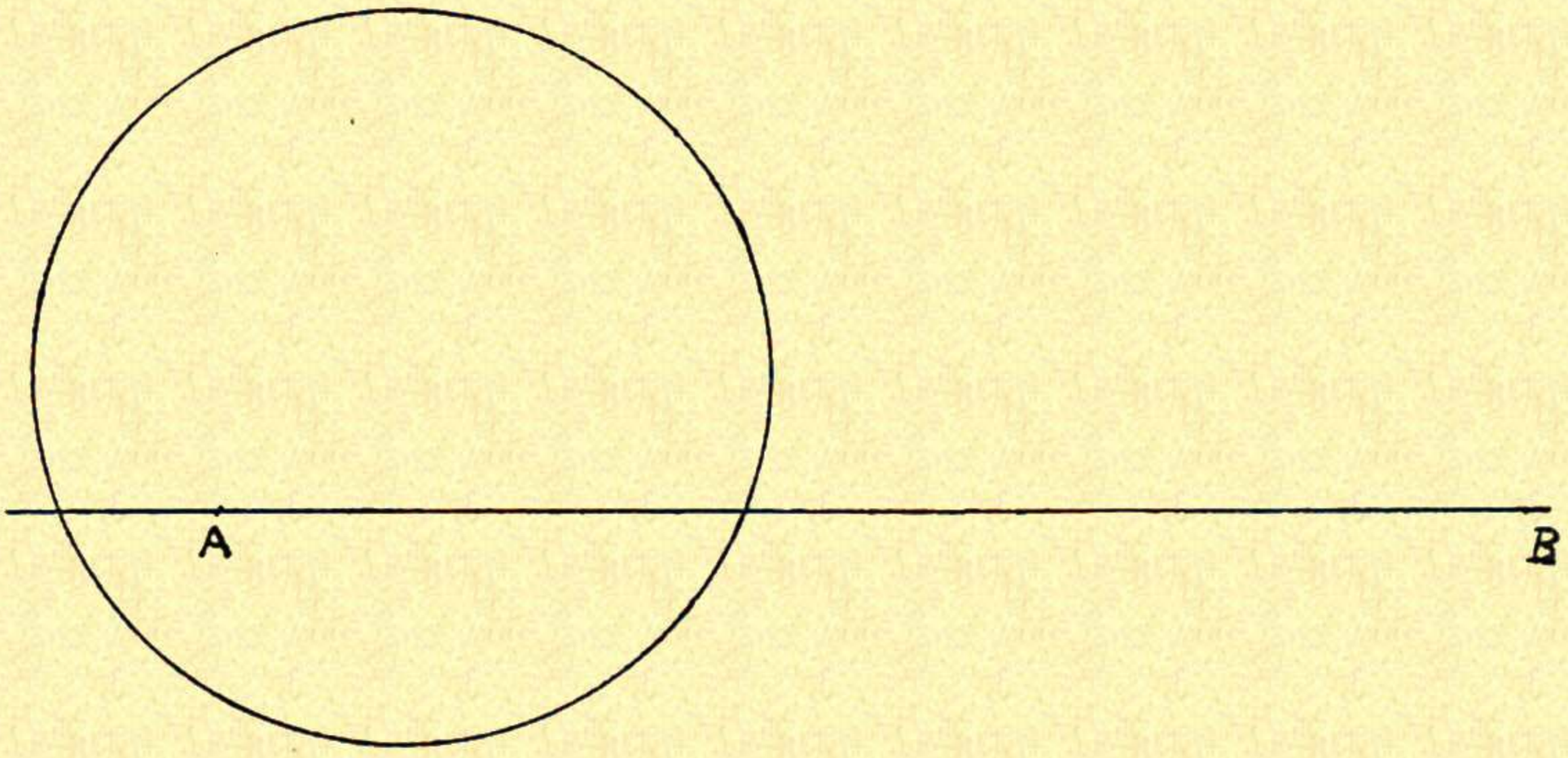
24. P is a point $3\frac{1}{2}$″ from the centre of a circle of radius $2\frac{1}{2}$″. Draw a chord through P, so that the portion of it intercepted by the circle may be 3″.

25. Take any two points C and P, $2\frac{3}{4}$″ apart ; with centre C, radius $1\frac{1}{4}$″, describe a circle, and from P draw a straight line PQR, cutting the circle in Q and R, so that the length of the chord QR may be $1\frac{1}{2}$″.

26. Draw a circle, radius 4·2 cm., and take any point 7·8 cm. from its centre. Through this point draw a straight line cutting the circle, so that the chord is 5 cm. long.

27. Draw two tangents to a circle, radius 2·73″, inclined at an angle of 50°. From the point where these tangents intersect draw a secant to the circle, so that the chord is 3·4″ long.

28. Find a point in AB such that a tangent to the circle drawn from that point will be 2″ long, and draw the tangent. State your construction.



29. The railway from P to Q consists of a circular arc AB, and two tangents PA and BQ. The radius of the arc is two miles, and the radii to the ends of the arc make an angle of 28°.

PA=1 mile, BQ=½ mile. Show the railway on a scale of 2″ to the mile, and measure the distance from P to Q as the crow flies. Ink-in the railway only.

30. ABC is a triangle in which AB=3″, BC=3·5″, AC=2·5″. With A and B as centres, two circles, radii ·8″, are described, and with C as centre, a circle, radius 1·4″, is described. Draw a triangle to circumscribe the three circles, each side touching two circles.

31. A tight endless band passes round two pulleys (without crossing). The centres of the pulleys are 2·7″ apart, and their radii are ·8″ and 1·2″. Draw a diagram of the pulleys and band.

32. Three circular pulleys, radii ½″, have their centres in a horizontal straight line and 2″ apart successively. A cord passes over the first pulley, under the middle pulley, and over the third pulley, and the ends hang down and the cord is kept taut by weights. Draw a diagram of the pulleys and cord.

33. Draw two circles, radii 2 cm. and 3·2 cm., touching one another. Draw two common tangents to these circles, and draw a straight line through both circles, so that the chord intercepted by each circle may be 4 cm.

34. Using ½″ as radius, describe three cuspidate trefoils, each touching both the other trefoils symmetrically at one point only; and draw a hexagon round the figure, touching the circles.

SERIES OF CIRCLES TOUCHING EACH OTHER CONSECUTIVELY AND TWO CONVERGING STRAIGHT LINES.

Exercise XX.

1. From a point 3″ from the centre of a circle whose radius is ·9″ two tangents are drawn. Draw a smaller circle to touch these two lines and the circle.

2. Two straight lines are inclined at an angle of 35°. Describe four circles touching both lines, and each touching the next circle, the radius of the smallest being ⅖″.

3. Draw two lines inclined at an angle of 37½° (to be made by construction). Describe a circle, radius ·8″, to touch the two lines, and describe a second circle to touch the first and the same two lines.

4. Describe a circle of 1″ radius touching two given straight lines inclined at an angle 30°; and draw two more circles, each touching the same two lines and the first circle.

5. A sector of a circle, whose radius is 10·3 cm. and angle 33°, is drawn. In it inscribe a circle, and then three others, consecutively, touching the former circle and the bounding radii of the sector.

6. Draw two straight lines meeting at an angle of 25°. Between them draw three circles to touch the straight lines; the middle circle also touching each of the others, and the smallest having a radius of $\frac{1}{4}''$.

7. Draw two lines which converge but do not meet. Between these lines describe three circles touching one another in succession, and also the converging lines.

8. Draw two straight lines (unlimited in length) inclined at any angle, and place a circle, having a radius of $1\frac{1}{2}''$, so that each of the lines may lie at a distance of $\frac{1}{2}''$ from the nearest points of the circle.

9. Draw two lines inclined at an angle of 75° (to be made by construction), and describe a circle, radius 1·5", to touch the two lines. Draw another circle to touch the former and the two straight lines.

CIRCLE TO TOUCH CIRCLE OR STRAIGHT LINE AT A GIVEN POINT, AND ALSO A GIVEN STRAIGHT LINE OR CIRCLE.

Exercise XXI.

"Tangents to a circle from an external point are equal."

1. A straight line AB is drawn, length 3", and a circle, radius 1·2", is drawn touching AB at A. Draw a second circle touching the first circle and AB at B.

2. AB is a straight line 3·5" long. Draw a circle, radius 2", touching AB at A, and another circle touching AB at B and this circle.

3. ABC is an isosceles triangle, right-angled at C, having AC 3" long. With centre A and radius $1\frac{1}{2}''$ a circle is described. Draw another circle touching this circle and BC at B. Give two solutions if possible.

4. ABC is a triangle, B is a right angle. AB=5 cm., BC=2·6 cm. With centre A and radius 2 cm. a circle is drawn. Draw two other circles to touch this circle and the straight line BC at the point C.

5. ABC is a triangle. BC=4″, AC=3″, AB=2″. P is a point in CB such that CP=$\frac{2}{3}$CB. A circle with centre A and radius 8″ is described. Draw two other circles to touch this circle and the straight line BC at the point P.

6. A circle, radius 1·5″, is drawn. Draw a tangent from a point A, 3·2″ from the centre. Draw a second circle touching the first circle and the tangent at the point A.

7. The centre A of a circle of 1·25″ radius is 1·75″ from a given straight line. Describe a circle that shall touch the given straight line in a point P, 2·5″ from A, and shall be touched internally by the given circle.

8. Draw two straight lines AB, AC of indefinite length, the ∠BAC being 50°. Take a point D in AB, 3″ from A, and a point E in AC 2·25″ from A. Describe a circle which shall touch AB in D, and cut AC in E, and a second point F (to be determined). Measure AF, and show that $AE \times AF = AD^2$.

9. Describe a circle, radius 1·3″, to touch at A a straight line AB 3″ long. Describe a second circle touching AB at B and the first circle.

10. Draw a straight line AB of indefinite length. At A erect a perpendicular AD 1$\frac{1}{2}$″ long. With centre D and radius $\frac{3}{4}$″ describe a circle. Describe a second circle that shall be touched internally by the first and by AB in a given point 2$\frac{1}{4}$″ from D.

11. Draw a straight line BC, and in it take any point D. 2$\frac{1}{2}$″ from D and 2″ from BC take a point A. Draw a circle to pass through A and to touch BC at D.

12. Draw a right-angled triangle ABC. B is the right angle, AB=5 cm, AC=8 cm. Draw a circle to pass through A and to touch BC at C.

13. AOB is a straight line of unlimited length. XOY cuts it at an angle of 37° and OX=1″. Draw two circles touching XOY at X and also touching AB.

14. Two straight lines converge at an angle of 55°. A point P is taken on one line 2·2 cm. from the point of section. Draw two circles to touch that line at P and also the other line.

15. A circle, with radius 2 cm., has its centre 3 cm. from a given straight line AB. X is a point on the circumference 3·7 cm. from AB. Draw two circles touching the given circle at X and also the given straight line.

16. ABC is a triangle. AB=3·3 cm., BC=4·4 cm., AC=5·5 cm. With centre A and radius 2 cm. a circle is described cutting AC at X. Draw two circles to touch this circle at X and also the straight line BC produced if necessary.

CONTINUOUS CURVES. TANGENTIAL ARCS.

Exercise XXII.

"The straight line which joins the centres of two circles, which touch externally or internally, passes through the point of contact."

1. Draw a straight line 5″ long and alternately above and below draw five semicircles (radii $\frac{1}{2}$″) to form a continuous curve.

2. A straight line ABC, 4″ long, is bisected at B. PQ is a parallel straight line of unlimited length $\frac{1}{2}$″ away from ABC. With centres A, B and C and radii ·8″ arcs of circles are drawn on one side of PQ. Connect these by circular arcs to form a continuous curve.

3. XY is a straight line 1 cm. long. With centres X and Y alternately, describe semicircles above and below XY to form a continuous spiral. The first semicircle is to have a radius 1 cm.

4. Draw a regular hexagon with sides 4 cm., and on alternate sides *outside* describe semicircles; connect these by circular arcs to form a continuous curve.

5. On a base 1·75″ describe a regular hexagon and on alternate sides *inside* describe semicircles. Connect these by circular arcs to form a continuous curve.

6. Draw a regular octagon in a circle (radius 2″) and on alternate sides describe, externally, semicircles: connect these by circular arcs to form a continuous curve.

7. Draw a regular octagon whose sides are 3 cm., and on alternate sides describe, internally, semicircles. Connect these by circular arcs to form a continuous curve.

8. Take a semicircular arc ABCD of radius $2\frac{1}{2}$″ and trisect it at B, C, and on AB and CD describe semicircles inwards. Connect their extremities with a continuous curve.

9. On alternate sides of a regular hexagon on a base of $1\frac{2}{3}$″ describe semicircles exterior to the hexagon. Join the nearest ends of the semicircles by arcs of circles, so as to form a continuous curve. Within this curve, and at a distance of $\frac{1}{8}$″ from it, place a similar and similarly situated curve. (The latter curve is to be a fine ink line, the first curve being heavier. The hexagon and the lines of construction are to be given in pencil.)

10. Draw a semicircle, and divide it into three equal parts. Draw round the semicircle three equal arcs of circles, so that they may touch at these points of division.

11. Two parallel straight lines $1\frac{1}{2}$″ apart are drawn, and another straight line ABCD cuts them perpendicularly at A

and D, and is trisected at B and C. With centres B and C and radii 1″, arcs of circles are drawn outside the parallel lines. Connect these arcs by circular arcs to form a continuous curve.

12. Two parallel straight lines XY and ZW, 2″ apart, are cut perpendicularly by PQRS at P and S. PQ=·4″=RS. With centres Q and R arcs of circles, radii 1″, are drawn outside the parallel lines. Connect them by circular arcs between the parallel lines to form a continuous curve.

13. Two parallel straight lines KL and MN, 2·3″ apart, are cut perpendicularly by STUV at T and U, and ST=UV=·5″. With centres S and V and radii 1″ arcs of circles are drawn outside the parallel lines. Connect these by circular arcs between the parallel lines to form a continuous curve.

[Patterns, pp. 21*, 22*, 33, 34, 35, 37, 38*, 39*, 40, 42, 59, 60, 61, 63, 64.]

FOILED FIGURES, COMPOSED OF SEMI-CIRCLES, HAVING ADJACENT DIAMETERS AND INSCRIBED IN REGULAR FIGURES, DEPENDING ON THE CONSTRUCTION OF HALF RIGHT ANGLES.

Exercise XXIII.

1. Draw an isosceles triangle with base 2·3″ and area 4·6 sq. inches. Inscribe in it a semicircle touching the middle point of the base.

2. Draw a sector of a circle with radii 3″ and angle 53°. Inscribe in it a semicircle so that it touches the middle point of the arc of the sector.

3. Draw an equilateral triangle with sides 7 cm., and in it inscribe three semicircles with adjacent diameters, each semicircle touching one side of the triangle.

4. Draw an equilateral triangle having sides 2″ and on the sides draw external semicircles. In a circle, whose radius is $1\frac{1}{2}$″, describe a similar figure.

5. In a square, side $3\frac{1}{2}$″, inscribe a quatrefoil of semicircles with adjacent diameters. Each semicircle is to touch one side of the square.

6. In a circle, radius 4·5 cm., inscribe a quatrefoil with adjacent diameters.

7. In a pentagon, whose sides are 2″, inscribe a cinquefoil of semicircles with adjacent diameters; each semicircle is to touch one side of the pentagon.

8. In a circle, radius 4·3 cm., inscribe a cinquefoil with adjacent diameters.

9. In a circle, radius 4 cm., inscribe a trefoil with adjacent diameters, the thickness throughout is to be 3 mm.

10. Inscribe a pentagon in a circle whose radius is 5 cm., and in the pentagon inscribe a cinquefoil of semicircles with adjacent diameters; each semicircle is to touch one side of the pentagon.

11. Inscribe a foiled figure composed of six semicircles with adjacent diameters in a circle whose radius is $2\frac{1}{4}$″.

12. In a circle, radius 2·25″, inscribe five equal semicircles having adjacent diameters.

FIGURES, SIMILAR TO, AND EQUAL TO THE SUM OF SIMILAR FIGURES.

Exercise XXIV.

"In a right-angled triangle, any figure described on the hypotenuse is equal to the sum of two similar and similarly described figures on the sides containing the right angle."

1. Construct a square equal to the sum of the squares on straight lines 6 cm. and 8 cm. Measure its side.

2. Construct a square equal to the sum of squares on lines 1·3″, 2·4″, 1·8″. Measure its diagonal.

3. Construct a circle equal to the sum of circles whose radii are 1·75″, 1·79″. Measure its radius.

4. Construct an equilateral triangle equal in area to two equilateral triangles, one on a base 2″ and the other of altitude 2″. Measure its side.

5. Draw two concentric circles with radii 5 cm. and 3 cm., and construct a third concentric circle equal in area to the sum of the other two. Measure its radius.

6. Draw three concentric regular hexagons whose sides are 1·3″, 1·47″ and 1·73″. Construct a third concentric hexagon equal in area to the sum of the other three. Measure its side.

7. Draw four concentric pentagons, the side of the largest being 2·1″ and the sides of the others being each $\frac{1}{5}$″ from the former pentagon. Construct a pentagon equal in area to the sum of the two smallest pentagons. Reduce it to a triangle and find its area.

8. Draw a semicircle equal in area to four semicircles whose diameters are ·5″, ·6″, 1·2″, 1·31″. What is its radius?

9. Construct an isosceles triangle equal in area to the sum of the areas of four squares whose sides are $\frac{1}{2}$″, $\frac{3}{4}$″, 1″ and $1\frac{1}{4}$″ respectively.

10. Construct an isosceles triangle equal to the sum of four squares whose sides are respectively $\frac{1}{2}$″, $\frac{5}{8}$″, $\frac{7}{8}$″ and $1\frac{1}{4}$″.

11. Describe two circles, diameters 1·8″ and 2·4″, to touch one another, and describe two other circles to touch these externally ; one having its area equal to the sum of their areas, and the other having its perimeter equal to the sum of their perimeters.

FIGURES, SIMILAR TO, AND EQUAL TO THE DIFFERENCE OF SIMILAR FIGURES.

Exercise XXV.

1. Construct a square equal to the difference of squares on lines 3·7″ and 2·5″. Measure its side.

2. Construct an equilateral triangle equal to the differences of two equilateral triangles, one on a base 4″ and the other of altitude 1·7″. Measure its side.

3. Draw two concentric circles, radii 7·3 cm. and 5·3 cm., and draw a concentric circle equal in area to the annulus. Measure its radius.

4. Draw two concentric regular hexagons whose sides are 2·03″ and 1·73″ and construct another hexagon equal in area to the difference of the two hexagons. Measure its side.

5. Describe two regular pentagons. One with sides 2″ and the other in a circle, radius 2·4″, and construct a regular pentagon equal in area to their difference. Reduce it to a triangle and determine its area.

6. Find an equilateral triangle with area equal to the difference between two other equilateral triangles having side 3″ and altitude 3″ respectively. Inscribe in it a circle and measure its diameter.

DIVISION OF TRIANGLES, &c., INTO ANY NUMBER OF EQUAL PARTS BY STRAIGHT LINES DRAWN THROUGH A GIVEN POINT.

Exercise XXVI.

"Triangles on the same base and between the same parallels are equal."

1. ABC is a triangle in which AB=7 cm., BC=8 cm., CA=9 cm.; P is a point in AB such that PA=2 cm. Bisect the triangle by a straight line drawn through P.

2. Draw the triangle DEF in which EF=3·72″, ∠DEF=75°, ∠DFE=45° (constructing the angles). Q is a point in EF such that QE=1·11″. Bisect the triangle by a straight line through Q.

3. Draw the same triangle as in question 1, and trisect it by lines drawn through P.

4. Draw the same triangle as in question 2, and divide it into four equal parts by straight lines drawn through Q.

5. Draw the triangle ABC in which ∠ABC=74°, ∠BCA=31°, AC=3·2″. Divide it into five equal parts by straight lines drawn from the point P in AB, such that AP=$\frac{2}{3}$ AB.

6. Draw a triangle having given two sides 9·3 cm. and 8·4 cm. and included angle 82$\frac{1}{2}$° (to be obtained by construction). Bisect it by a straight line drawn through one of the points of trisection of the base.

7. Construct a rhombus 2$\frac{1}{2}$″ side, and having one angle of 70°, and divide it into three equal parts by lines drawn from one angular point.

8. Divide a square, whose sides are 7 cm., into three equal parts by three straight lines drawn from the centre of the square.

9. Divide the kite ABCD into two equal parts by a straight line drawn through B. AB=AD=2″, CB=CD=2·73″, diagonal AC=3·2″.

10. Bisect the kite PQRS by a straight line drawn through Q. PQ=PS=5·5 cm., RQ=RS=7·5 cm., QS=8·1 cm.

11. Draw a parallelogram ABCD. AB=3″, AD=2″, ∠BAD=60°. Bisect it by a straight line perpendicular to AB.

12. Draw the parallelogram PQRS in which ∠SPQ=105°, PS=5·3 cm., PQ=7·3 cm. In SR take a point X such that SX=3 cm. Trisect the parallelogram by straight lines drawn through X.

13. ABCD is a quadrilateral in which AB=2·5 cm., BC=5·1 cm., AC=6·5 cm., AD=3 cm., DC=6 cm. Bisect it by a straight line drawn through B.

14. Construct a regular pentagon of 2″ side. From any angle draw a straight line dividing the pentagon into two parts, one of which is twice as large as the other.

15. The sides of a triangle ABC are as follows: AB=$3\frac{3}{4}$″, BC=$4\frac{1}{2}$″, CA=$2\frac{1}{2}$″. Draw the triangle and trisect it by lines drawn from a point P in AB, 1″ from A.

CONSTRUCTION OF SEGMENT OF THE CIRCLE CONTAINING GIVEN ANGLE.

Exercise XXVII.

1. On a base 3″ describe a segment containing an angle 53°.

2. On a base 7 cm. *construct* a segment containing an angle 75°.

3. On a base 2·93″ describe a segment containing an angle 125°. In it inscribe a circle whose radius is ·3″.

4. On a base 7·3 cm. *construct* a segment of a circle containing an angle 112$\frac{1}{2}$°.

5. Draw a triangle, base 2·75″, altitude 1·37″, vertical angle 73°.

6. On a base 3″ long, and on the same side of it, draw segments containing angles 50° and 127°. Draw a circle, with radius 1″, to touch the arcs of these two segments.

7. Draw a circle, with radius 5 cm., and cut off a segment containing an angle 100°. In the other segment inscribe a circle with radius 1·5 cm.

8. Draw a segment of a circle on a base of 2″, containing an angle of 80°. Draw an angle in the segment.

9. On the same side of a line 2″ long describe two segments of circles, one containing an angle of 60° and the other containing a right angle. Describe two circles, with radii $\frac{1}{4}$″, to touch the two arcs.

10. Divide a line 5″ long into six equal parts: on each part draw a segment of a circle containing an angle of 60°, alternately above and below the line.

11. A, B and C form the angular points of a triangle, of which the side AB=620 yards, BC=513 yards, and AC=995

yards. From a given point D you observe that the angle between A and B is 46°, and the angle between B and C is 49°. Find the position of D, and its distance from B. [Scale 300 yards to an inch.]

12. Draw a figure ABCD. Given AB=2″, ∠ACB=∠ADB=30°, CD=3″, and ∠ABC=∠DAB. Find its area.

INSCRIPTION OF SQUARES DEPENDING ON THE CONSTRUCTION OF HALF-RIGHT ANGLES.

Exercise XXVIII.

1. Draw a rhombus, with sides $2\frac{1}{2}$″ and one angle 53°. Inscribe in it a square.

2. Draw a regular hexagon, with sides 2·3″, and in it inscribe a square.

3. Draw a regular decagon in a circle with radius 5 cm. and inscribe a square.

4. Draw a rhombus, with diagonals 8 cm. and 6 cm., and inscribe a square.

5. Inscribe a regular hexagon in a circle, with radius 4·7 cm., and inscribe a square in the hexagon.

6. Draw a rhombus, with one side 6 cm. and area 30 sq. cm. Inscribe in it a square. Measure its diagonals.

7. Draw a regular hexagon, with sides 1·98″, and inscribe a square. What is the area of the square?

8. Draw a rhombus, with diagonals 5″ and 3·78″. Inscribe a square. Measure its diagonals.

9. Construct a parallelogram having each of its sides $2\frac{1}{2}$″ long and its two larger angles each equal to 120°. In this figure inscribe a square having its four angles in the four sides of the parallelogram.

10. Draw a regular hexagon in a circle, radius 5·3 cm. Inscribe a square. Measure its sides.

11. Draw a hexagon, with sides 5·2 cm. Inscribe a square. Measure its diagonals.

12. Construct a rhombus of 2·5″ side, one of the angles being 55°, and in it inscribe a square. Measure its sides.

13. Construct a rhombus, the two diagonals being 3″ and 5″. Inscribe a square in it. Measure its sides.

14. ABCD is a space enclosed by two equal arcs, ABC and ADC, each arc being a quarter of a circle, radius 3″. Construct the figure, and inscribe a square in it. Measure its diagonals.

15. Two circles, whose radii are 1·7″, have their centres 2·3″ apart. Inscribe a square in the area common to both circles. Measure its diagonals.

INSCRIPTION OF RECTANGLES IN TRIANGLES.

Exercise XXIX.

1. The three sides of a triangle are 3·75″, 3·5″ and 1·62″ respectively. Draw the triangle and in it inscribe a rectangle having each of its longer sides 2·75″ long.

2. Describe a triangle having its sides 3″, 2½″ and 2″ long; in it inscribe a rectangle having one of its sides 2·4″ long. Explain your construction.

3. Draw a triangle with base 10 cm. and base angles (to be constructed) 45° and 60°. Inscribe in it a rectangle having one of its sides 7 cm. long, and parallel to the base. Measure its diagonals.

INSCRIPTION OF OCTAGONS IN SQUARES.

Exercise XXX.

1. On a straight line 3″ long describe a square and in it inscribe a regular octagon.

2. On a straight line 8 cm. long describe a square and in it inscribe a regular octagon. Measure its sides.

3. Construct a square with diagonals 10 cm. and in it inscribe a regular octagon. Measure its sides.

4. Construct a square of area 8 sq. inches and in it inscribe a regular octagon. Measure its sides.

PART II.

FOURTH AND THIRD PROPORTIONALS.

Exercise XXXI.

In every case measure and write down the length of line constructed.

1. Find a fourth proportional to the lines 1·6″, 2″ and 2·5″.

2. Determine a fourtll proportional to three straight lines of lengths 2·67″, 1·83″ and 2·77″.

3. Find a third proportional to 3·35″ and 3·8″.

4. Find a third proportional to the lines 7·3 cm. and 8·4 cm., and then a fourth proportional to these three lines.

5. Construct a line whose length is $\frac{2{\cdot}37 \times 5{\cdot}27}{4{\cdot}77}$ inches.

6. Construct a line whose length is $\frac{(3{\cdot}75)^2}{4{\cdot}21}$ inches.

7. Construct a line whose length is $\frac{8{\cdot}9 \times 9{\cdot}8}{7{\cdot}3}$ cm.

8. If A=7·8 cm., B=9·3 cm., C=8·5 cm.; find X, Y and Z in the following cases:—

(1) A : B=C : X. (2) A : Y=C : B. (3) B : Z=A : B.

9. If P=4·73″, Q=3·79″, R=4·23″; find L, M and N in the following cases:—

(1) P : Q=Q : L. (2) M : P=Q : R. (3) R : Q=N : P.

10. Find a third proportional to lengths of $\frac{3}{4}$″ and 1″.

11. Determine the lengths of third proportionals (greater and less) to lengths of 3·5 cm. and 5·1 cm.

12. Taking $\frac{1}{4}$″ as the unit find two third proportionals to lines of 7 and 10 units.

13. Construct a rectangle on a side 1″ equal to a square on a side 1·73″.

14. Construct a rectangle on a side 5 cm. equal to a square on side 4 cm.

15. Draw a rectangle whose sides are 2·73″ and 3·14″. On a straight line whose length is 2·35″ construct a rectangle of the same area. Measure its diagonal.

16. Draw a triangle whose sides are 10·2 cm., 8·7 cm. and 7·3 cm. Construct a rectangle equal to it, and on a straight line 2·5 cm. long construct another rectangle equal to the former. Measure its diagonal.

17. Draw a triangle with sides 3·7″ and 3·17″ and included angle 57°. Construct a triangle equal to it in area, with the same included angle, but one side 2·7″.

18. Inscribe a square and an equilateral triangle in a circle of 1·73″ radius, and find by construction and write down the ratio that the area of the square inscribed in the circle bears to the area of the square described on one of the sides of the equilateral triangle.

19. Construct a rhombus with a side of 2″ equal to a square on the side $1\frac{1}{2}$″.

MEAN PROPORTIONALS.

Exercise XXXII.

1. Find a mean proportional to the lines 2″ and 2·8″.

2. Find a mean proportional between 4″ and 3″.

3. Find a mean proportional between 2·35″ and 3·8″.

4. If P=4·73″ and Q=3·71″; find Y, when $Y^2=PQ$.

5. If A=3·21″, B=4·31″; find X, when A : X=X : B.

6. Find Z when Z : A=B : Z, if A=8·3 cm., B=7·2 cm.

7. Find by geometrical constructions (to two places of decimals) $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$, $\frac{2}{\sqrt{3}}$, $\frac{7}{\sqrt{5}}$, $\frac{2}{3}\sqrt{11}$, using suitable units.

8. Find a mean proportional between 19 and 37.

9. Find the value of $\sqrt{323\times497}$.

10. Find a mean proportional between 8·3 cm. and 9·5 cm.

11. Draw a square whose side is $\sqrt{5}$″; measure its diagonals.

12. Draw a rectangle having sides 3·12″ and 1·28″. Construct a square equal to it. Measure and write down the lengths of one side and of the diagonal of the square.

13. Construct a square whose area is 7 sq. inches, and trisect it by straight lines drawn from the middle point of one of the sides.

14. Make a rectangle whose area is $7\frac{1}{2}$ sq. inches, and reduce it to a square of the same area. Measure the diagonal of the square.

15. Construct a triangle whose angles are in the ratio of 3 : 4 : 5 and perimeter 7″, and bisect by a line perpendicular to its longest side.

16. Draw a triangle whose sides are $4\frac{1}{4}$″, $2\frac{1}{4}$″ and 3″ respectively, and bisect it by a line drawn perpendicular to its longest side. What is the area of the whole triangle?

17. Construct a quadrant of a circle equal to a given semi-circle, whose diameter is 2″.

MISCELLANEOUS QUESTIONS ON PROPORTIONALS.

Exercise XXXIII.

1. Divide a line $4\frac{1}{2}''$ long into three parts in the ratio 1 : 2 : 3, and find a fourth proportional to the three parts.

2. Find a 4th proportional to the lines 3·3", 2·1" and 2·2"; also a mean proportional between the lines 2·4" and 1·8". Measure them.

3. Taking the line one inch as the unit, obtain, by geometrical construction, lines equal to $\sqrt{5}$ and $2\sqrt{2}$. Scale and write down against each line so found its length in inches and decimals of an inch.

4. Find a third proportional (greater) to lines of $1\frac{1}{4}''$, 2". Draw a line of length $\sqrt{7}''$.

5. Find by construction a third proportional to two straight lines $1\frac{1}{3}''$ and $1\frac{3}{4}''$ long; and a fourth proportional to three straight lines $1\frac{1}{4}''$, $2\frac{1}{3}''$, $1\frac{5}{8}''$ long. Write down the length of each line so found.

6. PQ, QR, RT are respectively $2\frac{1}{2}''$, 2" and $1\frac{1}{2}''$. Find a line AB so that PQ : QR : : RT : AB, and construct a four-sided figure having one angle 45°, and its sides equal to these four lines.

7. If (*a*) 3·5" : 2·75" : : 2·5" : AB,
and (*b*) 2·8" : BC : : BC : 1·3",
find AB and BC by geometrical constructions.

8.

A ——————————————————————

B ——————————————————

C ——————————————————————————

D ——————————

E ——————————

F ——————————————————

G ——————————————————————————

(α) Find a 4th proportional to A, B and C in inches.
(β) „ 4th „ „ C, D and E „ „
(γ) „ 4th „ „ D, E and F „ centimetres.
(δ) „ 3rd „ „ A and B „ „
(ε) „ 3rd „ „ B and A „ „
(ζ) „ 3rd „ „ G and F „ inches.
(η) „ mean „ between C and D „ „
(θ) „ mean „ „ E and G „ „
(ι) „ mean „ „ A and F „ centimetres.

CONSTRUCTION OF ANGLES.

Exercise XXXIV.

In this exercise the construction must be STRICTLY *geometrical. Division by trial, or use of the protractor, is not allowed.*

1. Construct an isosceles triangle whose base angles are double of the vertical angle.

2. Construct angles 18°, 36°, 54°, 72°.

3. Construct angles 48°, 66°, 84°, 102°.

4. Construct angles 63°, 87°, 153°.

5. Construct a triangle with base angles 51° and 69° and base 3″. Determine its area.

6. Construct a regular pentagon in a circle, radius 2″.

7. Construct a regular pentagon with sides 5 cm.

INCREASING AND DECREASING GIVEN FIGURES TO SIMILAR FIGURES IN A GIVEN RATIO.

Exercise XXXV.

"The homologous sides of similar figures are to one another in the subduplicate ratio of their areas."

1. Draw a triangle with sides 10 cm., 9 cm. and 7 cm., and construct a similar triangle $\frac{2}{3}$ the area.

2. Draw a triangle with two sides 3·63″ and 2·95″ and included angle 60°. Construct a similar triangle $\frac{4}{5}$ the area.

3. Draw a triangle with perimeter 10″ and sides in the ratio 2 : 4 : 5. Divide it into three equal parts by lines parallel to the shortest side.

4. Make an isosceles triangle with base 3″ and vertical angle 50° and construct another similar triangle $1\frac{1}{3}$ times as large.

5. Draw an isosceles triangle on a base 7·3 cm., having vertical angle 40°. Construct also a similar triangle $\frac{5}{4}$ the size.

6. Construct an isosceles triangle whose base is 2·4″ and area 5 sq. inches. Divide it in the ratio of 2 : 3 by lines parallel to the base.

7. Describe a triangle with perimeter 6·8″ and its sides in the ratio 2 : 3 : 4 ; and make a similar triangle of half its area.

8. Draw an isosceles triangle whose base is 2·5″, and vertical angle is 48°. Divide it into three equal parts by lines parallel to the base.

9. Describe an isosceles triangle having a base 3″ and altitude $\sqrt{10}$″; and cut off a quarter of the triangle by a line parallel to the base.

10. Draw a circle whose diameter is 7 cm. and divide it into four equal concentric annuli.

11. Construct a quadrant of a circle, radius 2·5″, and divide it into three equal parts by arcs having the same centre as the arc of the original quadrant.

12. Describe a circle of 3·2″ diameter, and another concentric circle of $\frac{3}{4}$ of its area.

13. Construct a parallelogram, given two sides 2″ and 2·75″ and included angle 60°. Divide it into three equal parts by lines parallel to the shorter diagonal.

14. In a square, side 2·8″, inscribe a second square whose area is $\frac{2}{3}$ that of the other.

15. Construct a quadrilateral ABCD, given AB=3·2″, BC=2·15″, CD=3·7″, AC=4·3″ and the angle CAD=42°. Make a similar quadrilateral $\frac{2}{3}$ its area.

16. Draw a regular pentagon on a side of 2″, and place centrally inside it another having an area $\frac{1}{3}$ of the area of the former.

17. In a circle, radius 2·7″, inscribe a regular pentagon, and draw a second whose area shall be $\frac{2}{3}$ of the former.

18. On a line 2·1″ construct a regular pentagon, and make a similar figure half as large again.

19. Make a regular pentagon whose sides are 4·8 cm. and another whose area is half as large again.

20. Describe a regular hexagon in a circle whose radius is 5·1 cm. and construct a similar figure half the area.

21. Construct the irregular figure ABDC, having given AB=2·73″, ∠BAC=120°, ∠BAD=45°, ∠ABC=30°, and ∠ABD=90° (all angles to be made without the use of the protractor) and make a figure similar to it, and half as large again.

REDUCTION OF RECTILINEAL FIGURES TO SQUARES OF EQUAL AREA.

Exercise XXXVI.

1. Draw a triangle with sides 10·3 cm., 7·3 cm. and 5·8 cm. and construct a square of equal area. Measure its sides.

2. Draw a triangle with base 7·9 cm. and base angles 75° and 60°. Make a square of equal area. Measure its diagonals.

3. A figure is composed of a square, whose sides are 7·2 cm., surmounted by an equilateral triangle. Construct a square of equal area. Measure its perimeter.

4. Construct an equilateral triangle, the vertical height of which is 2·5″. Draw a square equal to it in area. Measure its diagonals.

5. Construct a rectangular parallelogram with sides of 2·4″ and 1·8″, and construct a square of equal area. Measure its sides.

6. Construct a parallelogram with sides of 3·4″ and 2·6″, one angle being 40°. Construct a square of equal area with the parallelogram. Measure its sides.

7. Describe a triangle having its sides $2\frac{3}{4}''$, $2\frac{1}{2}''$, and $1\frac{3}{4}''$ respectively, and construct a square equal to it in area. Measure its diagonals.

8. Construct the figure ABCD, having given AB=3″, ∠ABC=108°, ∠ABD=42°, ∠BAC=30° and ∠BAD=82°. Measure and write down the lengths of AD, DC and CB. Find the length of a side of a square of equal area.

9. Construct two different triangles with data:—AB=$4\frac{1}{4}''$, AC=3″, angle ABC=35°; and make a square equal to the sum of their areas, measure its sides.

10. On one side of a line 1″ long construct a regular heptagon, and on the other side a square, and make a square equal to the sum of the areas of these figures, measure its diagonals.

11. Construct the figure ABCDE, having given AB=2·5″, AD=3·6″, AE=2·46″, DE=2·58″, ∠BAC=27°, ∠DAC=25°, ∠ABC=45°. Reduce it to an equivalent triangle and make a square equal to this triangle. Measure the side of the square.

REDUCTION OF RECTILINEAL FIGURES TO TRIANGLES OF GIVEN SHAPE.

Exercise XXXVII.

1. Draw a triangle with sides 7, 8 and 9 cm., and construct an equilateral triangle of the same area. Measure its side.

2. Draw a triangle with two sides 4·31″ and 3·25″ and included angle 30°. Construct an isosceles triangle, with vertical angle 75°, of the same area.

3. Draw a triangle whose *angles* are in the ratio 2 : 3 : 4 and whose perimeter is 7″. Construct another triangle, whose *sides* are in the ratio 2 : 3 : 4, and of the same area as the former.

4. Convert a rhombus, sides 5·7 cm., containing an angle 40°, into an equilateral triangle of the same area. What is the length of the side of the triangle?

5. Reduce a square, with diagonal $2\frac{1}{2}$″, to an equilateral triangle of the same area.

6. Construct a rhombus, with one angle double of the other, equal in area to a square whose sides are 2·3″.

7. A figure is formed of a square of side $2\frac{1}{4}$″, surmounted by an equilateral triangle. Reduce the figure to an equilateral triangle of the same area. Measure its altitude.

8. Describe a rhombus, side $2\frac{1}{2}$″, and one angle 45°. Construct an equal equilateral triangle.

9. Describe a regular pentagon of $6\frac{1}{4}$ sq. inches area. Mark all measurements.

10. Construct a rectangle whose sides are $\sqrt{5}$″ and $\sqrt{6}$″ and reduce it to an isosceles triangle, of equal area, whose vertical angle is 40°. Measure its base.

11. Construct a square equal in area to two other squares, whose sides are 1·8″ and 2·2″, and make an equilateral triangle of one quarter the area. Measure its sides.

12. Make a parallelogram having sides $3\frac{1}{4}$″ and $1\frac{1}{4}$″, and the angle between them 45°. Construct a triangle, of equal area, with sides in ratio 8 : 9 : 7. Measure the longest side.

13. Construct an equilateral triangle equal in area to $\frac{1}{4}$ of the heptagon, whose sides are 1·75″.

14. Draw a regular pentagon with sides 2″, and construct a regular hexagon of equal area. Measure its sides.

15. Draw a regular pentagon in a circle, radius 5 cm., and construct a regular hexagon of equal area. Measure its sides.

16. Draw a regular hexagon with sides 2·1″, and construct a regular octagon of equal area. Measure its sides.

17. Draw a regular heptagon in a circle, radius 6 cm., and make a regular pentagon of the same area. Measure its sides.

18. Draw a regular pentagon of area 36 sq. cm. Measure its sides.

19. Draw a regular hexagon of area 45 sq. cm. Measure its sides.

20. Construct a regular heptagon of area 12 sq. in. Measure its sides.

21. Construct a regular octagon whose area is 11·2 sq. in. Measure its sides.

CIRCLES PASSING THROUGH TWO GIVEN POINTS AND TOUCHING A GIVEN STRAIGHT LINE OR CIRCLE.

Exercise XXXVIII.

"If from any point without a circle a tangent and a secant are drawn, the rectangle contained by the whole secant and the part of it without the circle is equal to the square on the tangent."

1. Describe a circle passing through two given points, and touching a given straight line. Is this always possible?

2. AB is a straight line. P and Q are two points on the same side of AB, distant $\frac{3}{4}$″ and $1\frac{1}{2}$″ respectively from it. PQ=2″. Describe a circle which shall pass through P and Q and touch AB.

3. Two points, C and D (1·4″ apart) are respectively 1·6″ and ·45″ from the given straight line XY. Draw two circles to pass through C and D and to touch XY.

4. Draw two straight lines AB, AC of indefinite length, the $\angle$BAC being 50°. Take a point D in AB, 3″ from A, and a point E in AC 2·25″ from A. Describe a circle which shall touch AB in D, and cut AC in E, and a second point F (to be determined). Measure AF, and show that $AE \times AF = AD^2$.

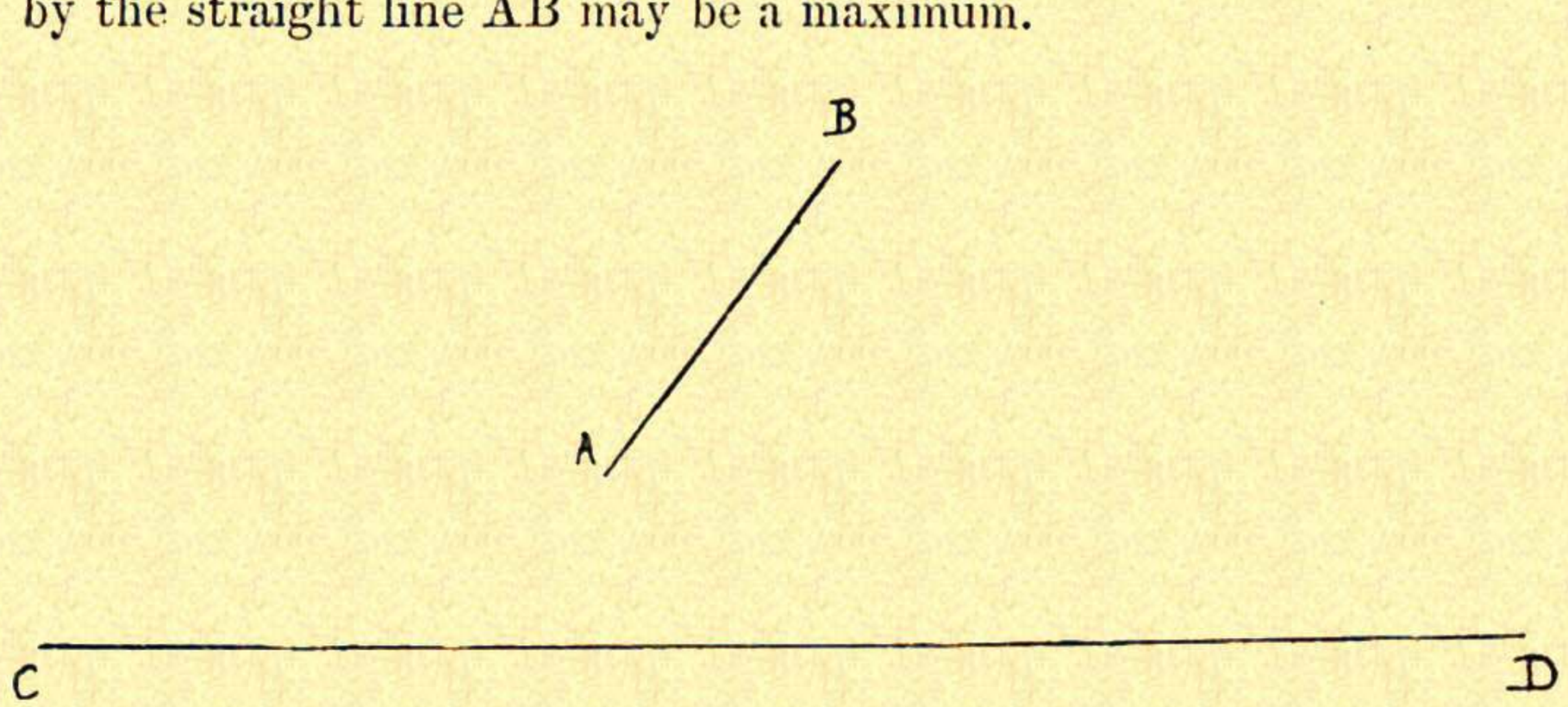
5. Draw a circle 3″ diameter. On a diameter produced mark two points 1·75″ and 3·5″ respectively from the centre, on opposite sides of it. Draw a circle to touch the first circle and pass through the two points.

6. Two points are at distances of 2·5 cm. and 3·8 cm. from a given straight line, such that their distance from each other is 3·2 cm. Draw a circle through the two points so as to touch the given straight line.

7. Show the construction for drawing a tangent to a circular arc of large radius (whose centre is out of reach) from a point outside the circumference.

8. AP (2″) touches a circle at P. ABC is a secant, AB being $1\frac{1}{2}$″, B and C being on the circle. Give a construction for finding C.

9. Find a point in CD such that the angle subtended thereat by the straight line AB may be a maximum.



10. Draw a circle (centre O) with radius 2 cm. and take two points A and B such that OA=3 cm., OB=4 cm. AB=3·7 cm. Draw two circles to pass through A and B and touch the given circle.

11. Draw a circle (centre O) with radius 1″, and take two points P and Q such that OP=1·6″, OQ=2·3″ and PQ=2″. Draw two circles to pass through P and Q and to touch the given circle.

CENTRE OF SIMILITUDE. APPLICATIONS.

Exercise XXXIX.

1. Draw a triangle ABC in which AB=10 cm., BC=9 cm., CA=8 cm. In it inscribe an equilateral triangle with one side parallel to AB.

2. Draw a triangle DEF.

DE=4″, EF=3·5″, ∠DEF=55°.

In it inscribe a triangle similar to ABC in the previous question, having its longest side inclined at an angle 50° to DE.

3. Describe a triangle having given the base 2″, the sum of the two sides $4\frac{1}{2}$″, and one angle at the base 55°. In this triangle inscribe an equilateral triangle, with one side inclined at an angle 25° to its base.

4. Construct a triangle whose sides are in the ratio of 4 : 5 : 6, and whose inscribed circle has a radius of 1·2″.

5. Draw a right-angled triangle, with sides 2″ and 3″. Inscribe in it an equilateral triangle situated so that one side is parallel to the hypotenuse of the right-angled triangle.

6. The sides of a triangle are in the ratio of the numbers 7, 10 and 12, and the radius of the inscribed circle is 1″. Draw the triangle.

CENTRE OF SIMILITUDE. INSCRIPTION OF SQUARES IN VARIOUS FIGURES.

Exercise XL.

1. Draw a triangle with sides 10·3 cm., 8·7 cm., 7·2 cm. Inscribe the largest possible square. Measure its diagonal.

2. Draw a triangle, base 3″, base angles 75°, 60°. Inscribe a square, one of whose sides is in the base of the triangle. Measure the diagonal.

3. Draw the triangle ABC.

AB=7·9 cm., BC=8·4 cm., ∠ABC=45°.

Inscribe a square, one of whose sides is in AC. Measure its side.

4. Draw a segment of a circle on a chord $3\frac{1}{2}$″, with angle 105°. Inscribe a square. Measure its side.

5. Draw a circle with radius 5 cm. In it place a chord 8 cms long. In the larger segment inscribe a square. Measure it. perimeter.

6. Draw a sector, radius $3\frac{1}{2}''$, angle 30°. Inscribe a square. Measure its diagonals.

7. Draw a sector, radius 8·4 cm., angle 120°. Inscribe a square. Measure the sides.

8. Draw a circle, radius $2\frac{1}{2}''$; inscribe an equilateral triangle. In the three segments inscribe squares.

9. Draw a circle, radius 8 cm.; inscribe in it six squares. Each square has two corners on the circle, all the remaining corners form the angles of a regular hexagon.

10. Draw an equilateral triangle with side 3″. On each side, externally, draw a semicircle. In each semicircle inscribe a square.

11. Draw a regular pentagon in a circle, radius 7 cm. In it inscribe a square. Measure the sides.

12. Draw a regular pentagon with side 2″. In it inscribe a square. Measure the diagonals.

13. Construct a triangle with sides $\sqrt{7}''$, $\sqrt{5}''$, $\sqrt{8}''$ (obtained geometrically) and inscribe a square.

14. Construct a kite with two of its sides 3″ and two 2″ respectively, and with its longer diagonal $3\frac{3}{4}''$; inscribe within it a square.

15. Draw a quadrant of a circle, radius 3″. Inscribe a square in it. Measure its diagonals.

16. Describe the isosceles triangle ABC, having given that the base BC=2·49″ and $\angle$BAC=45°. In it inscribe a square.

17. In an equilateral triangle whose side is 3″, inscribe a square, and in the square inscribe an equilateral triangle.

18. Construct a kite ABCD having BD=2·9″, the sides AB, AD=2·6″ each, and the sides CB, CD=4·1″ each. Inscribe a square in the figure.

19. On a line 2·35″ as base describe a regular pentagon and in it inscribe a square.

20. Draw a rhombus having sides $3\frac{1}{2}''$ and one angle 60°. In this place a square. On each side of this square and outside it draw another square as large as the bounding lines of the rhombus will permit.

21. Draw the kite PQRS.

. PR=8 cm., PQ=PS=4 cm., RQ=RS=6·2 cm.

In it inscribe a square. Measure its sides.

22. Construct a triangle, with perimeter 7″, and sides in the ratio of 2 : 3 : 4. In it inscribe a square.

23. On a base 4″, describe a segment of a circle containing 65°. In this segment inscribe a square.

24. Describe a quadrant with a radius of 3·15″, and in it inscribe a square having two of its corners in the arc of the quadrant.

25. With a radius of $2\frac{1}{4}$″ describe a semicircle, and in it inscribe a square having two of its corners in the arc of the semicircle, and the side non-adjacent to these upon the bounding diameter.

26. Describe a triangle having its sides $4\frac{1}{2}$″, $3\frac{3}{4}$″ and $2\frac{1}{2}$″ respectively, and in it inscribe a square.

27. In a circle, radius 2·34″, inscribe the regular hexagon ABCDEF, and the equilateral triangle BDF. In each of the triangles FAB, BCD, DEF and BDF inscribe squares.

CENTRE OF SIMILITUDE. CIRCLE TO TOUCH TWO GIVEN STRAIGHT LINES AND TO PASS THROUGH A GIVEN POINT.

Exercise XLI.

1. Two straight lines meet each other at an angle of 36°, and a point lies between them not in the bisector of the angle. Draw a circle so as to touch the given straight lines, and pass through the given point.

2. Draw two straight lines meeting at an angle 75°. Through a point distant ·9″ from the vertex and ·3″ from one line draw two circles each touching both lines.

3. Draw two straight lines at an angle of $52\frac{1}{2}$° (to be obtained by construction), and find a point distant 1·22″ and 1·97″ from them respectively. Draw a circle to pass through this point and to touch both lines.

4. Draw two straight lines inclined at an angle 35°. Using the marquois scales find a point P ·3″ from one and ·1″ from the other. Describe two circles to pass through P and to touch the given straight lines.

5. AB, AC are two straight lines inclined at an angle of 70°; P is a point 2″ from AB and 1″ from AC, describe a circle to pass through P and touch AB and AC.

CENTRE OF SIMILITUDE. INSCRIPTION OF SEMICIRCLES IN VARIOUS FIGURES.

Exercise XLII.

1. Draw a square with side 8·9 cm. and inscribe a semicircle whose diameter is parallel to one of the diagonals of the square.

2. Draw a square with diagonals 11·3 cm. and inscribe a semicircle in it with diameter parallel to one of the diagonals.

3. Draw a kite PQRS. Diagonal PR=3″. PQ=PS=$1\frac{3}{4}$″, QR=RS=$2\frac{1}{4}$″. Inscribe in it a semicircle touching PQ and PS, with the extremities of its diameter on QR and RS.

4. Draw a regular pentagon with side 2″. Inscribe in it a semicircle, with its diameter parallel to one side.

5. Draw a regular pentagon in a circle, radius 5 cm. Inscribe in it a semicircle, with its diameter parallel to one side.

FOILED FIGURES, COMPOSED OF SEMICIRCLES WITH ADJACENT DIAMETERS, INSCRIBED IN REGULAR FIGURES, EACH SEMICIRCLE TOUCHING TWO SIDES OF THE REGULAR FIGURE.

Exercise XLIII.

1. In an equilateral triangle, sides 4″, inscribe a trefoil of semicircles with adjacent diameters, each semicircle touching two sides of the triangle.

2. In an equilateral triangle whose sides are $\sqrt{90}$ cm. (obtained geometrically) inscribe three equal semicircles, with their diameters adjacent, each touching two sides of the triangle.

3. In a square, side 3·5″, place four equal semicircles with their diameters adjacent, each semicircle touching two sides of the square.

4. In a square, whose area is 10 sq. inches, inscribe four equal semicircles with their diameters adjacent, each semicircle touching two sides of the square.

5. In a regular pentagon with sides 6 cm. inscribe a cinquefoil of semicircles with adjacent diameters, each semicircle touching two sides of the pentagon.

6. Inscribe six equal semicircles in a regular hexagon (sides 7 cm.) with diameters adjacent, each semicircle touching two sides of the hexagon.

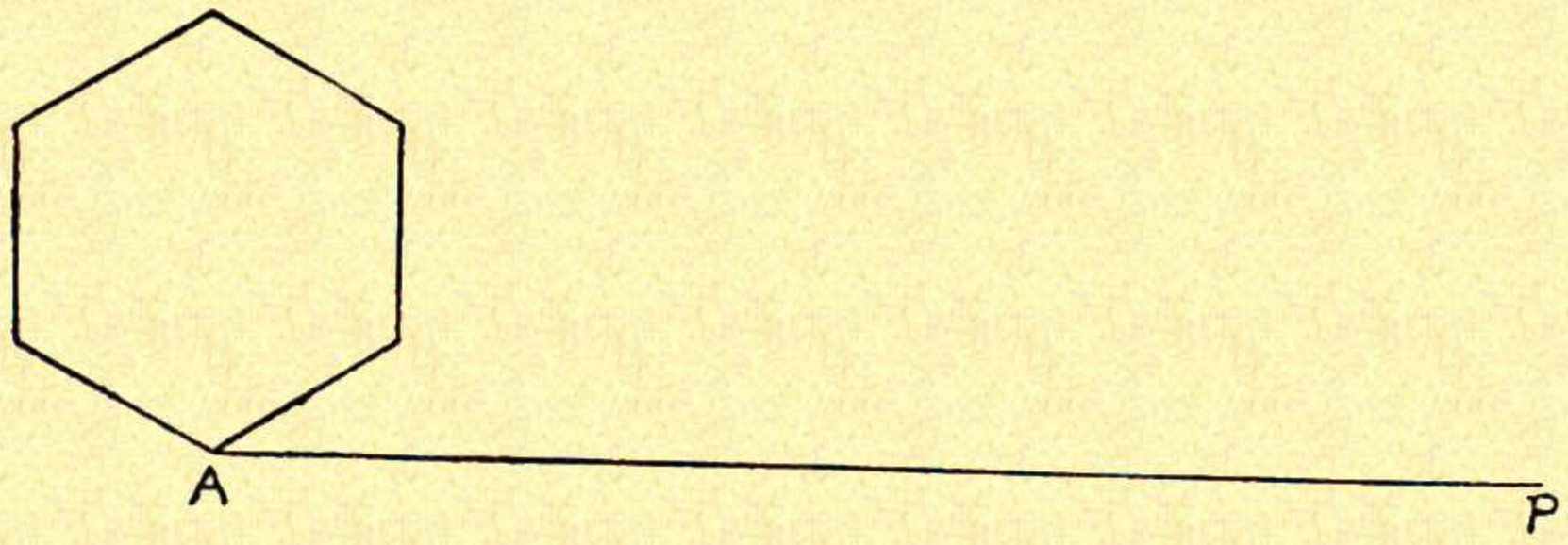
7. Inscribe seven equal semicircles (with diameters adjacent) in a regular heptagon whose sides are 2·4″, each semicircle touching two sides of the heptagon.

LOCI.

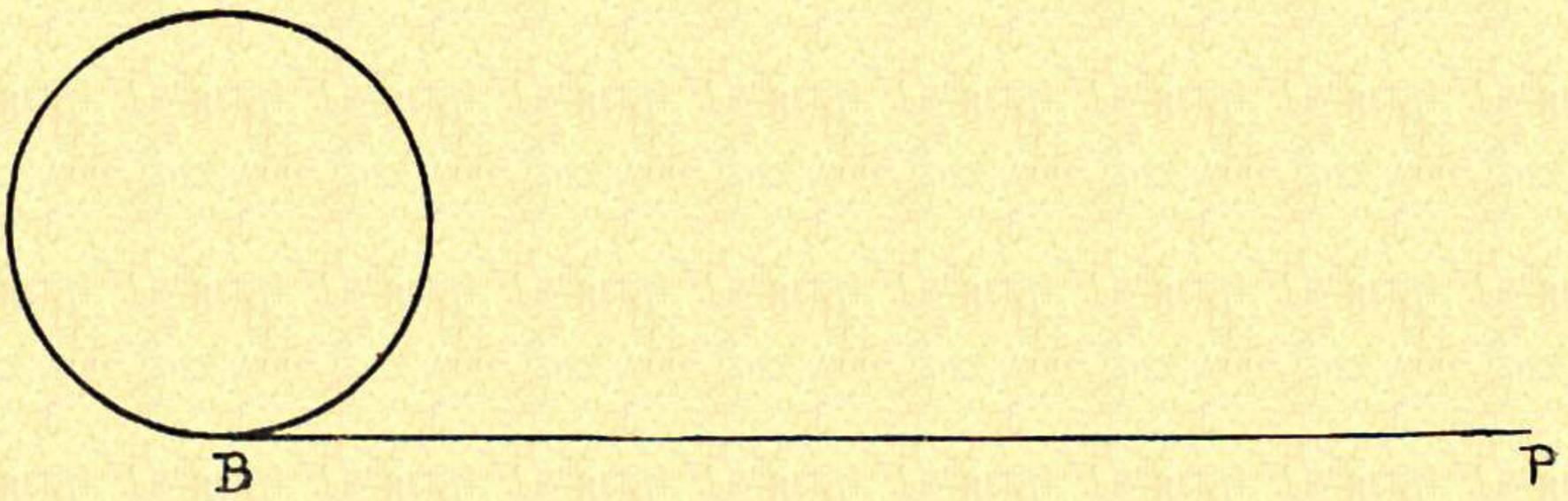
Exercise XLIV.

The constructions should be in pencil. Only the original figure and the locus should be inked in.

1. A string AP, equal in length to the perimeter of the regular hexagon, is attached to the angle A. It is kept taut and wound round the circumference of the hexagon. Find the locus of its extremity P.



2. A string PB, equal in length to the circumference of the circle (diameter 1″) is attached to the point B. It is kept taut and wound round the circumference. Find the locus of its extremity P.

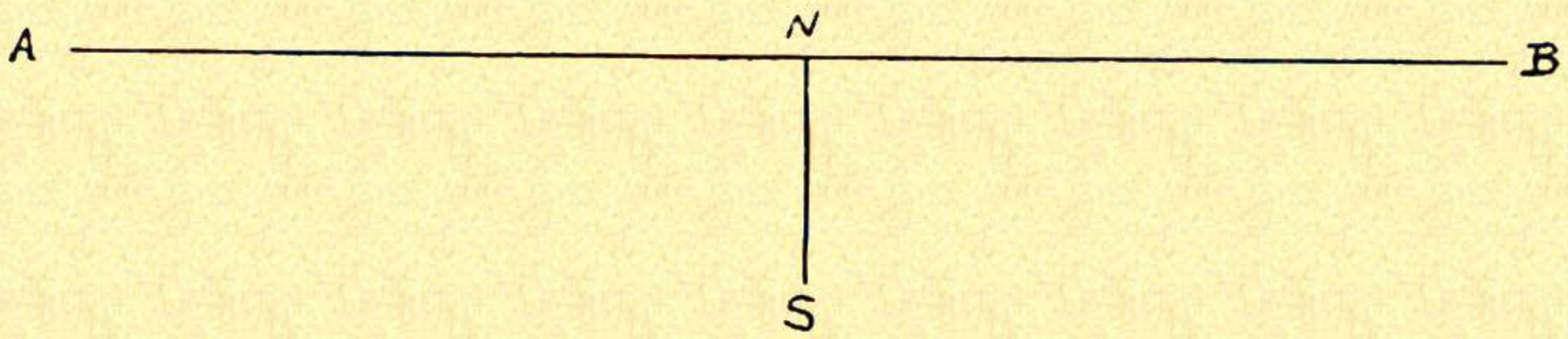


3. A and B are fixed points $4\frac{1}{2}$″ apart. Find the locus of P, a point which moves so that the sum of its distances from A and B is always 6″.

4. Draw a circle with radius 2·3″. Take a point R 1·3″ from its centre. Find the locus of the middle points of all chords of the circle which pass through R.

5. Draw a circle with radius 5 cm. Take a point S 7 cm. from its centre. Find the locus of the middle points of all chords which (when produced) pass through S.

6. Find the locus of a point which moves so that its distance from the given straight unlimited line AB is equal to its distance from the given point S. [Take SN $\frac{3}{4}''$ long.]



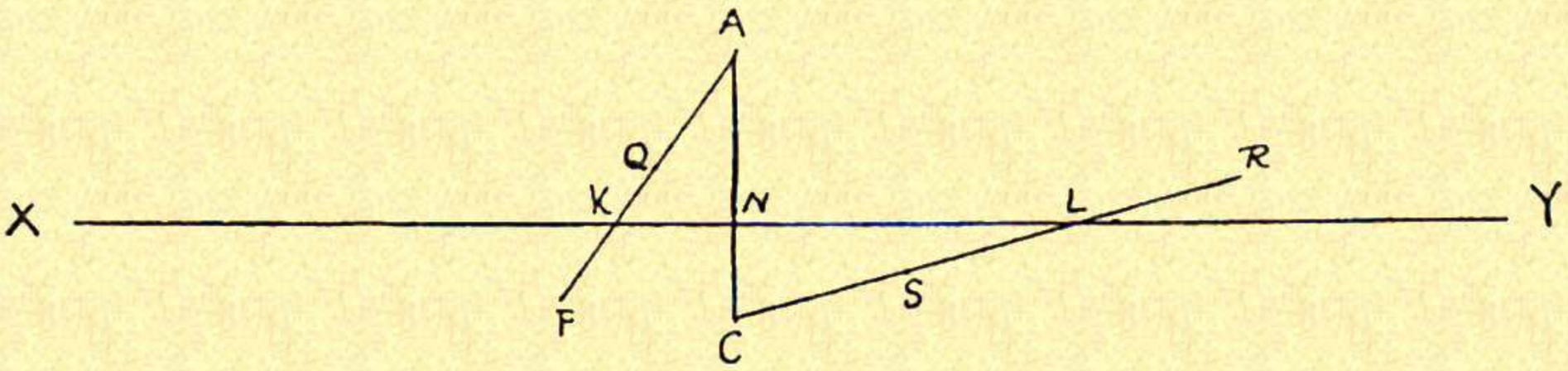
7. Find the locus of a point P, which moves so that its distance SP from the fixed point S is equal to *half* of its distance from the given straight line AB.

[Use the figure of question 6, but take SN 3″ long.]

8. Find the locus of a point P, which moves so that its distance SP from the fixed point S is equal to *twice* its distance from the given straight line AB.

[Use the figure of question 6, but take SN $\frac{1}{2}''$ long.]

9. XY and ANC are fixed straight lines. AQKP is a variable straight line which passes through A and cuts XY at K. If KP=KQ=NC, find the loci of P and Q. [Take AN=1″, NC=·5″.]

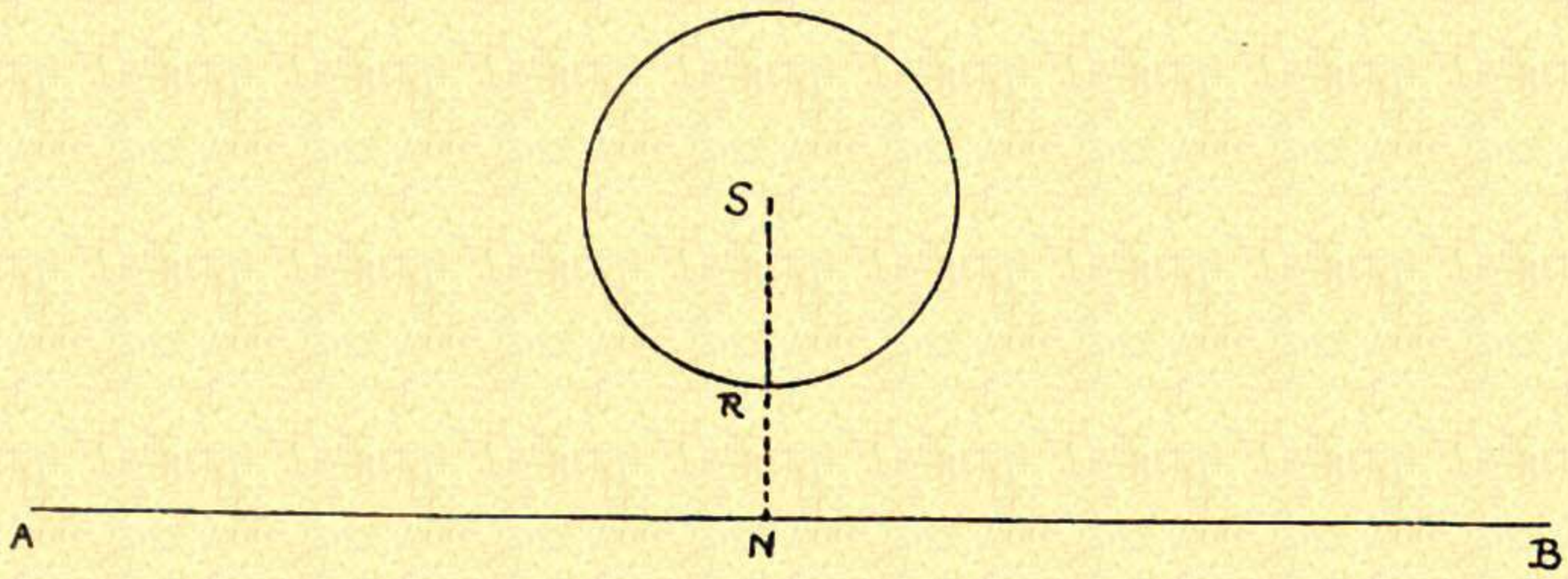


10. Using the figure of the previous question, CSLR is a variable straight line passing through C and cutting XY at L. If LS=LR=AN, find the loci of R and S.

11. If S and H are two fixed points 4″ apart, find the locus of P which moves so that the area of the rectangle contained by SP and HP $\doteq$ 5 sq. inches.

12. Find the locus of a point which moves so that the rectangle under its distances from two given straight lines, at right angles to each other, is constant and equal to $\frac{3}{10}$ of a sq. inch.

13. A point P moves so that its distance from a fixed straight line AB=its distance from the circumference of the circle whose centre is S. Find the locus of P. (SR=1·2″ and SN=2″.)

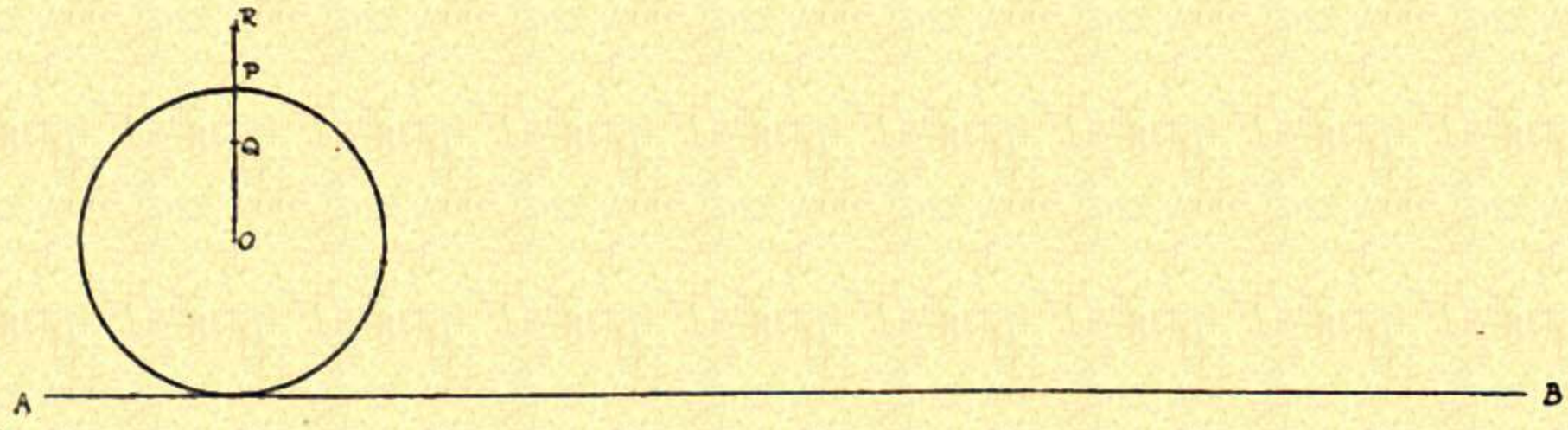


14. AB, BC represent two rods hinged together at B. AB revolves about a pivot at A, and at the same time BC revolves in the same direction with double the speed, about the hinge B; consequently BC always makes with AB produced the same angle that AB makes with the fixed line AX.

Draw lines showing the positions of the rods when AB has revolved 15°, 30°, 45°, 60°, 75° and 90°; and draw a fair curve through the successive positions of C.

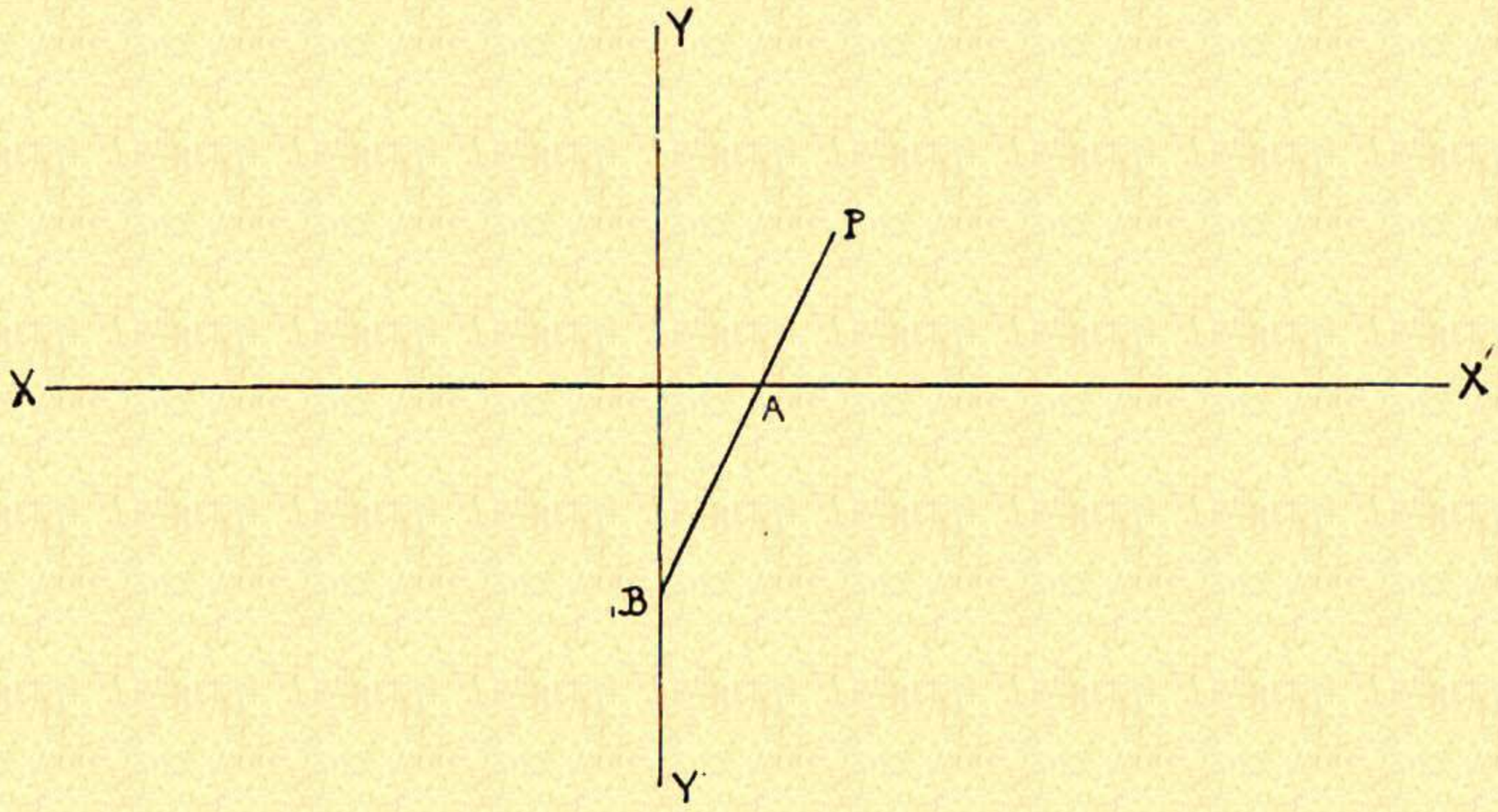
A B C X

15. Find the locus of P on the circumference of the given circle which rolls along the given straight line AB, without sliding, for one complete revolution. [Radius of the circle=2 cm. $\pi=\frac{22}{7}$.]



16. A ladder, 25′ long, resting against a vertical wall begins to slip so that its extremities keep in contact with the wall and the ground. Find the locus of a point P on a rung 10′ from the bottom of the ladder. (The ladder is supposed to start slipping from the vertical position.) Scale 5′ to 1″.

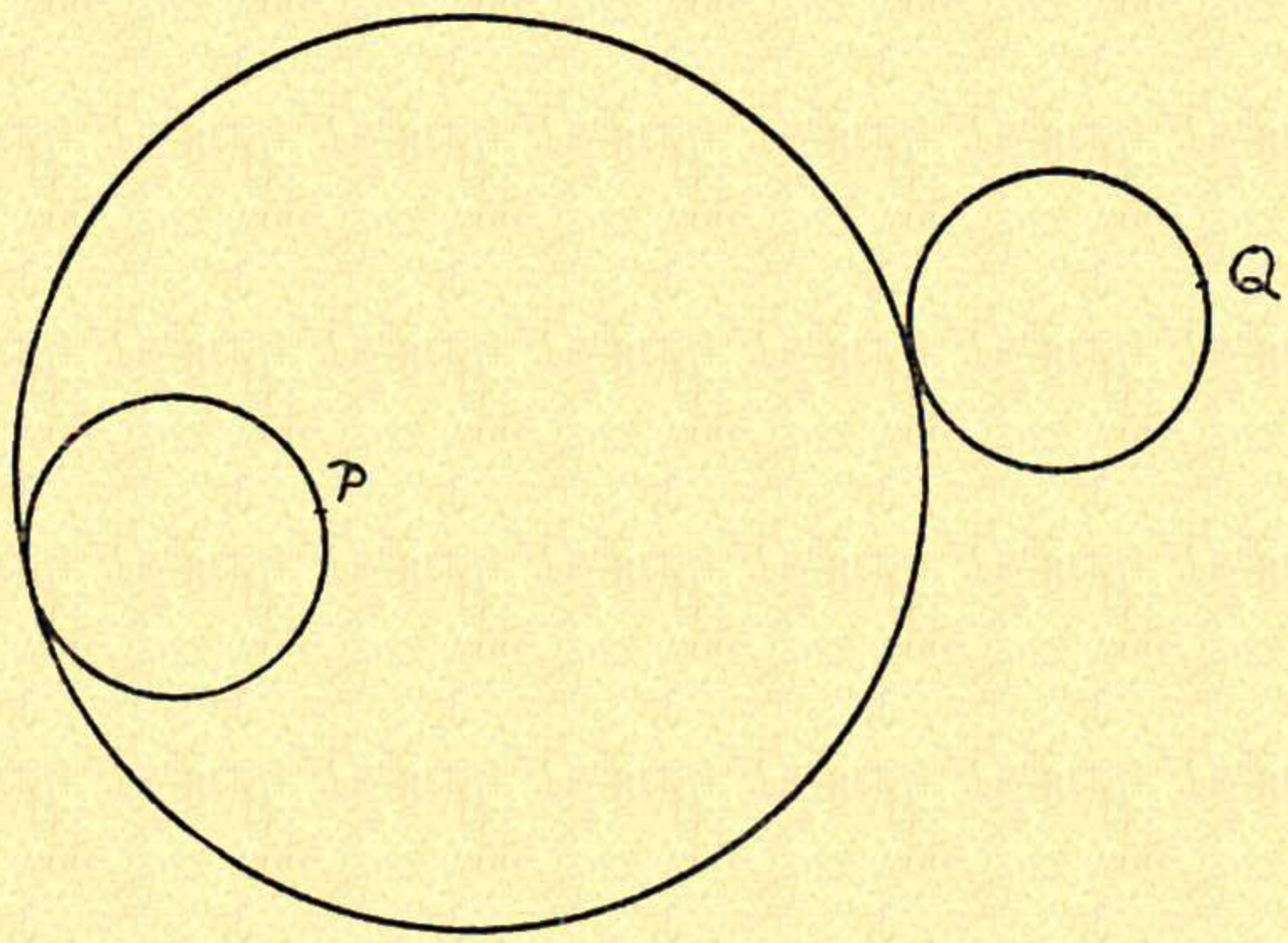
17. A and B (fixed points on the straight line BAP) move along the straight lines XX′, YY′, respectively. If P is also a fixed point on the line, find its locus. [BA=4 cm. and AP=3 cm.]



18. Find the locus of Q on a fixed radius of the circle in the figure of question 17. [PQ=7 mm.]

19. Find the locus of R on a fixed radius (produced) of the circle in the figure of question 17. [PR=7 mm.]

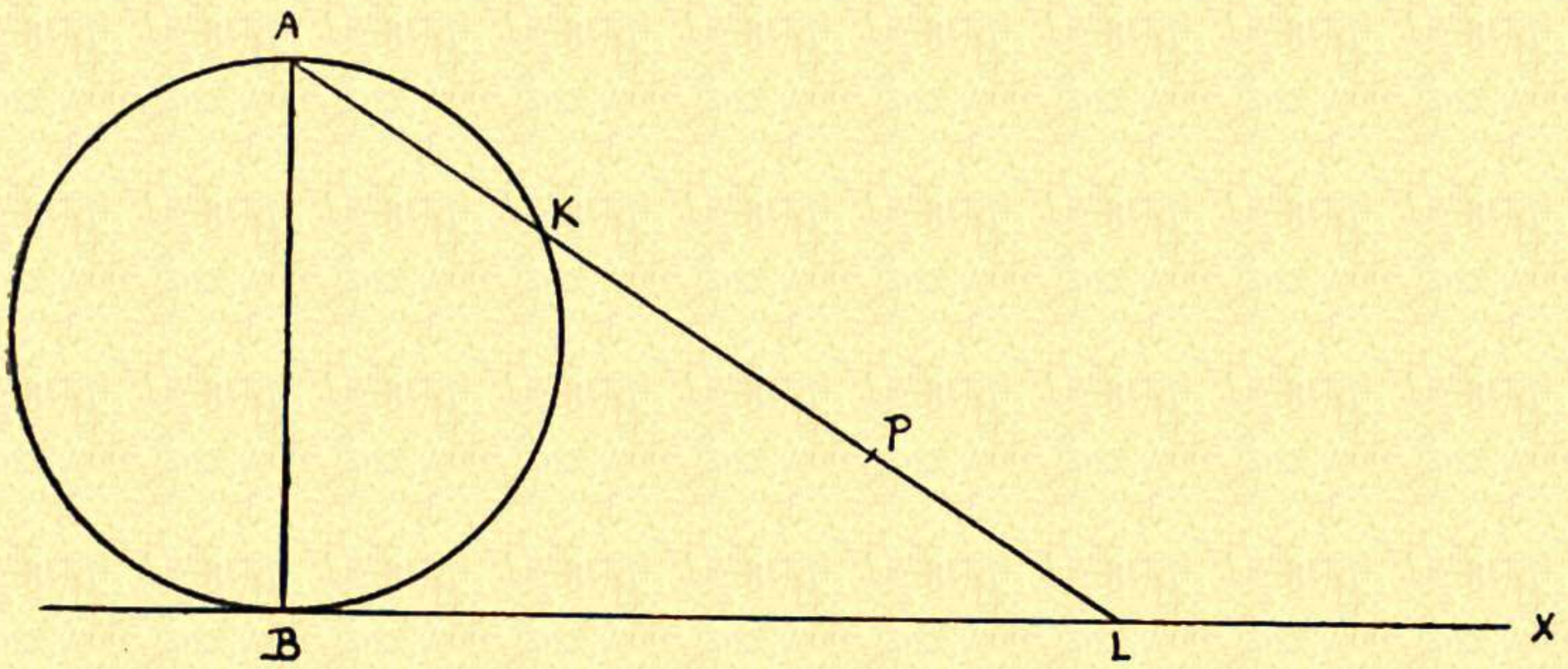
20. The larger circle (radius 2·4″) is fixed and the smaller circle (radius ·8″) rolls on the *inside* of the large circumference. Find the locus of P, a fixed point on the circumference of the smaller circle.



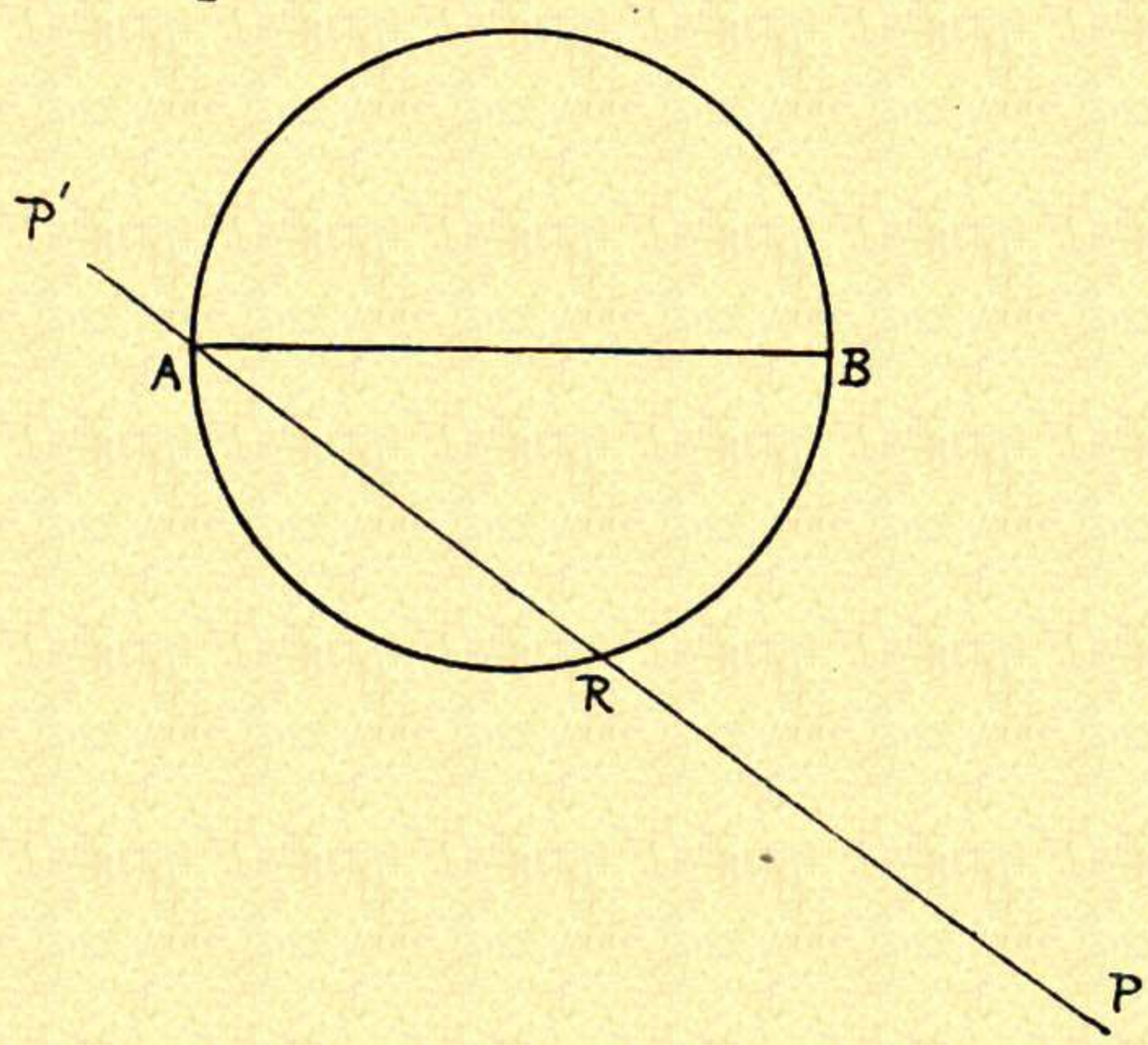
21. Using the figure of question 20, find the locus of Q, a fixed point on the circumference, of the small circle which rolls on the *outside* of the large circumference. [Take radii 4·2 cm. and 1·4 cm.]

22. Using the figure of question 17. If the circle represents a cartwheel rolling on the straight road AB, find the path (in space) of a fly which starts from the hub O and walks along the spoke OP reaching the rim of the wheel, when the latter has made one complete revolution.

23. AB is a fixed diameter of a circle and BX a fixed tangent. Through A a variable straight line AKPL is drawn meeting BX in L and the circle in K. On it PL is set off equal to AK. Find the locus of P. [AB=2″.]



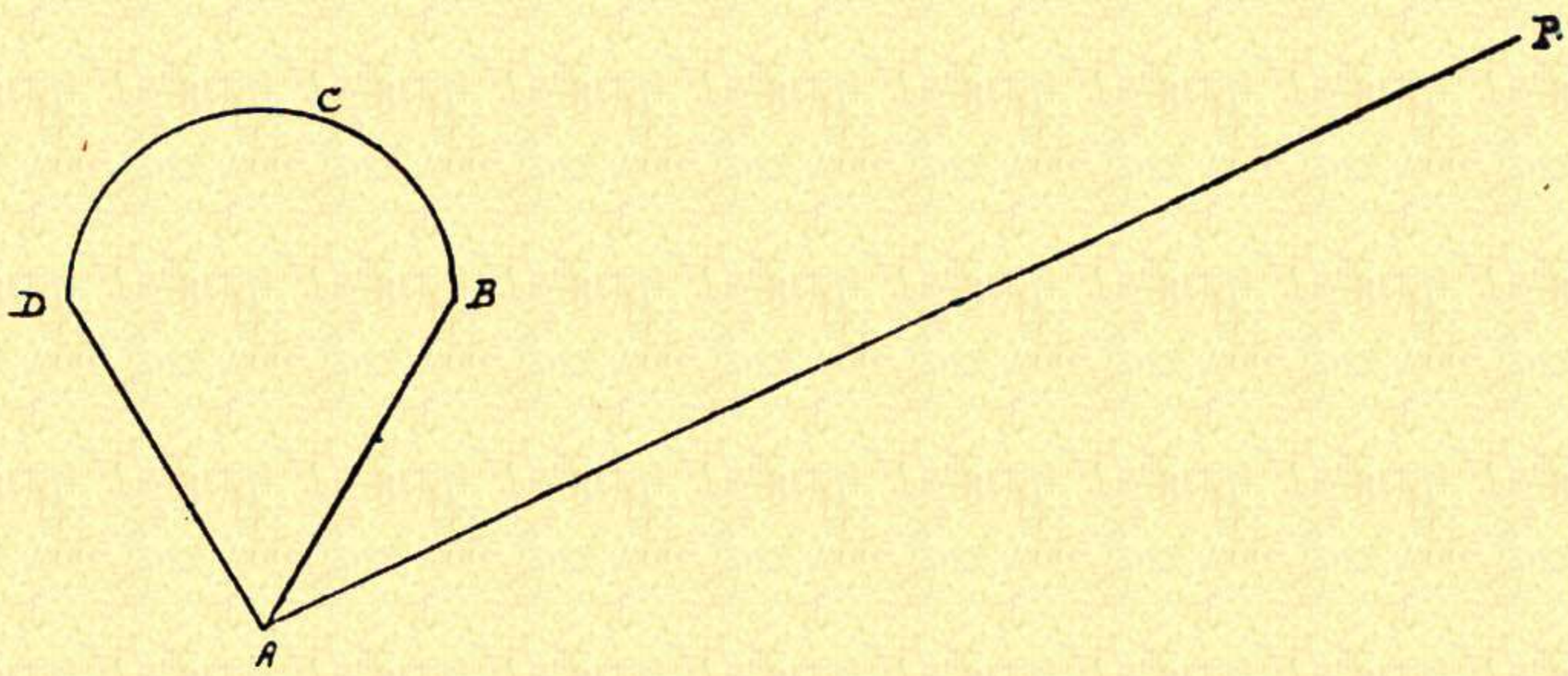
24. AB is a fixed diameter of a circle. Through A a variable straight line ARP is drawn and RP=AB. Find the locus of P. [N.B.—RP may be measured either way, so that the locus of P′ must be included.]



25. A train is travelling along a straight line at the rate of 9 miles an hour: a carriage door whose width is 30″ swings open uniformly until it makes an angle of 150° with the train, and then closes uniformly, the whole process taking 1 second. Trace the

path of a point on edge of the door (away from the hinge) during this second.

26. A string AP is attached to the point A of the figure, shown below, composed of an equilateral triangle and a semicircle. The string is kept taut and is wound round the perimeter of ABCD. The string is equal in length to the perimeter and starts from the position shown. Find the locus of P.



CONSTRUCTION OF TRIANGLES, &c., WITH GIVEN PARTS.

Exercise XLV.

1. The base of a triangle is 3″, its height is 1·9″, its vertical angle is 60°. Construct the triangle.

2. On a base of $2\frac{1}{2}$″ describe a triangle having its vertical angle 40° and its altitude $1\frac{3}{4}$″.

3. Construct a triangle on a base of 3·5″, having its vertical angle 70°, and its altitude 2·25″.

4. Construct a triangle, whose base is 3″, altitude 2″, and vertical angle 40°. Divide it in the ratio of 2 : 3 by a line parallel to the base.

5. The base of a triangle is 3·25″, one of the angles at the base 45°, and the sum of the other two sides 5·6″. Construct the triangle.

6. Draw a triangle with area 5 sq. inches and base 3″, vertical angle 39°. Measure the longest side.

7. Construct a triangle on base 3″, with vertical angle 75° and perimeter $7\frac{1}{2}$″. Measure the base angles.

8. Construct a triangle having the base 2·7″, the altitude 2·3″ and the angle at the vertex 32°.

9. Describe a triangle on base $2\frac{1}{2}$″, the sum of the other two sides being $4\frac{1}{4}$″ and the vertical angle being 60°.

10. Construct a triangle having given the base 2·7″, the altitude 2·5″, and the angle at the vertex 50°.

11. Describe a triangle whose base is 3″, vertical angle 45°, and area equal to 4 sq. inches.

12. The angles of a triangle are proportional to 8, 7, 15, and its perimeter is 5·75″. Construct the triangle.

13. Construct a triangle, the perimeter being 7″ and the sides in the proportion of $2 : 3\frac{1}{4} : 2\frac{3}{8}$.

14. Construct a right-angled triangle with perimeter 23 cm. and one acute angle 30°. Find the radius of the circumcircle.

15. The angles of a triangle are in the ratio of the numbers 2, 3, 4, and its perimeter is 7″. Construct the triangle.

16. Describe a circle of 1″ radius, and about it construct a triangle whose three sides touch the circle and are to each other as 2 : 3 : 4.

17. Construct a right-angled triangle having given the hypotenuse 3·79″ and the difference of the other two sides ·38″.

18. Construct a right-angled triangle having given the hypotenuse 3·92″ and the sum of the two other sides 5·13″.

19. Construct a right-angled triangle with hypotenuse 3·7″ and perpendicular from the right angle on the hypotenuse 1·34″.

20. Construct a right-angled triangle with hypotenuse 5″ and difference of other two sides ·75″. Measure the radius of the in-circle.

21. Construct an isosceles right-angled triangle, when the sum of the hypotenuse and one side is 5″. Find its area.

22. Construct a triangle with angles in the ratio of 2 : 3 : 4 and the radius of the inscribed circle 1·23″.

23. Construct a triangle having given the altitude 2″ and the base angles 70° and 35°.

24. Construct a triangle with radius of inscribed circle 1·1″ and base angles 72° and 53°.

25. Construct a triangle with radius of circumcircle 1·75″ and base angles 49° and 57°.

26. Construct a triangle, given the base 3″, one of the angles at base 75° and sum of remaining sides 5″. Find its area.

27. Draw a triangle with sides proportional to 11, 12, 13, and with radius of circumcircle 5 cm. What is its area?

28. Construct a triangle with base 9 cm., vertical angle 45°, and difference of sides 5·5 cm. Find radius of the in-circle.

29. Draw a triangle with base 10·6 cm., and vertical angle 72°, with other sides in ratio 4 : 3. Measure the median drawn from vertical angle to the mid-point of base.

30. A, B and C are the middle points of the sides EF, FD, DE of the triangle DEF, if AB=4·1 cm., BC=3·1 cm., CA=5·1 cm. Draw the triangle DEF.

31. Construct a triangle whose medians are 10·4 cm., 7·4 cm. and 8·2 cm. respectively.

32. Construct a square with diagonals 3″. What is its area?

33. Construct a rectangle having its diameter $3\frac{1}{2}$″ long, and one of its sides $2\frac{3}{4}$″. Measure the other side.

34. Construct a triangle with two sides 3″ and 4″, and included angle 30°. Find another triangle of equal area but having two sides 2·37″ and 3·75″ respectively. Measure the included angle.

VERIFICATION OF WELL-KNOWN GEOMETRICAL PROPOSITIONS.

Exercise XLVI.

1. Draw any five straight lines AO, BO, CO, DO, EO meeting at the point O. Measure each of the angles AOB, BOC, COD, DOE, EOA. What is their sum?

2. Draw any irregular pentagon. Measure its angles. What is their sum?

3. Draw any irregular hexagon. Produce all the sides. Find the sum of the exterior angles so formed.

4. Draw any quadrilateral in a circle. Measure all its angles. What are the sums of each pair of opposite angles?

5. Circumscribe a circle with a quadrilateral. Measure it sides. What are the sums of each pair of opposite sides?

6. Draw a circle with radius 7 cm., and in it draw four chords AB, CD, EF and GH intersecting at any point X. Measure AX, XB, CX, XD, &c. Find the values of $AX \times XB$, $CX \times XD$, $EX \times XF$ and $GX \times XH$. Quote the enunciation of the proposition you are verifying.

7. Draw a triangle with sides AB=65 mm., BC=52 mm., CA=39 mm. Find the areas of squares on AB, BC, CA. Measure the angle C.

8. Draw a triangle PQR with sides PQ=65 mm., QR=60 mm., RP=25 mm. Find the areas of squares on PQ, QR, RP respectively, and measure the angle R.

9. ABC is any triangle, say with sides 8, 9 and 10 cm., and O the point of intersection of the medians. Compare the values of $AB^2+BC^2+CA^2$ and $3(OA^2+OB^2+OC^2)$.

10. Draw a circle with radius 2″ and take a point P 3″ from its centre. Draw a tangent PQ and three secants PAB, PCD and PEF. Measure PQ, PA, PB, . . . &c. Find the values of PQ^2, $PA \times PB$, $PC \times PD$ and $PE \times PF$. Quote the enunciation of the proposition you are verifying.

11. Draw a circle with radius 2·5″ and in it place three chords each 3″ long. Draw perpendiculars from the centre on the chords and measure their lengths. Quote the enunciation of the proposition you are verifying.

12. Draw a triangle ABC with sides 10 cm., 8·1 cm., and 7·1 cm. Circumscribe it with a circle. Take any point P in the circumference and draw perpendiculars PD on BC, PE on CA, PF on AB (producing the sides of ABC if necessary). What do you notice about the points D, E, F?

13. *The Nine-points Circle.* Draw a large triangle ABC, say with sides 5″, 4·5″ and 3·9″. Draw perpendiculars AD, BE, CF to the opposite sides; these meet at O. Bisect AO at L, BO at M and CO at N. Draw a circle through L, M and N. If it is well drawn it should also pass through the points D, E, F. At what other points does it cut the sides of the triangle ABC?

14. Taking the same triangle as in the previous question, bisect one side BC at K and join AK. Measure accurately AB, AC, AK and KB. Calculate AB^2+AC^2 and AK^2+KB^2. What is the connection between these two? The straight line which joins an angle of a triangle to the middle point of an opposite side is called a "median." Enunciate the proposition you are verifying.

15. ACDB are points on a straight line AB such that AC=3·7″, CD=2·5″, DB=1·2″. Draw the rectangle AD.DB and the square on CD and the square on CB. Find the area of each and show their connection. Enunciate the proposition you are verifying.

16. APBQ are points on a straight line such that AP=PB=7 cm. and BQ=2 cm. Draw the rectangle AQ . QB and the squares on PB and on PQ. Find the area of each and show their connection. Enunciate the proposition you are verifying.

17. Draw any quadrilateral in a circle whose radius is 7·2 cm. Find the sum of the areas of the rectangles contained by its opposite sides, also find the area of the rectangle contained by its diagonals. Compare your results.

18. Draw a right-angled triangle with two sides 3″ and 4″. On each side describe an isosceles triangle with vertical angle 100°. Compare the sum of the areas of the triangles on the sides with that of a similar triangle on the hypotenuse.

MISCELLANEOUS PROBLEMS.

Exercise XLVII.

1. Draw two straight lines, which, though not parallel, do not meet on your paper. Construct a straight line to bisect the angle between them.

2. ABCD is a quadrilateral in which AB=4″, BC=2·75″, AD=3″ and the angles ABC, BAD are 90° and 60° respectively. Draw the figure (obtaining the angles by construction), and construct a line which will, when produced, bisect the angle between BA and CD, without producing BA and CD to meet.

3. At the two extremities of a line AB 3″ long, on the same side of it, erect perpendiculars AC=2″ and BD=1½″. Bisect the angle made by CD and AB without producing them to meet. Measure accurately and write down the distance from A at which the bisecting line cuts the diagonal AD.

4. A and B are two points on the same side of a straight road, and equal distances from it. A person has to go from A to B, depositing a parcel on the way at some point in the road. Show by a plan the point in the road which will give him least walking.

5. Draw a scale of miles (2 miles to an inch). Two villages A, B, are 3 miles and 5 miles respectively from a straight main road, and 4 miles apart. Find the shortest course for a postman who has to start from A, leave a parcel on the road, and proceed to B. Measure the length of his walk.

6. A lighted candle of height 9″ is standing on a plane mirror. The light of the candle is reflected by the mirror and a reflected ray passes through a point 14″ above the mirror and 17″ from the flame of the candle. Draw a diagram of the path of the ray of light on a reduced scale. [When light is reflected by a plane surface the angle of incidence = the angle of reflection.]

7. Draw a straight line 3″ long and divide it into two parts so that the square on one part may be double the square on the other part. Measure the parts.

8. Divide a straight line 2·9″ long into two parts so that the square on one part may be equal to the rectangle contained by the whole line and the other part. Measure the parts.

9. AB is a straight line 3·7″ long. Produce it to C, so that the rectangle AC . CB is equal to may be equal to 3·8 sq. inches. Measure AC.

10. Divide a straight line AB (10 cm. long) into two parts at C, such that the rectangle contained by BC and another straight line X (7·2 cm. long) may be equal to the square on AC. Measure AC.

11. PQ is a straight line of unlimited length. A and B are two points 2·4″ apart, the former being 3″ from PQ and the latter 1·04″. Find the points on PQ at which AB subtends the greatest angle.

PART III.

PRACTICAL SOLID GEOMETRY.

REGULAR SOLIDS IN SIMPLE POSITIONS.

Exercise XLVIII.

Draw the plan and elevation of the following solids:—

1. A cube, sides 2·3″, standing on one of its faces, one edge of that face parallel to the XY line.

2. A rectangular parallelepiped, sides 2·1″, 1·7″ and 1·2″, with its longest edge parallel to the XY line, and its shortest edge vertical.

3. A right circular cylinder, of height 4 cm., standing on one end, the radius of its base being 2·2 cm.

4. A sphere, whose diameter is 8 cm.

5. A right circular cone, 8 cm. high, standing on its base 5 cm. in diameter.

6. A pyramid, 2·7″ high, standing on a square base, one edge of which is 2″ long and parallel to the XY line.

7. A hexagonal prism, standing on one of its ends, with one edge 1″ long and parallel to the XY line, the height of the prism being 2·3″.

8. An octahedron, edges 5 cm., with a vertical axis, and with two edges parallel to the XY line.

9. A hexagonal prism, resting on one of its rectangular faces, which measure 3″ by 1·2″, the longer edge being parallel to the XY line.

10. A pyramid of four equal spherical shot (radii $\frac{3}{4}$″), resting on the ground, the line joining the centres of one pair of adjacent shot being parallel to the XY line.

11. A pyramid of five equal spherical shot (diameters $1\frac{3}{4}$″), resting on the ground, the line joining the centres of one pair of base shot *which do not touch each other* being parallel to the XY line.

12. A pyramid of five equal spherical shot (diameters 4 cm.), resting on the ground, the lines joining the centres of two pairs of adjacent base shot being parallel to the XY line.

REGULAR SOLIDS WITH SIDES OBLIQUE TO THE V.P.

Exercise XLIX.

Draw the plan and elevation of the following solids:—

1. A brick, $9'' \times 4\frac{1}{2}'' \times 3\frac{1}{2}''$, with its largest faces parallel to the H.P. and its longest edge inclined at an angle of 30° to the V.P., on a suitable scale.

2. An equilateral triangular prism, resting on one of its rectangular faces (whose sides are 7 cm. and 3 cm.), the longer edge being inclined at an angle of 17° to the XY line.

3. A right-circular cone, with its axis 3″ long, inclined at an angle of 30° to the V.P., but parallel to the H.P., its circular ends being 1·2″ in diameter.

4. A tetrahedron, with edges 6 cm., with one face in the H.P., one edge of that face being inclined at an angle of 23° to the XY line.

5. A pyramid, of height 2·7″, on an equilateral triangle, (sides 1·5″) as base, standing on the H.P., one of the sides of the base being inclined at an angle of 37° to the XY line.

6. A round table, diameter 4′, standing on four legs of 3′, one side of the square formed by the sockets of the legs being inclined to the vertica plane at an angle 20°.

7. An octahedron, edges 5 cm., with a vertical axis and with two horizontal edges inclined at an angle of 20° to the V.P.

8. A pyramid of four equal spherical shot (diameters 3·6 cm.), resting on the ground, the line joining the centres of the two hinder base shot being inclined at an angle of 15° to the V.P.

9. A pyramid of five equal spherical shot (diameters 3·8 cm.), resting on the ground, the line joining the centres of one pair of base shot *which do not touch one another* inclined at an angle of 20° to the XY line.

REGULAR SOLIDS WITH SIDES OBLIQUE TO THE H.P.

Exercise L.

Draw the plan and elevation of the following solids:—

1. A cube, edges 2·3″, with two faces parallel to the V.P. and two faces inclined at an angle of 30° to the H.P. and one edge in the H.P. at right angles to the XY line.

2. A brick, $9'' \times 4\frac{1}{2}'' \times 3\frac{1}{2}''$, with its short edges perpendicular to the V.P. and its long edges inclined at an angle of 50° to the H.P. (on a suitable scale).

3. A cylinder, with its axis 2·7″ long parallel to the V.P. and inclined at an angle of 50° to the H.P. The diameters of the circular ends being 1·8″.

4. A pyramid, on a square base, sides 1·7″, with axis 2·4″ parallel to the V.P. and inclined at an angle of 15° to the H.P., two sides of the base being perpendicular to the V.P.

5. An equilateral triangular prism, with long edges 2·8″ inclined at an angle of 40° to the H.P., and one edge of a triangular end 1·4″ perpendicular to the V.P.

6. A right circular cone poised on its vertex and its axis inclined to the H.P. at an angle of 80°, but parallel to the V.P. The axis is 2·8″ long and the radius of the circular base is 1·1″.

7. A truncated cone, with axis parallel to the V.P., but inclined at an angle of 70° to the H.P. The axis is 5·5 cm. long and the radii of the circular ends are 3·7 cm. and 2·5 cm.

8. A pentagonal pyramid, (sides of base 1″ and height 3″), poised on its vertex and with its axis inclined to the H.P. at an angle of 82°, and with one edge of the base perpendicular to the V.P.

9. A pentagonal pyramid, (sides of base 2·3 cm. and height 7 cm.) with the base inclined at an angle of 35° to the H.P. and with one edge of the base, nearest to the V.P., also parallel to it.

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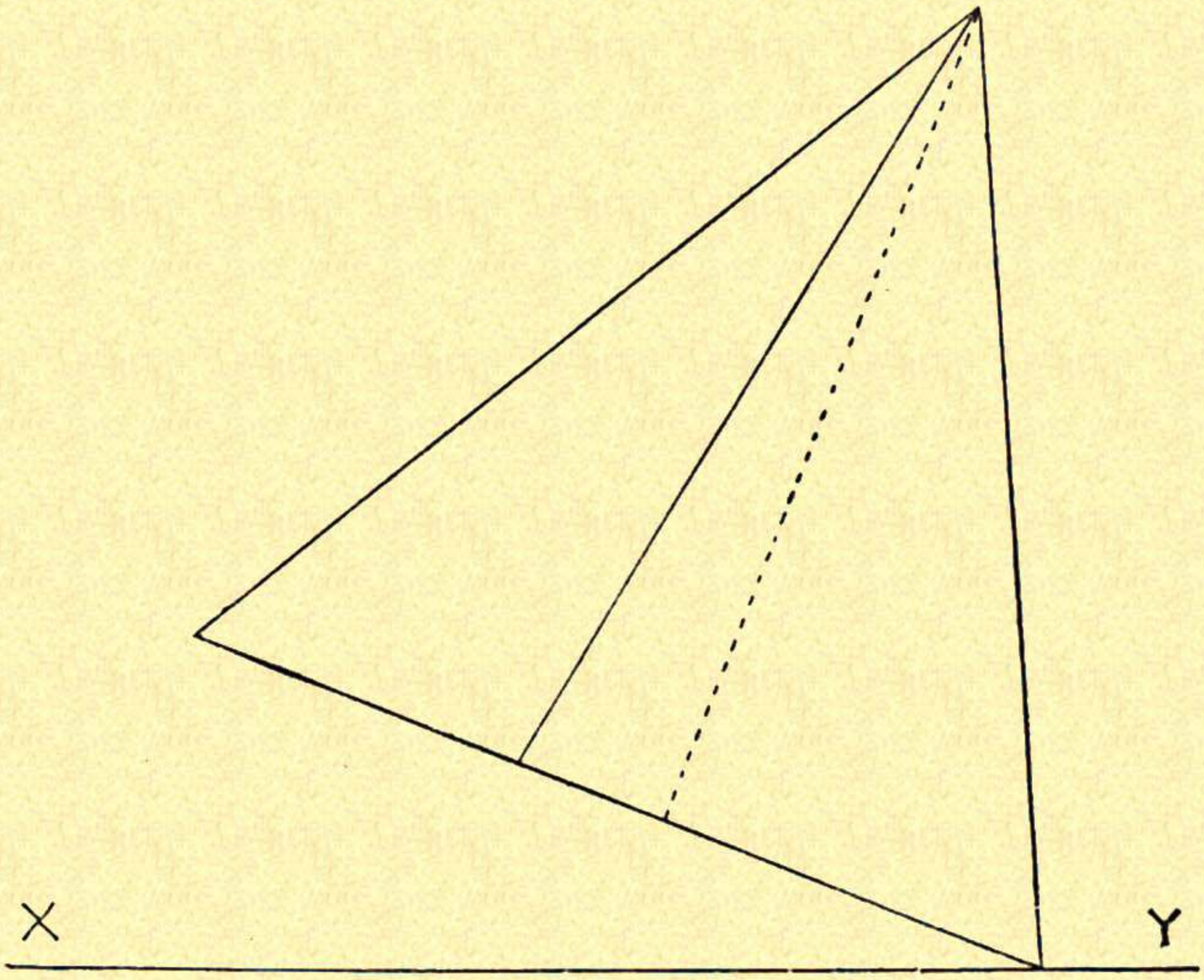
10. The figure represents the elevation of a pyramid on a



regular pentagonal base, the dotted line being the axis. The view is taken looking North. Draw (α) the elevation looking East, (β) the elevation looking West.

REGULAR SOLIDS WITH SIDES OBLIQUE TO BOTH PLANES.

Exercise LI.

Draw the plan and elevation of the following:—

1. A cube, edges 6·7 cm., with one edge in the H.P., but inclined at an angle of 20° to the XY line, and one face inclined at an angle of 30° to the H.P.

2. A brick, $9'' \times 4\frac{1}{2}'' \times 3\frac{1}{2}''$, with one shortest edge in the H.P., but inclined at an angle of 15° to the XY line, and smallest faces inclined at an angle of 40° to the H.P.

3. A tetrahedron, edges 3″, with one edge parallel to the H.P., but inclined at an angle of 35° to the V.P., and one face (which contains this edge) inclined at an angle of 15° to the H.P.

4. A cone, with circular base (diameter 2·8″), inclined at an angle of 40° to the H.P. and with vertical plane containing the axis (length 3·5″) inclined at an angle of 10° to the V.P.

5. A brick, $3'' \times 1\frac{1}{2}'' \times 1\frac{1}{8}''$, poised on one corner, with one diagonal vertical and another parallel to the V.P.

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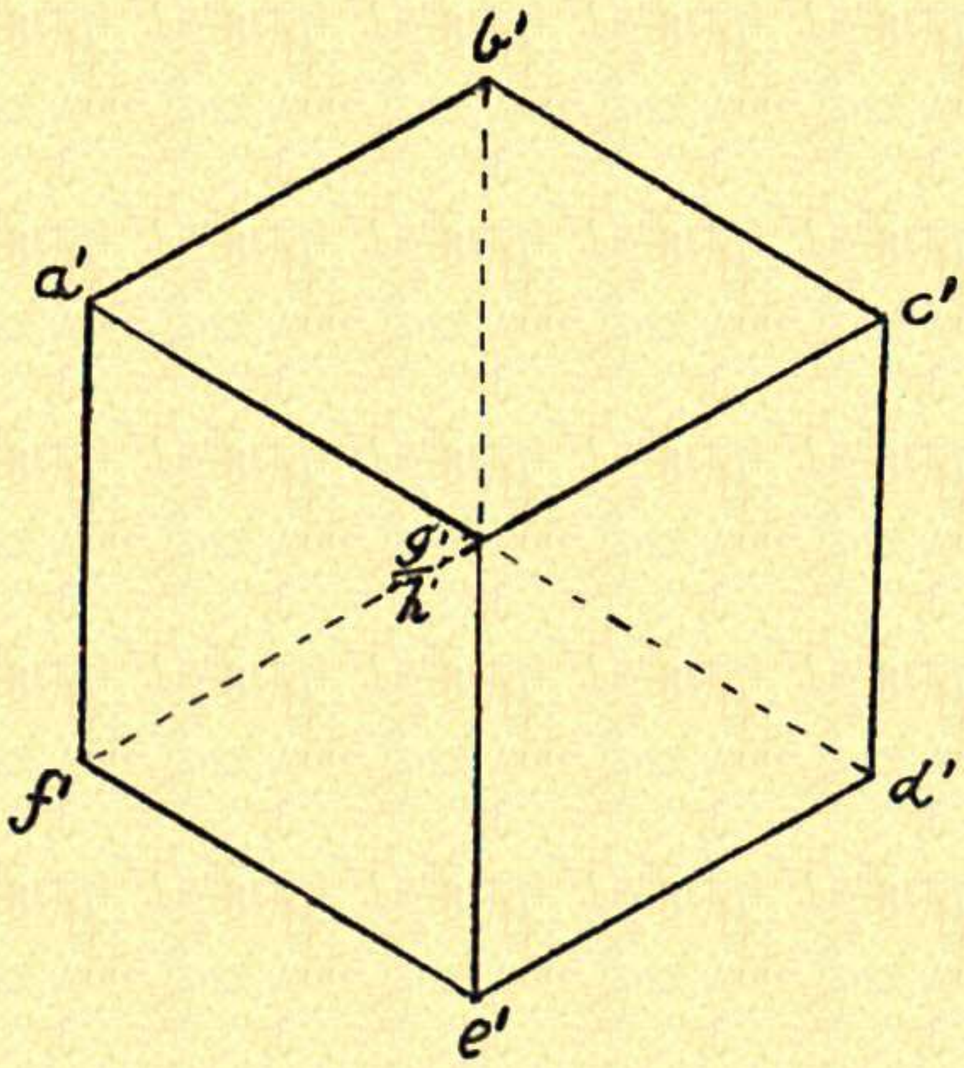
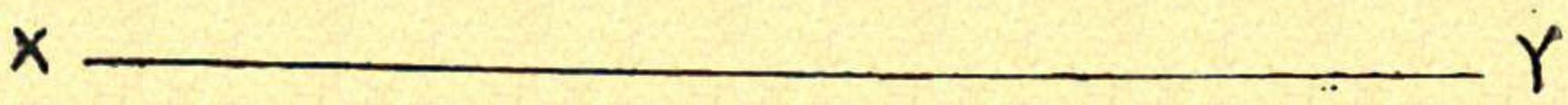
6. The elevation of a cube poised on one corner is shown by $a'b'c'd'e'f'g'h'$. Draw its plan.





7. A right circular cone is poised on its vertex, and its axis, 3″ long, is inclined at an angle of 75° to the H.P., the horizontal projection of the axis being inclined at 40° to the V.P. If the radius of the base is 1·2″, draw its plan and elevation.

SECTIONS OF SOLIDS MADE BY HORIZONTAL AND VERTICAL PLANES.

Exercise LII.

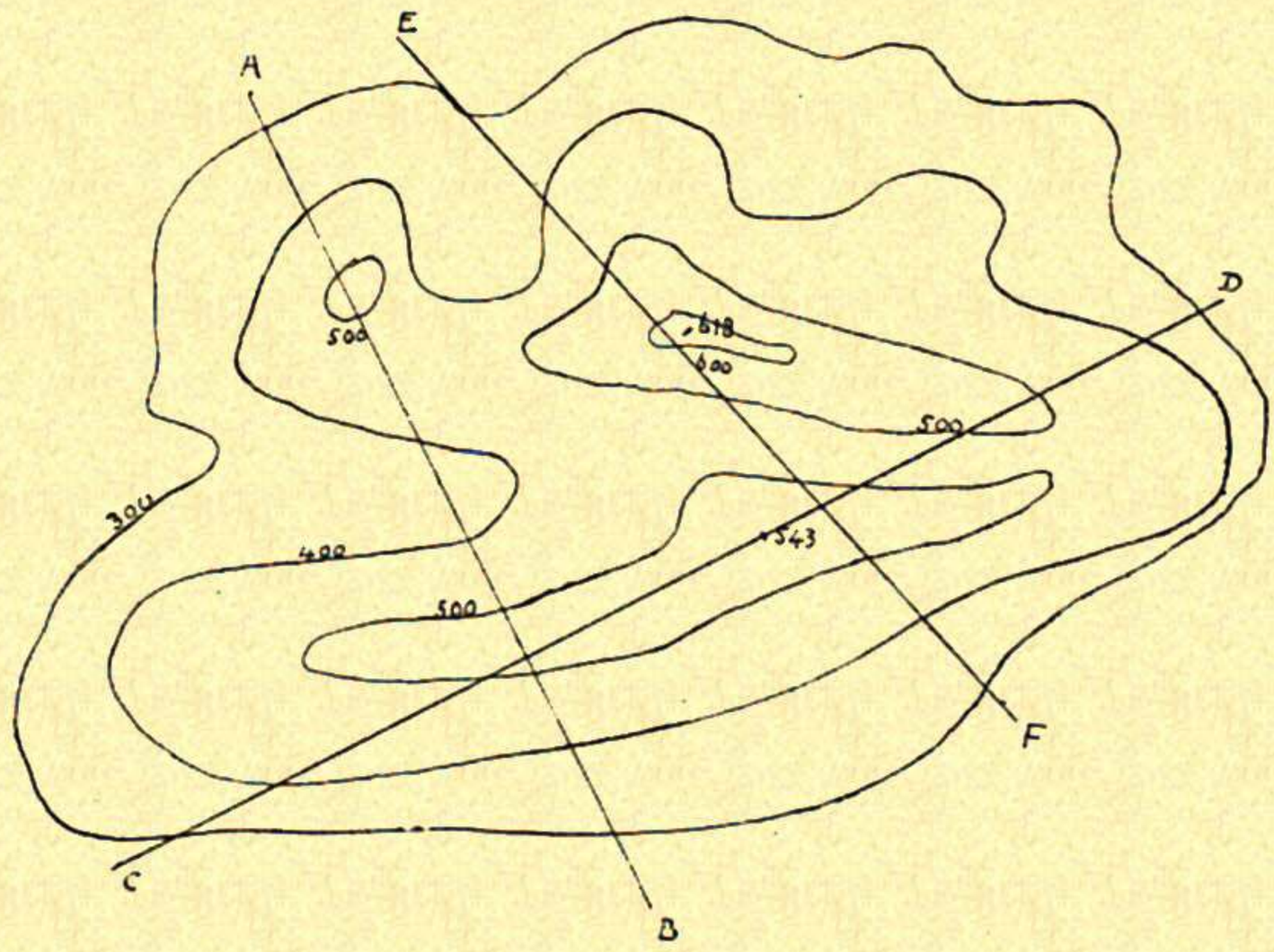
1. A cone, whose height is 8 cm. and radius of base 2·5 cm. in a vertical position is cut by a H.P. 3 cm. from the base. Determine the section.

2. A pentagonal pyramid, with axis 8 cm. vertical, and one side of the base 3 cm. parallel to the XY line is cut by a H.P. 5 cm. from the base. Determine the section.

3. An equilateral triangular prism, resting on a triangular end (sides 2″ and height 3″), with the face nearest the V.P. inclined at an angle of 15° to the V.P., is cut by a plane parallel to the V.P. and nearer to it, by ·4″, than the central axis. Determine the section.

4. A square prism, sides 2·8″ and length 4″, with one long edge in the H.P. parallel to the XY line and with one large face inclined at 30° to the H.P. is cut by a plane parallel to the V.P. and nearer to it by 1·2″ than the central axis. Determine the section.

5. A sphere, diameter 2·8″ is cut by a V.P. 1″ from its centre. Find the section.



6. Draw vertical sections of the country, shown above through the lines AB, CD and EF. The contours are at 100′ interval. Vertical scale 250′ to 1″.

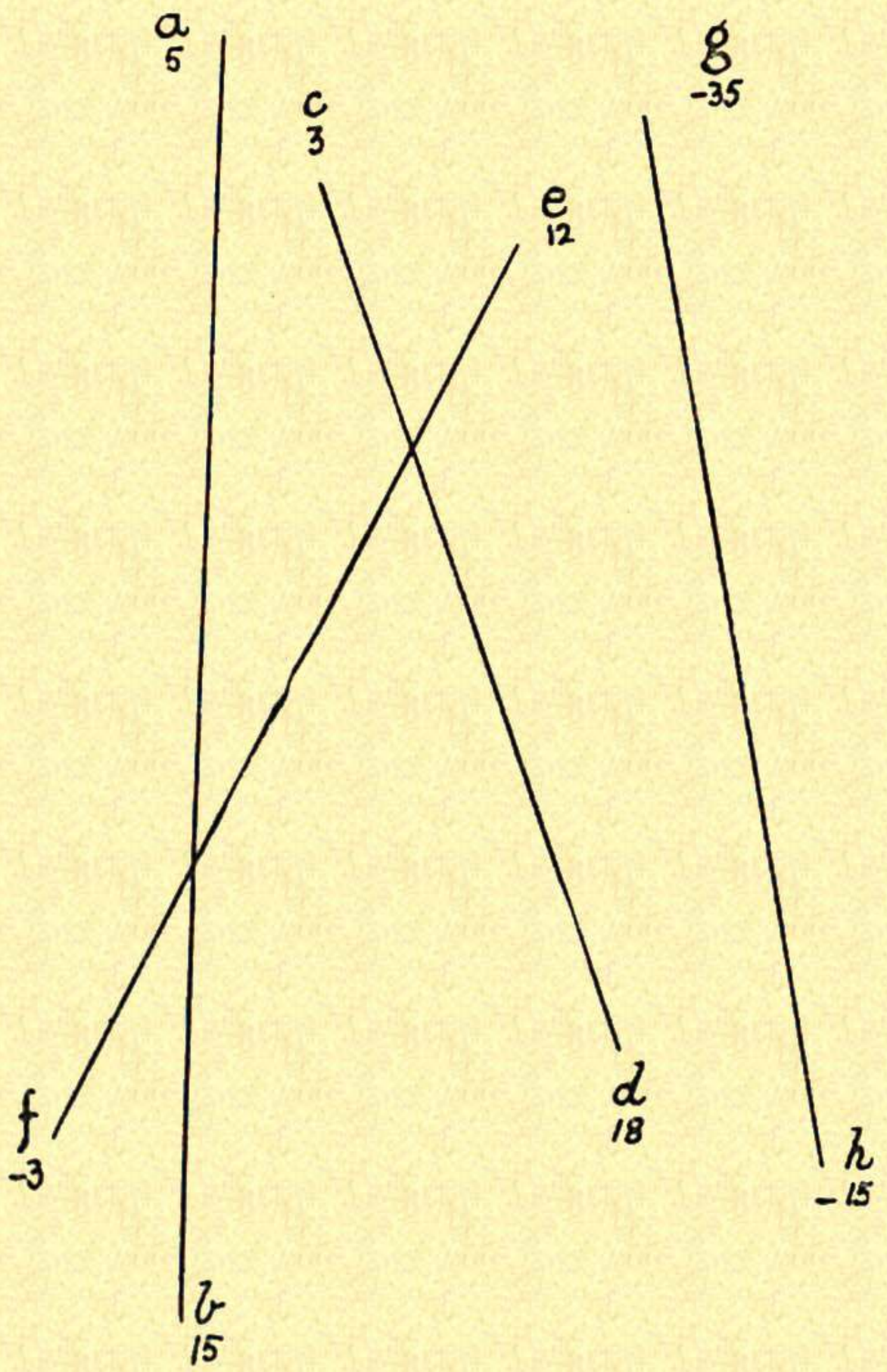
STRAIGHT LINES AND PLANES.

Exercise LIII.

1. A straight line, 10 cm. long, is inclined at an angle of 40° to the H.P., and its plan is inclined at an angle of 50° to the XY line. Draw its elevation.

2. The elevation of a straight line is 2·7″ and its plan is 1·2″. It is in a plane perpendicular to both the V.P. and H.P. Find its length and inclinations to the V.P. and H.P.

3. The point P is situated 2 cm. from both planes of projection and Q is situated 5 cm. from both planes. The planes containing the points and perpendicular to both planes of projection are 9 cm. apart. Determine the true length of the line PQ.



4. Find the true lengths of the straight lines AB, CD, EF, GH, shown above on the index system and their inclinations to the H.P. (10 units to the inch.)

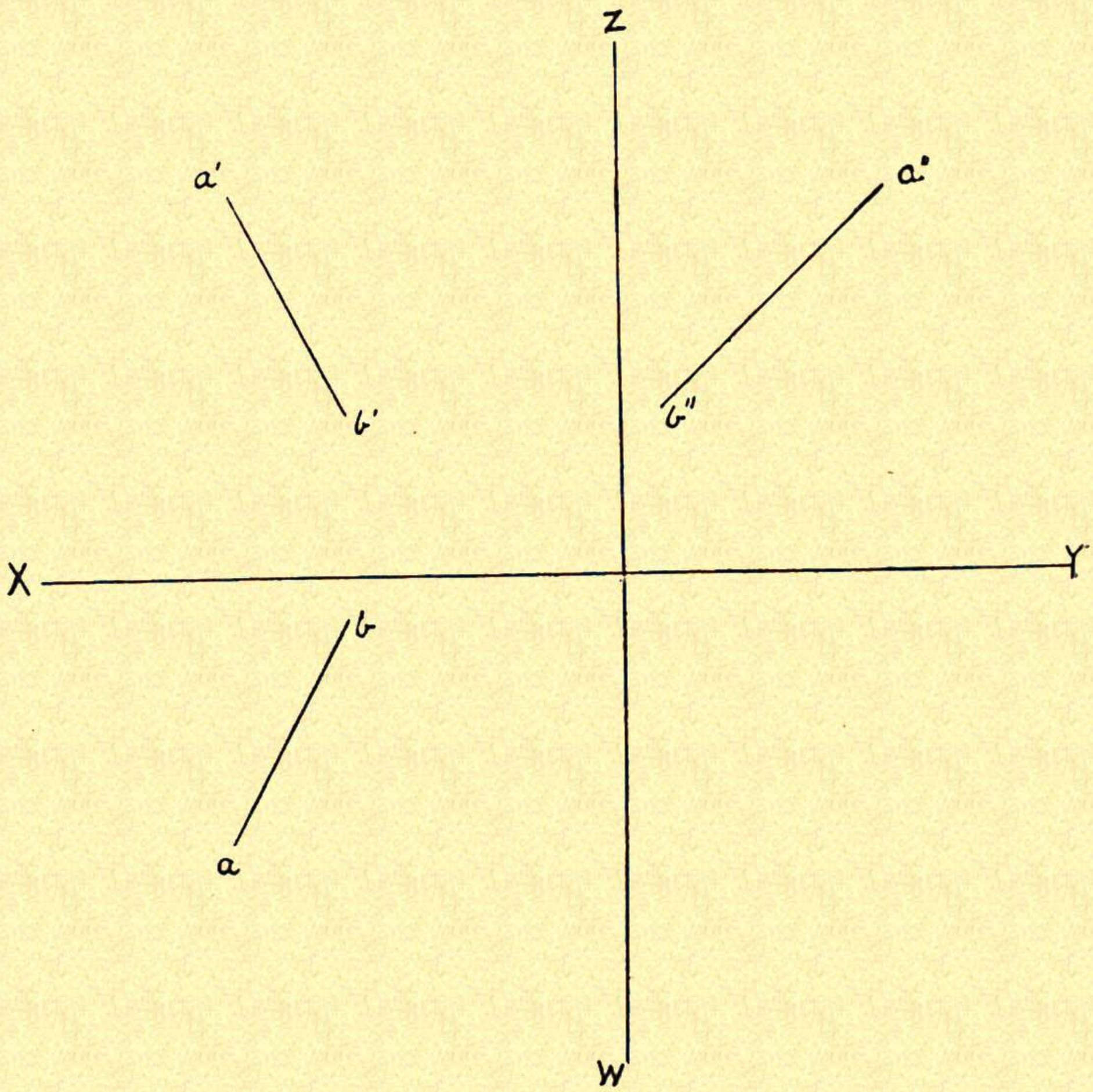
5. The plan of a straight line AB is 7·2 cm. long, its elevation 8 cm. long, and its projection on XY=5 cm. Find its true length and inclination to the H.P.

6. A straight line 9 cm. long has one extremity in XY, and the other extremity 2·1 cm. in front of the V.P., and 5 cm. above the H.P. Draw its plan and elevation.

7. A straight line, 3″ long, has one end on the H.P. ·7″ in front of the V.P,, and the other end ·4″ in front of the V.P., and 1·7″ above the H.P. Draw its plan and elevation.

8. AB, the elevation of a straight line, is 4″ long. A is $\frac{1}{2}$″ and B is $2\frac{1}{2}$″ from XY. CD, its plan, is 4·3″ long. C is 2·8″ from XY. Determine the length of the line and its inclinations to both planes of projection.

9. Find the length of the diagonal of a brick $9'' \times 4\frac{1}{2}'' \times 3\frac{1}{2}''$. (Scale $\frac{1}{3}$.)



10. *ab*, *a′b′*, and *a″b″* are the projections of AB on three mutually perpendicular planes. Find the true length of AB and its inclinations to those planes.

11. The contours of a plane are $\frac{3}{4}''$ apart in plan and at 10 units vertical interval. Find the inclination of the plane.

12. The projections of the line AB and CD are shown in figures 1 and 2. Determine (α) their lengths, (β) their inclinations to the two planes, (γ) their traces. The figures to be drawn on double the scale.

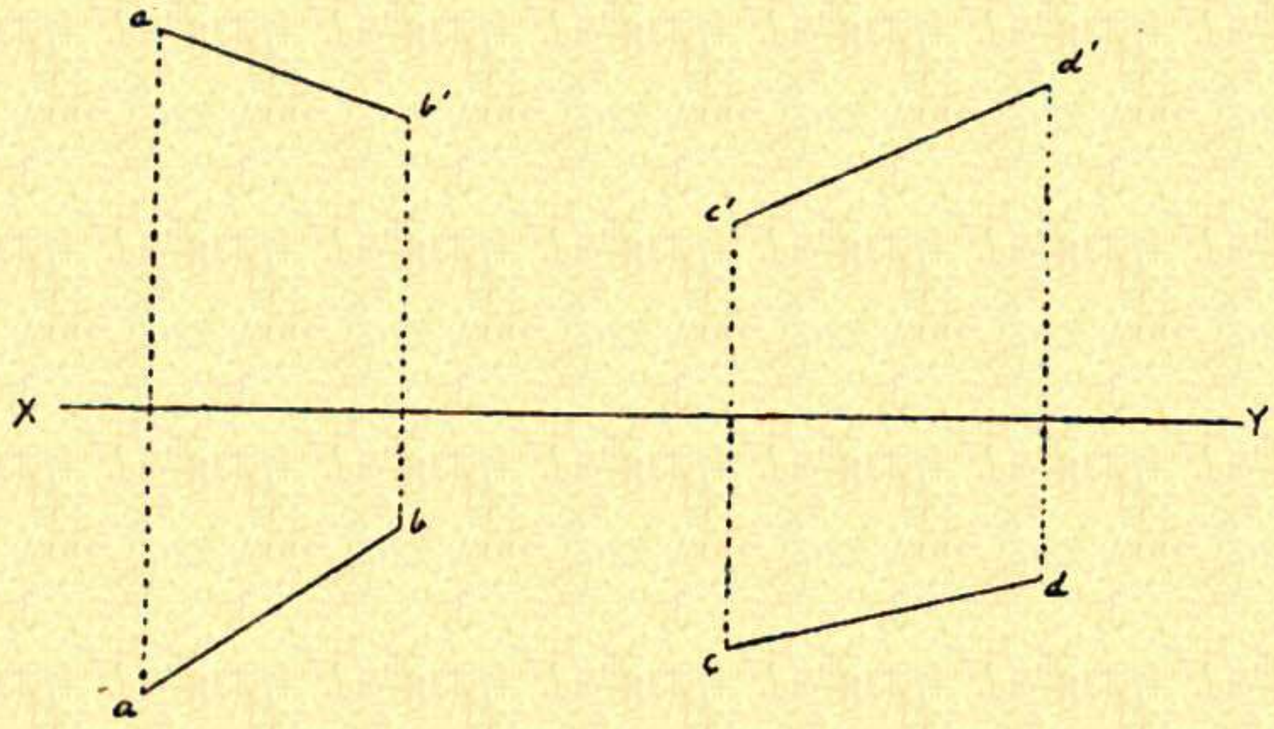


FIG. 1

FIG. 2.

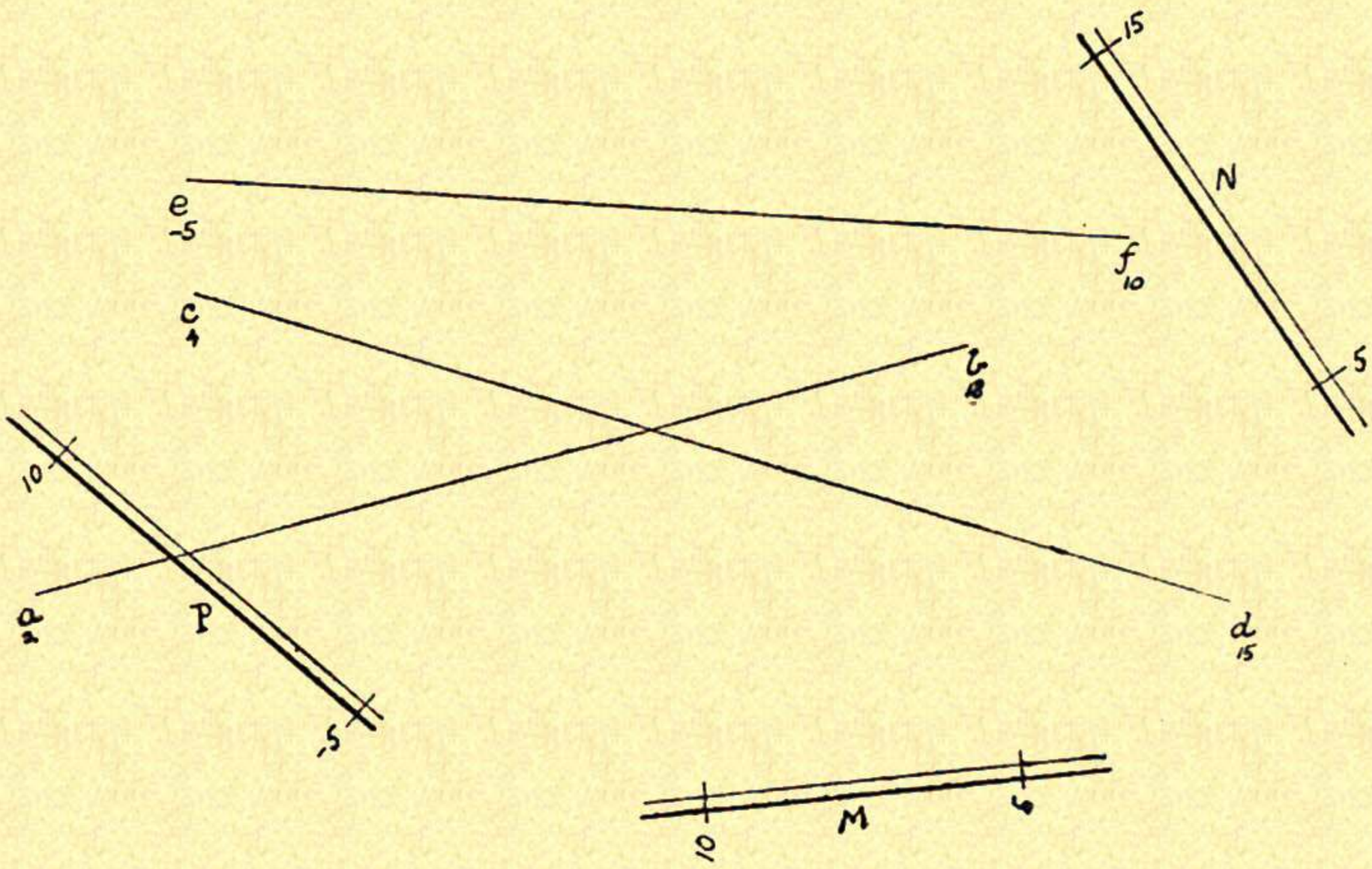
13. Determine the planes containing the points (1) ABC, (2) DEF, (3) GHK, (4) LMN. Scale 10 units to 1″.

14. In the same figure: (1) Through the points MK draw a plane inclined at an angle of 50°. (2) Through the points AN draw a plane inclined at an angle of 70°. (3) Through the points GE draw a plane inclined at an angle of 20°.

15. Two lines inclined at an angle of 120° in plan intersect at a point whose index is 25. The one is inclined at an angle 30° and the other at 20° to the H.P. Find the inclination of the plane containing the lines.

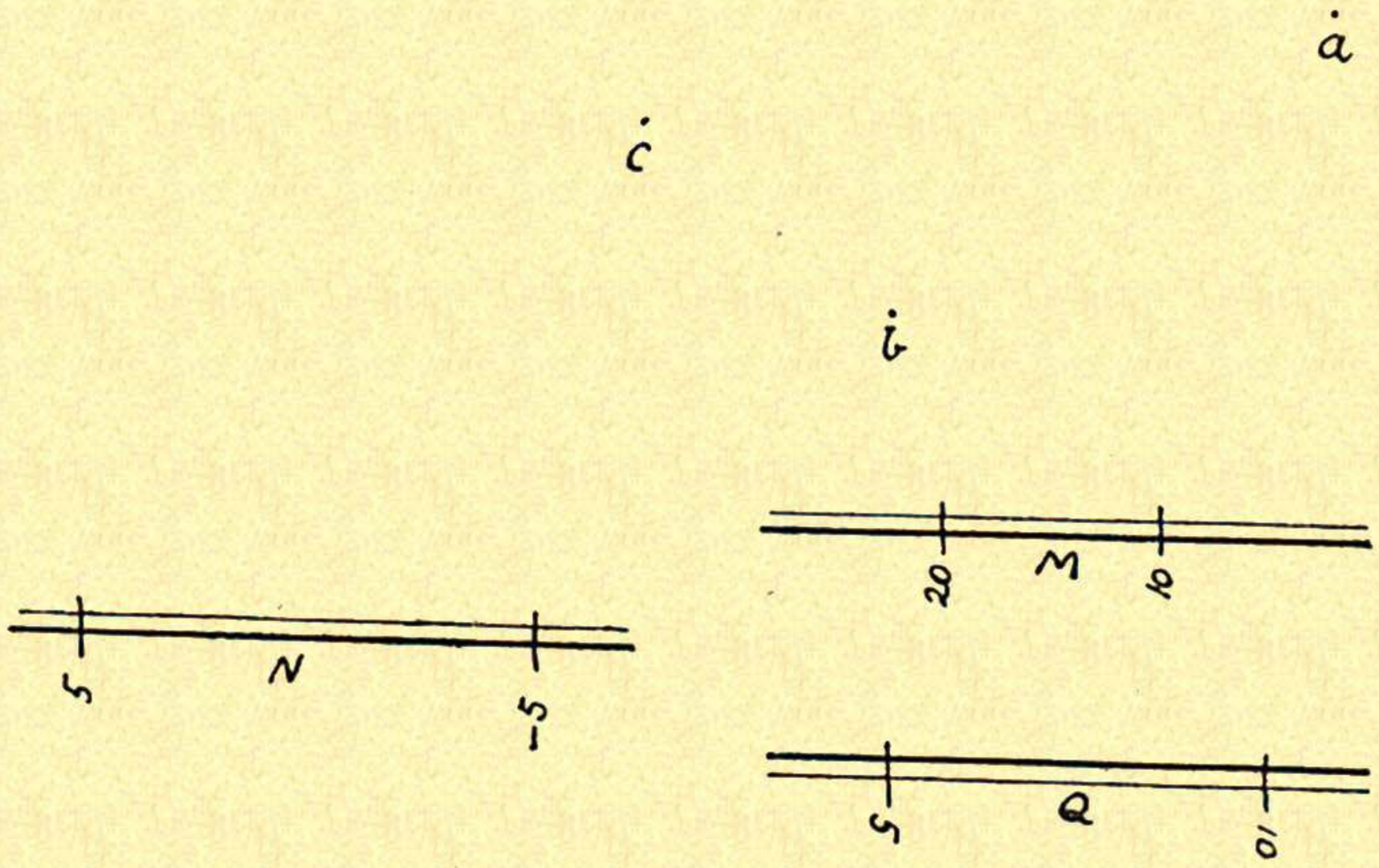
16. Determine the intersections of

(1) The line AB and the plane M.
(2) „ „ CD „ „ N.
(3) „ „ EF „ „ P.



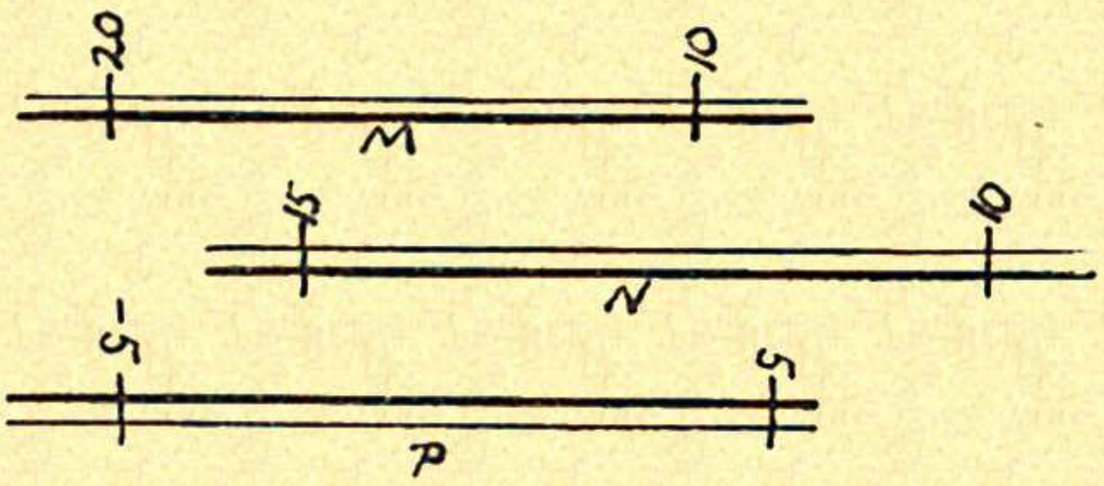
17. From B in the plane N draw a straight line inclined at 30° to the H.P. (See fig. question 18).

18. From C in the plane Q draw a straight line inclined at 20° to the H.P.

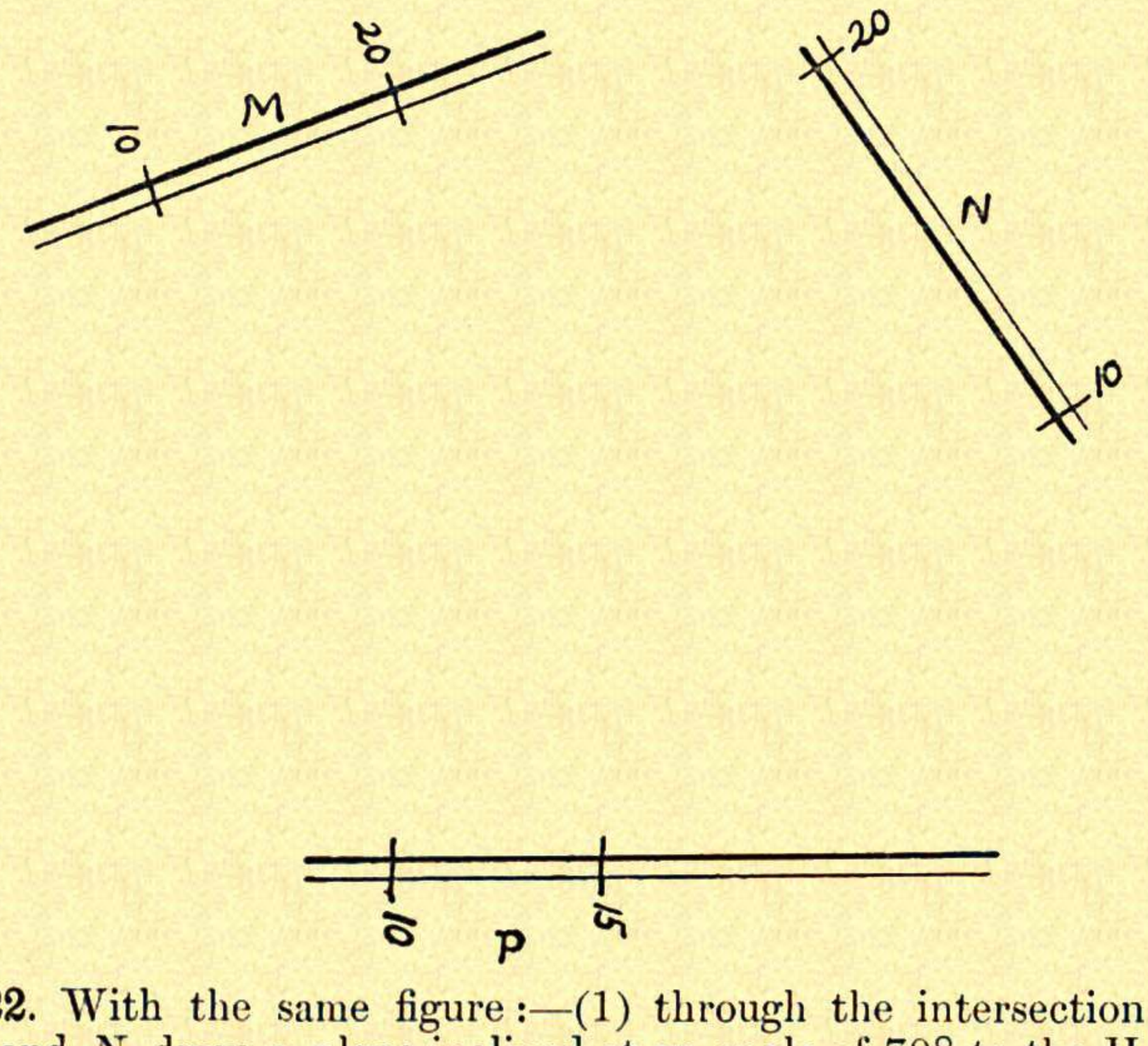


19. From the given point A in the plane M draw a straight line inclined to the H.P. at an angle of 40°. (See fig. question 18.)

20. Find the intersections of the planes (1) M and N, (2) N and P, (3) P and M.



21. Find the intersection of the planes (1) M and N, (2) N and P, (3) P and M.



22. With the same figure:—(1) through the intersection of M and N draw a plane inclined at an angle of 70° to the H.P., (2) through the intersection of N and P draw a plane inclined at an angle of 30°, (3) through the intersection of P and M draw a plane inclined at an angle of 50°.

PART IV.

MILITARY EXAMINATION PAPERS.

MILITARY ENTRANCE EXAMINATION,

JUNE 1900.

Including (1) the construction of scales, and (2) practical plane and solid geometry.

N.B.—*All the questions in this paper may be attempted.*

Each figure should be neatly drawn in clear, fine pencil lines on the same page as the question; the figures should be inked in if time permits. Marks for neatness and for the manner in which the figures are inked in will be given.

The solutions must be strictly geometrical, and all lines of construction should be shown.

No writing whatever is allowed on the blotting paper.

1. Copy accurately the figure given on page 25.
(No marks will be given unless the figure is inked in.)

2. A map is on the scale of 1 : 60000; construct a scale of English miles and furlongs.
(Show the calculations on the same page as the scale. To obtain full marks the scale must be inked in.)

3. A map which was originally on the scale of 25″ to a mile has been reduced in scale so that the length of the map is 66″ instead of 80″; construct a plain scale of yards to read to 10 yards for the reduced map, and find its R.F.
(Show the calculations clearly on the same page as the scale. To obtain full marks the scale must be inked in.)

4. Draw two straight lines meeting at an angle of 35° (which may be protracted). Draw a third line, cutting these lines 3″ and 2″ respectively from their point of intersection. With centre in the third line, describe a circle touching the other two lines.

5. Construct a triangle with sides 2·4″, 3·6″, and 4·2″ long: and construct a square equal to it in area.

6. Draw and figure the scale of a plane inclined at 20° to the horizontal. Unit, 0·1″.

7. A right prism 3″ long, on a regular hexagon with edges half an inch long as base, rests with one rectangular face in the horizontal plane. Draw its plan.

MILITARY ENTRANCE EXAMINATION,

November 1900.

Including (1) the construction of scales, and (2) practical plane and solid geometry.

N.B.—*All the questions in this paper may be attempted.*
Each figure should be neatly drawn in clear, fine pencil lines on the same page as the question; the figures should be inked in if time permits. Marks for neatness and for the manner in which the figures are inked in will be given.
The solutions must be strictly geometrical, and all lines of construction should be shown.
No writing whatever is allowed on the blotting paper.

1. Copy accurately the figure given on page 37.
(To obtain full marks the figure must be inked in.)

2. Construct a regular pentagon of 2″ side. From any angle draw a line dividing the pentagon into two parts, one of which is twice as large as the other.
(To obtain full marks the figure and construction lines must be inked in.)

3. The representative fraction of a map is $\frac{1}{3520}$. Draw a scale of single yards showing on it a length of 600 yards.
(All calculations must be clearly shown. To obtain full marks the scale must be inked in.)

4. Draw a circle 3″ diameter. On a diameter produced mark two points 1·75″ and 2″ respectively from the centre, on opposite sides of it. Draw a circle to touch the first circle and pass through the two points.

5. Construct a triangle with two sides 4″ and 5″ long and the contained angle 30°. Find another triangle of equal area but having two sides 3·25″ and 4·5″ respectively.

6. The elevation of a line is 2·75″ long, its plan is 3″ long, and its projection on XY is 2″ long. Find its true length and inclination to the horizontal.

7. A right pyramid 3·5″ high, on a square base of 2″ side, lies with one triangular face in the horizontal, its axis being parallel to the vertical plane. Draw its elevation and plan.

MILITARY ENTRANCE EXAMINATION,
June 1901.

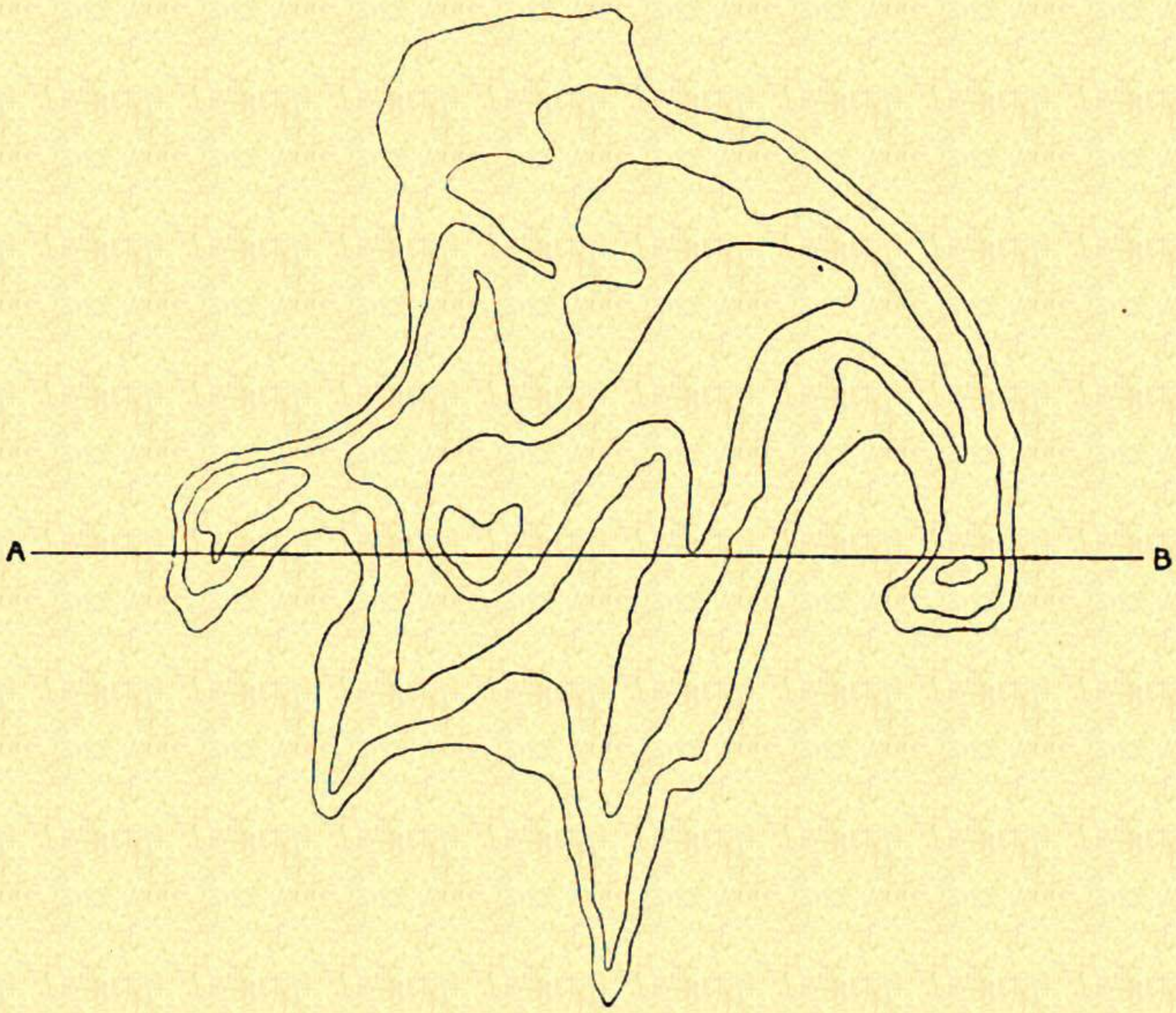
Including the construction of scales, and practical plane and solid geometry.

N.B.—*All lines of construction should be shown. The figures should be inked in if time permits.*

The methods used should be geometrical, except where other methods are expressly prescribed in the Question Paper. In the absence of special instructions, protractors, set-squares, and parallel rulers may be used. No credit will be given for trial methods.

No writing whatever is allowed on the blotting paper.

1. On the accompanying plan of a solid are shown the lines of intersection by horizontal planes of heights 0″, 1″, 2″, 3″, 4″ above the ground plane Show the elevation of the section by the vertical plane AB.



Determine as many points as possible, and then draw the section freehand.

(The figure is given on half the true scale.)

2. Copy accurately the figure given on page 32.

(To obtain full marks the figure must be inked in.)

3. On a certain map 1′ represents 1000 miles. Make a diagonal scale for the map, showing single miles. By two dots on your scale show a length of 473 miles. Calculate the R.F.

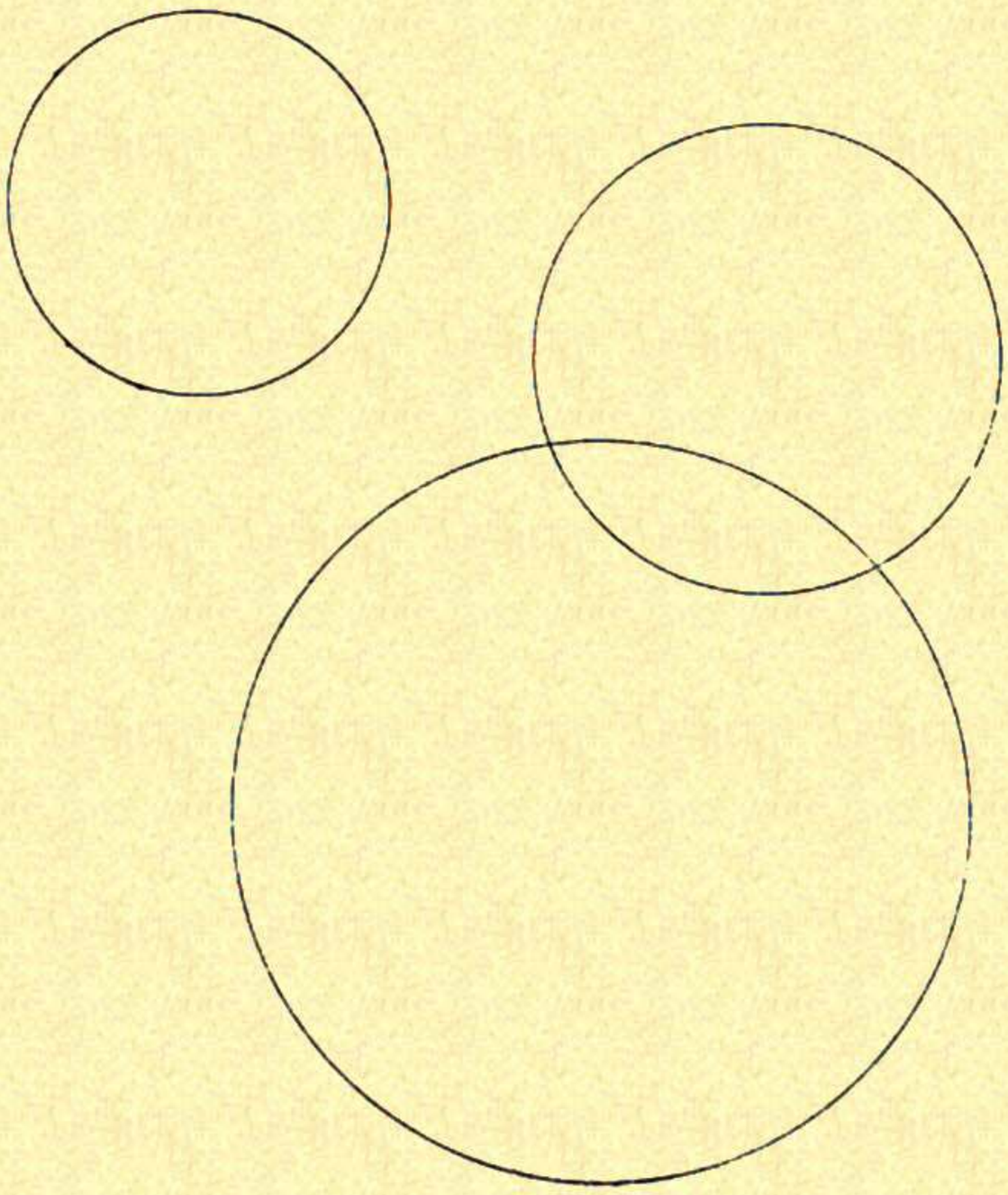
(Calculations must be clearly shown. To obtain full marks the scale must be inked in.)

4. A straight line, 3″ long, has one end on horizontal plane 1″ in front of the vertical plane, the other on the vertical plane 2″ above the horizontal plane. Draw its plan and elevation.

5. Draw a line AB 4″ long, and divide it in P so that the square on AP is equal to the rectangle contained by AB and BP. State your construction. Measure and write down the length of AP to the nearest hundredth of an inch.

6. Construct a triangle having a perimeter of $6\frac{1}{2}$″, a base of 2″, and a vertical angle of 45°. State your construction.

7 Draw a circle cutting the three given circles at right angles. State your construction. Show all lines of construction,



but ink in only the final circle. (The figure is given on half the true scale.)

MILITARY ENTRANCE EXAMINATION,
November and December 1901.

Including the constructions of scales, and practical plane and solid geometry.

N.B.—*All lines of construction should be shown.*

The methods used must be geometrical, except where other methods are expressly prescribed in the Question Paper. In the absence of special instructions, protractors, set-squares, and parallel rulers may be used. No credit will be given for trial methods.

1. Copy accurately the figure given on page 17.

(No marks will be given unless the work is inked in.)

2. On the scale of a certain French map, 1 cm. represents 78·5 m. Construct a scale of metres for the map, and indicate by two dots a length of 973 m.

(Show the calculation, and give the R.F. To obtain full marks, the scale must be inked in.)

3. The point A is situated 1″ from both planes of projection. B is situated 2″ from both planes. The planes containing the points and perpendicular to both planes of projections are 4″ apart. Determine the length of line joining them.

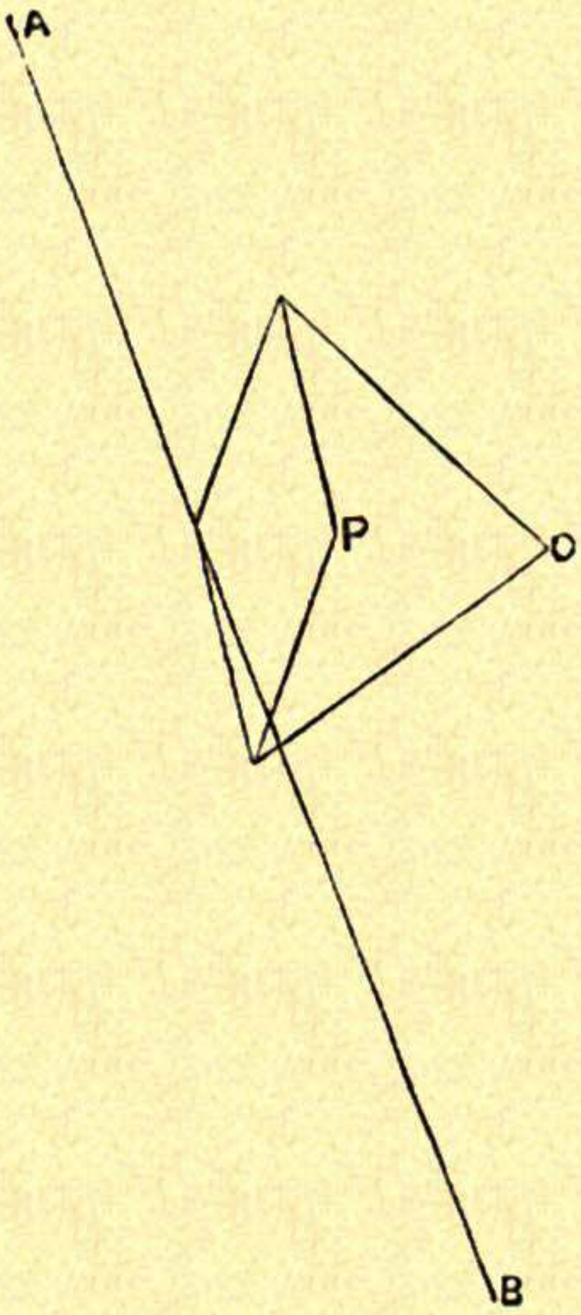
(Full marks will be given for pencil work. You may use any method you please.)

4. Three spherical balls, 2″ in diameter, are placed on level ground in contact, and a fourth, of the same diameter, on the top of them. Draw the plan and elevation of the balls (any elevation you like). Ink in the plan and elevation, leaving construction lines in pencil.

5. Draw a rectangle having sides 3·12″ and 1·28″. Construct a square equal to it. Measure and write down the length of one side and of the diagonal of the square (correct to two places of decimals.) State the construction. Ink in if you have time.

6. Draw a triangle having the given point A as vertex and the given line AM as altitude: and such that its vertical angle shall be 70° and its base 3·5″. State your construction. Ink in if time permits.

7. The diagram shows a system of six rods jointed together. The point O is fixed and Q moves on the straight line AB. Find how much of AB, Q runs over and draw the corresponding path traced out by P. Ink the path of P but nothing else.



(The four shorter rods are of the same length.)

(The figure is given on half the true scale).

N.B.—The six figures in this paper are drawn on half the true scale.

MILITARY ENTRANCE EXAMINATION,

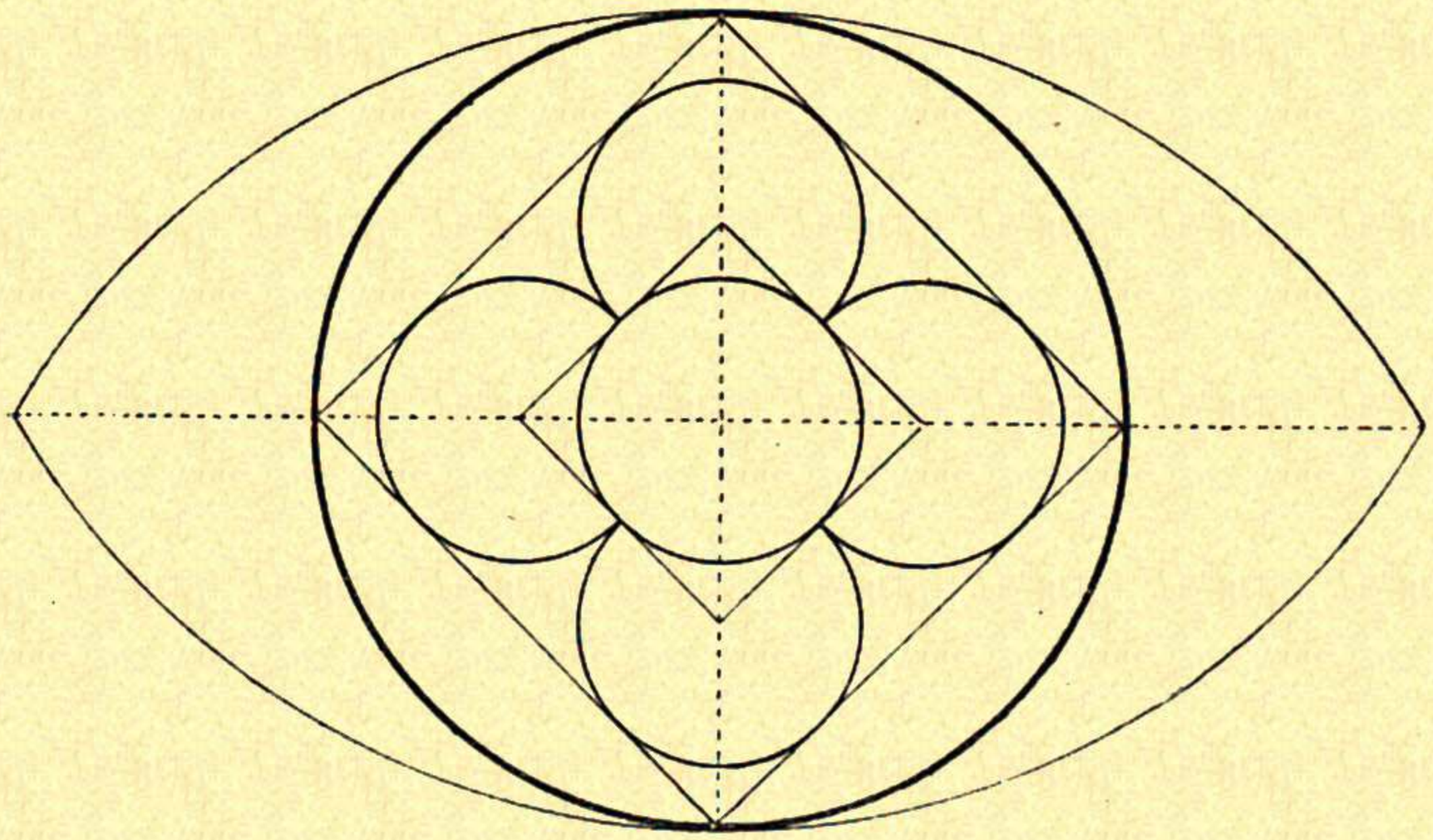
JULY 1902.

Including the construction of scales, and practical plane and solid geometry.

N.B.—*All lines of constructions should be shown.*
The methods used must be geometrical, except where other methods are expressly prescribed in the Question Paper. In the absence of special instructions, protractors, set-squares, and parallel rules may be used. No credit will be given for trial methods.
No writing whatever is allowed on the blotting paper.

1. Draw the figure given below, making your copy exactly similar in every respect.

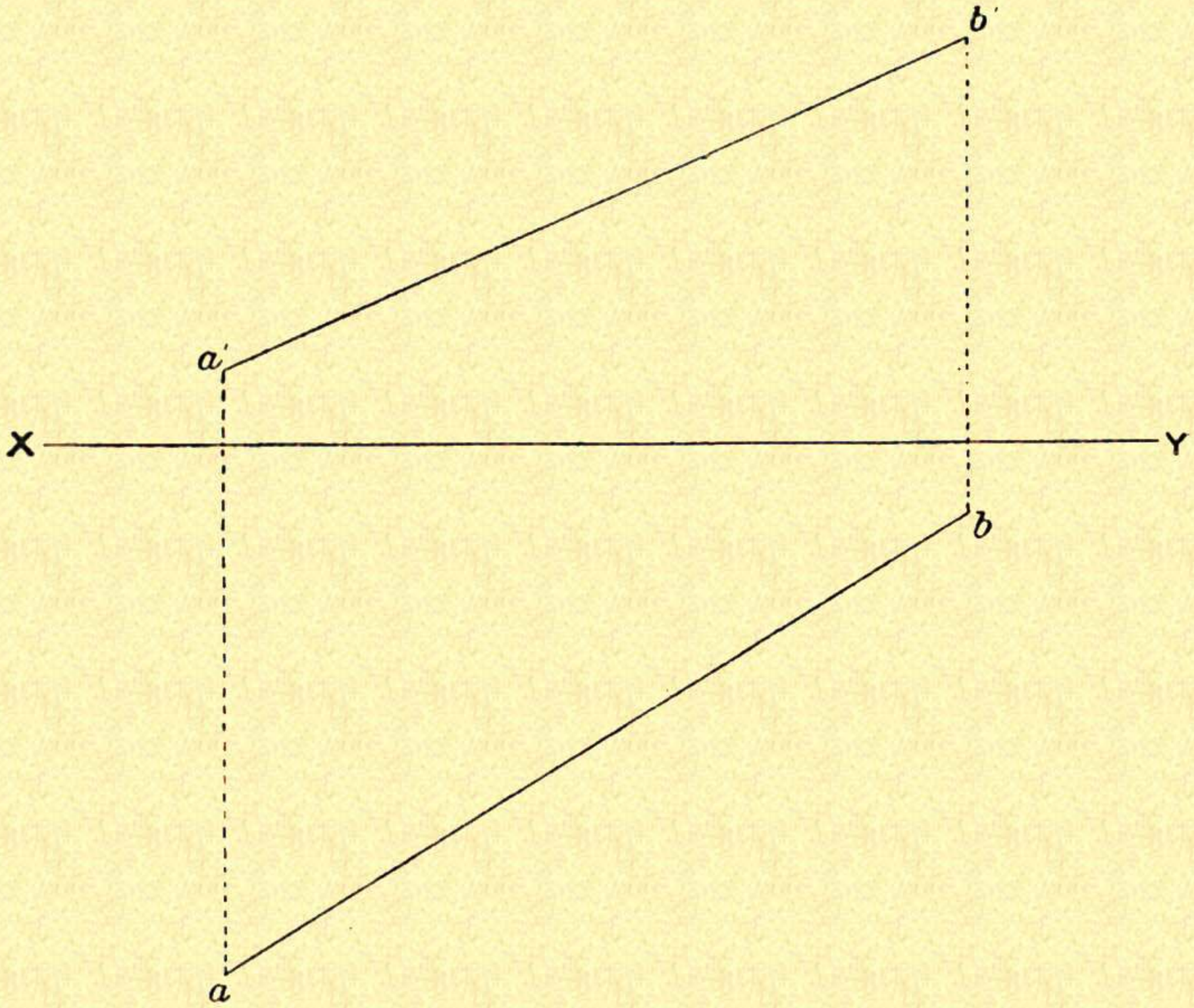
No marks will be given unless the work is inked in.



2. A line 6″ long represents 2 miles. Construct on it a diagonal scale to read miles, furlongs and poles. Give the representative fraction.

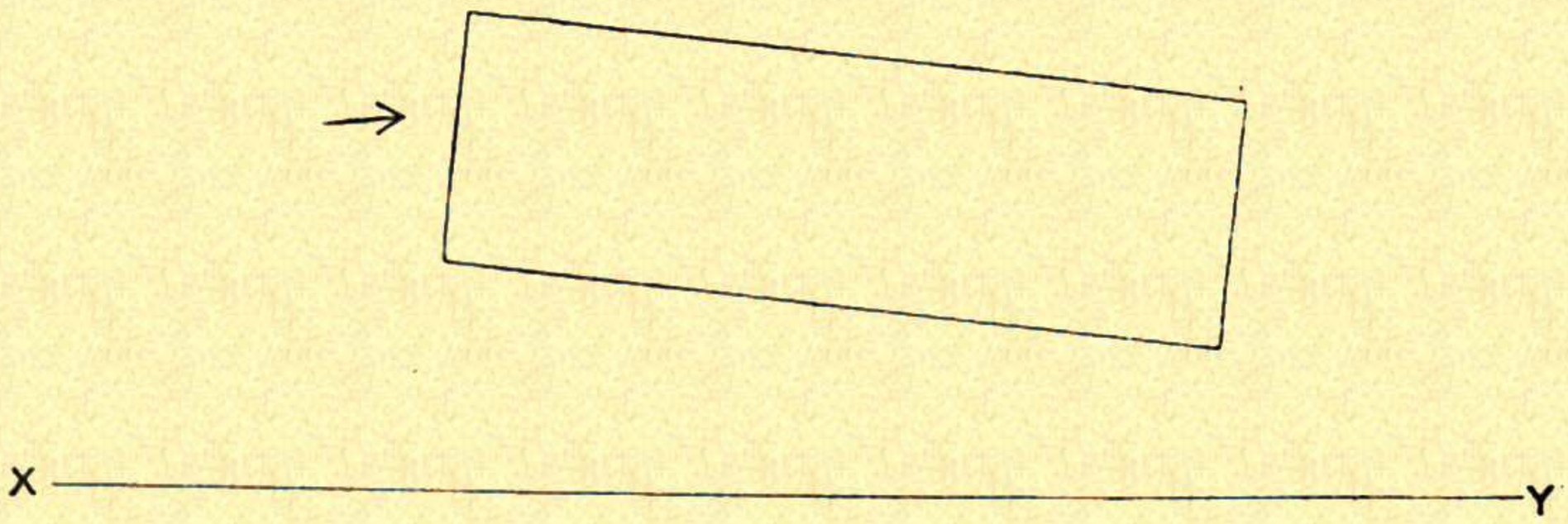
To obtain full marks the scale must be inked in.

3. The lines $a'b'$, ab are the projections of a line AB. Find (*a*) the length of the line; (*b*) its inclination to both planes of projection; (*c*) the traces of the line.



The work must be finished in pencil only. Indicate plainly the length in Question (*a*), the angles in (*b*), and the points in (*c*). Use the same figure for all three solutions.

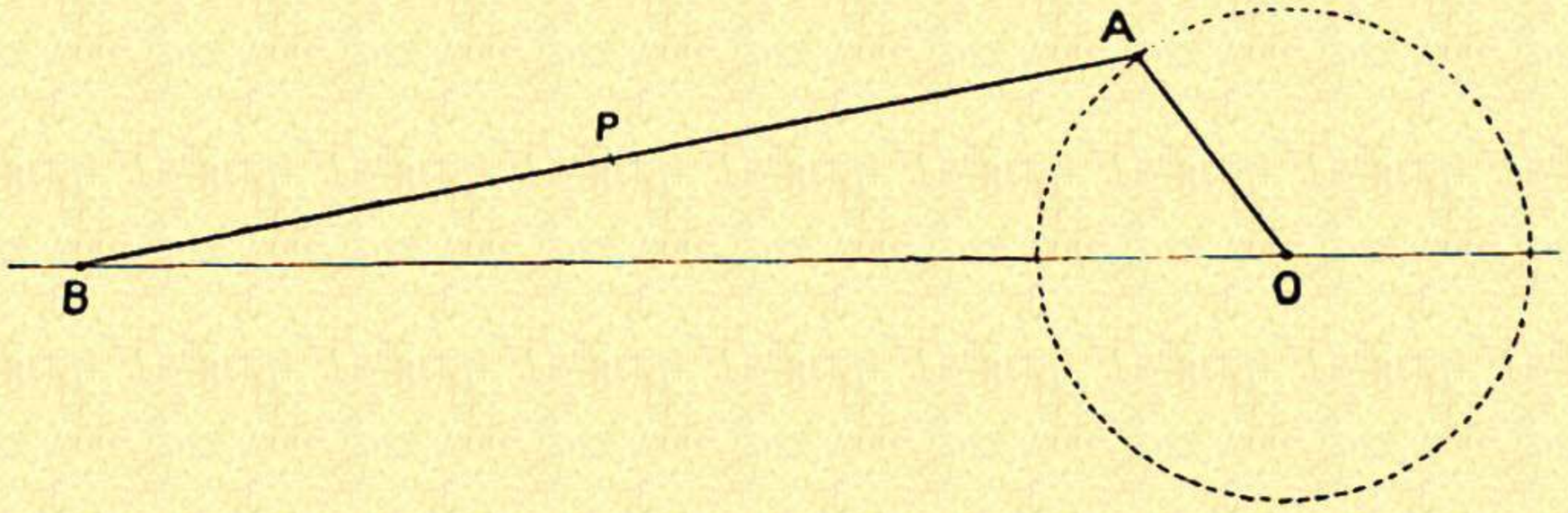
4. The figure represents the elevation of a brick-shaped piece of wood which is $1\frac{1}{2}''$ wide. Draw a plan, and also a side elevation, looking in the direction indicated by the arrow.



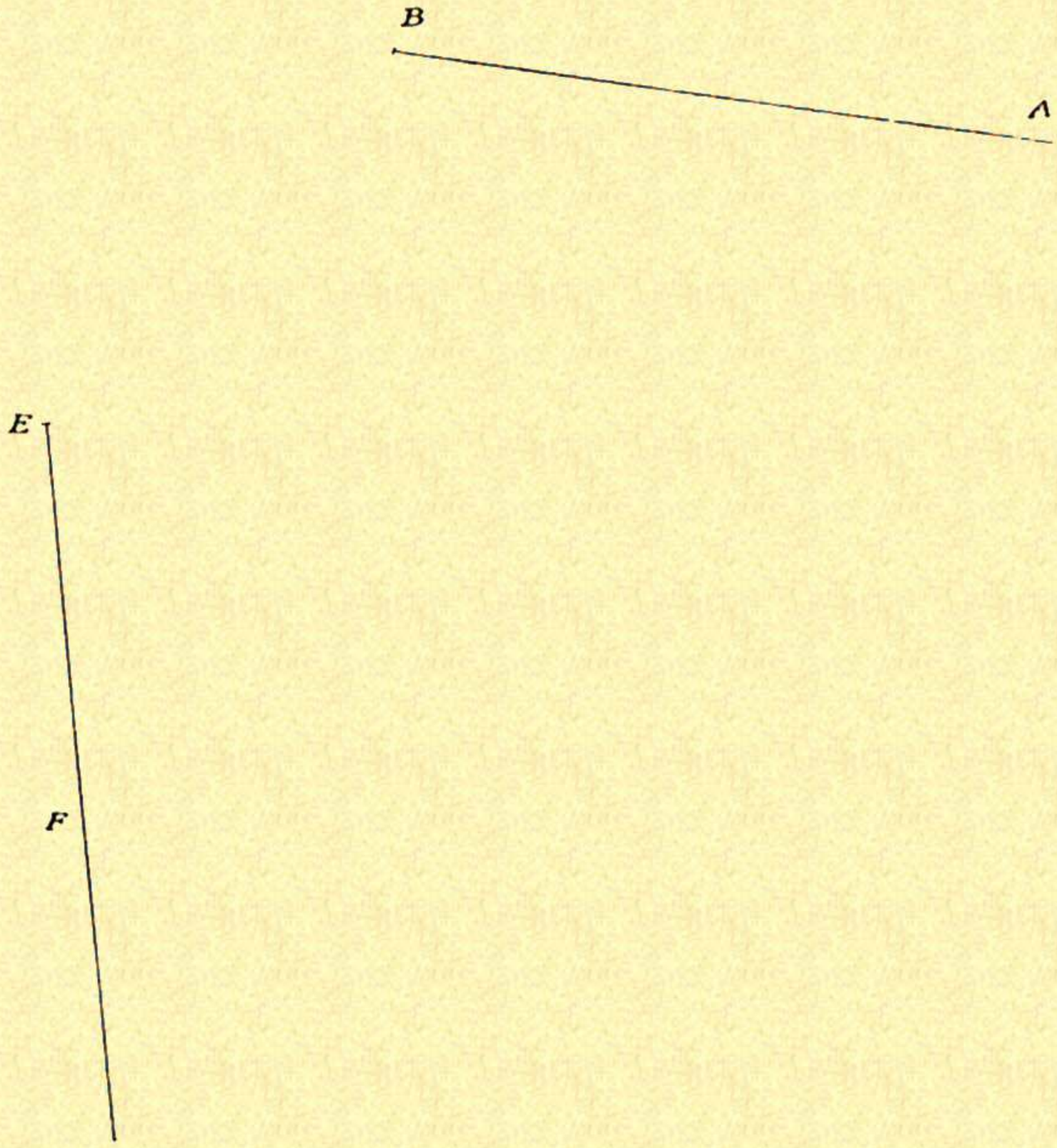
Ink in the plan and elevation, showing the construction lines dotted in ink.

5. The figure represents a simple mechanism in which the rod OA turns on the fixed centre O, A thus describing the circle indicated by the dotted line. The end B of the rod AB moves on the straight line OB. Trace the path of the point P.

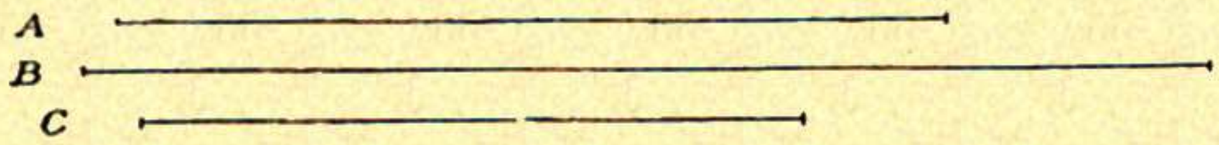
The drawing to be finished in pencil.



6. ABCDEF is to be a railway in which AB, CD, and EF are straight parts, and BC and DE are the arcs of circles of radii 7 chains and 16 chains respectively. AB and EF are shown below on a scale of 1″ to 5 chains. Draw the arcs, and determine the position of the straight part CD.



7. Construct the fourth proportional D to the three given lines, A, B, C. Measure the four lines carefully, and calculate and compare the areas of the rectangles A × D and B × C.



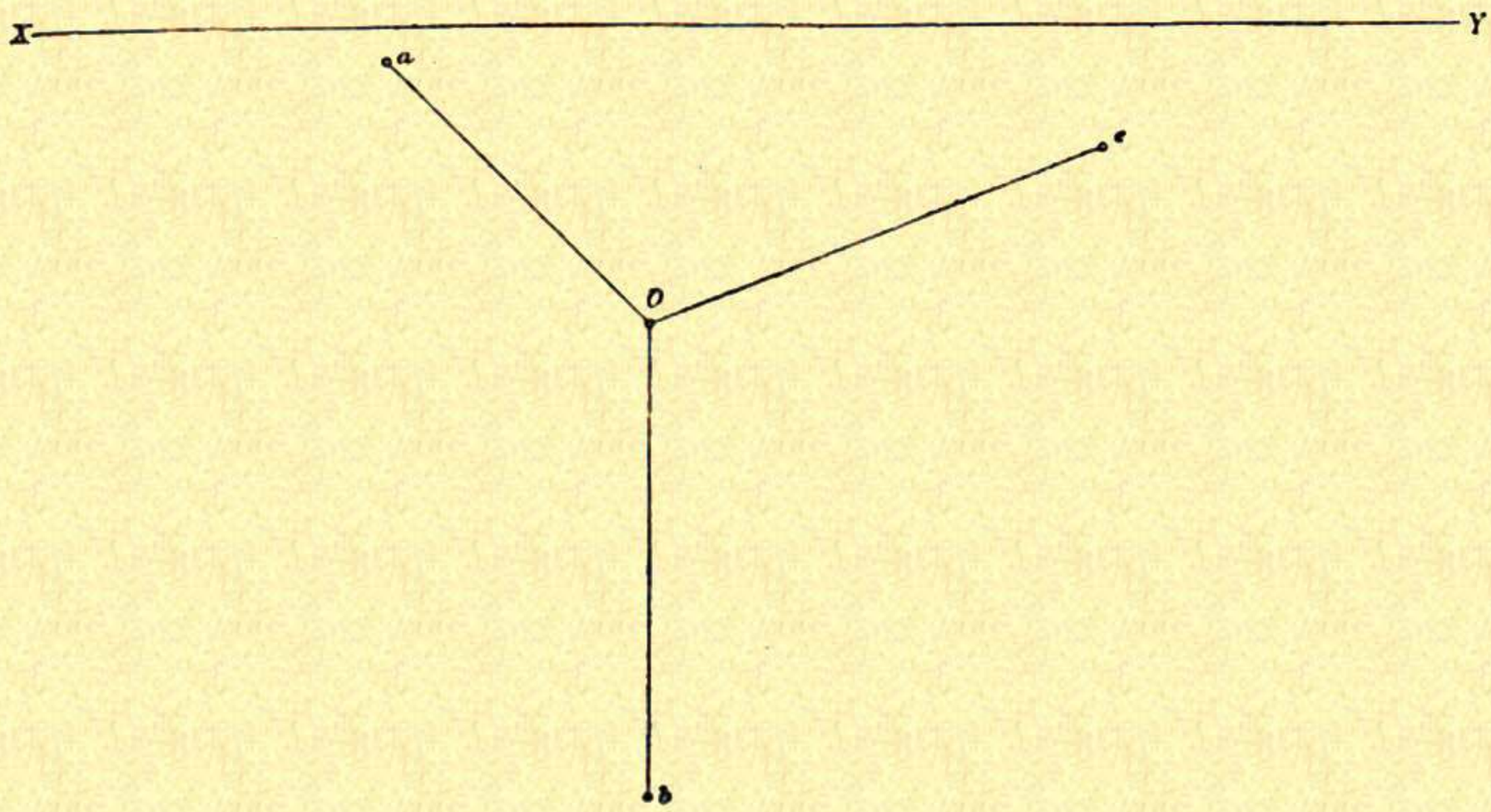
N.B.—The four figures in this paper are drawn on half the true scale.

MILITARY ENTRANCE EXAMINATION,
November and December 1902.

Including the construction of scales, and practical plane and solid geometry.

N.B.—*All lines of construction should be shown.*
The methods used must be geometrical, except where other methods are expressly prescribed in the Question Paper.
In the absence of special instructions, protractors, set-squares, and parallel rules may be used. No credit will be given for trial methods.
No writing whatever is allowed on the blotting paper.

1. Three poles, meeting in a point O, stand on a horizontal plane. O*a*, O*b*, O*c* represent their respective plans. The common

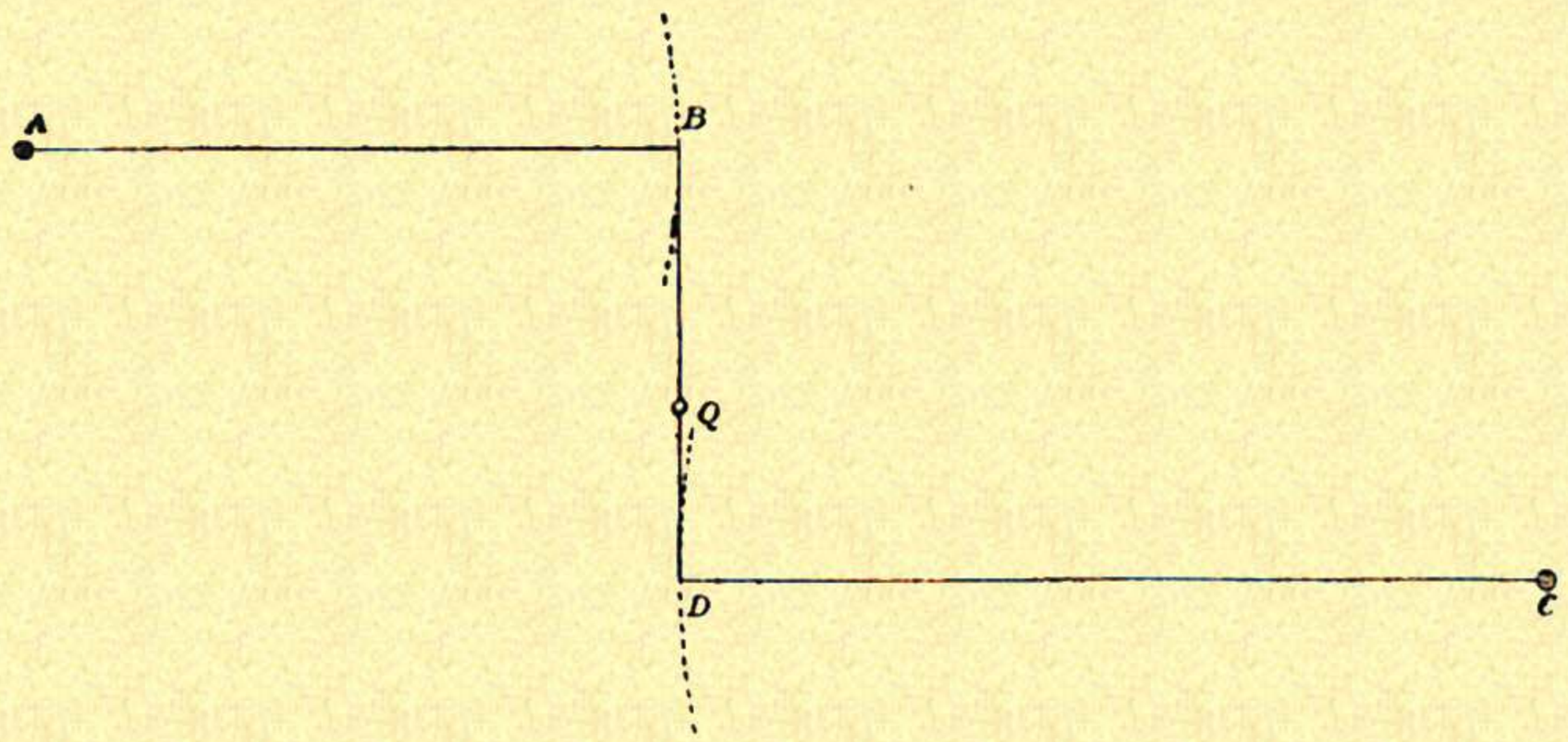


point O is 8′ vertically above the horizontal plane. Show them in elevation and find the actual length of the poles. Scale $\frac{1}{2}''=1'$.

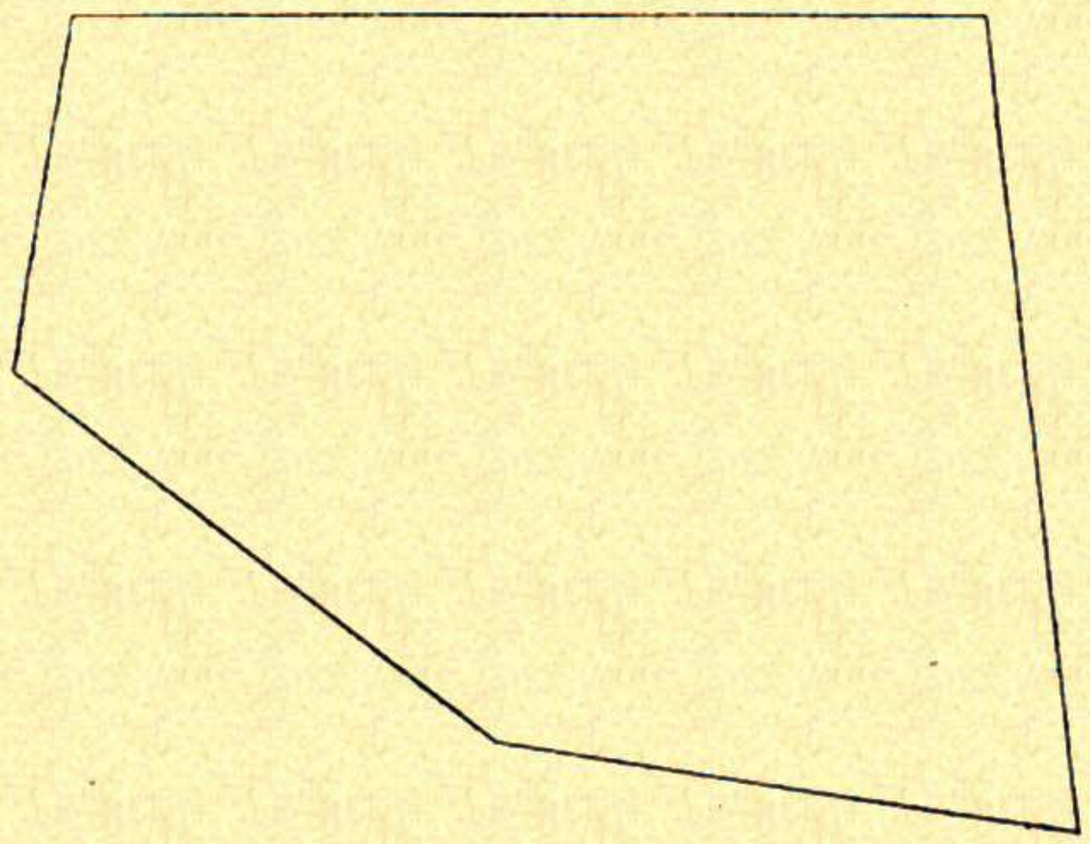
2. A certain distance on a drawing is marked 6′ 6¾″. The application of a rule to the drawing shows that the actual distance is 5″. Construct a scale for the drawing, long enough to measure 10′, sub-dividing one foot to inches.

3. Draw the plan and elevation of a cube, 2½″ edge, one face of which is inclined at 30° to the horizontal plane, another face being parallel to the vertical plane.

4. The rods AB and CD, centred at A and C, are connected by the link BD (*i.e.*, AB rotates about A, CD rotates about C,



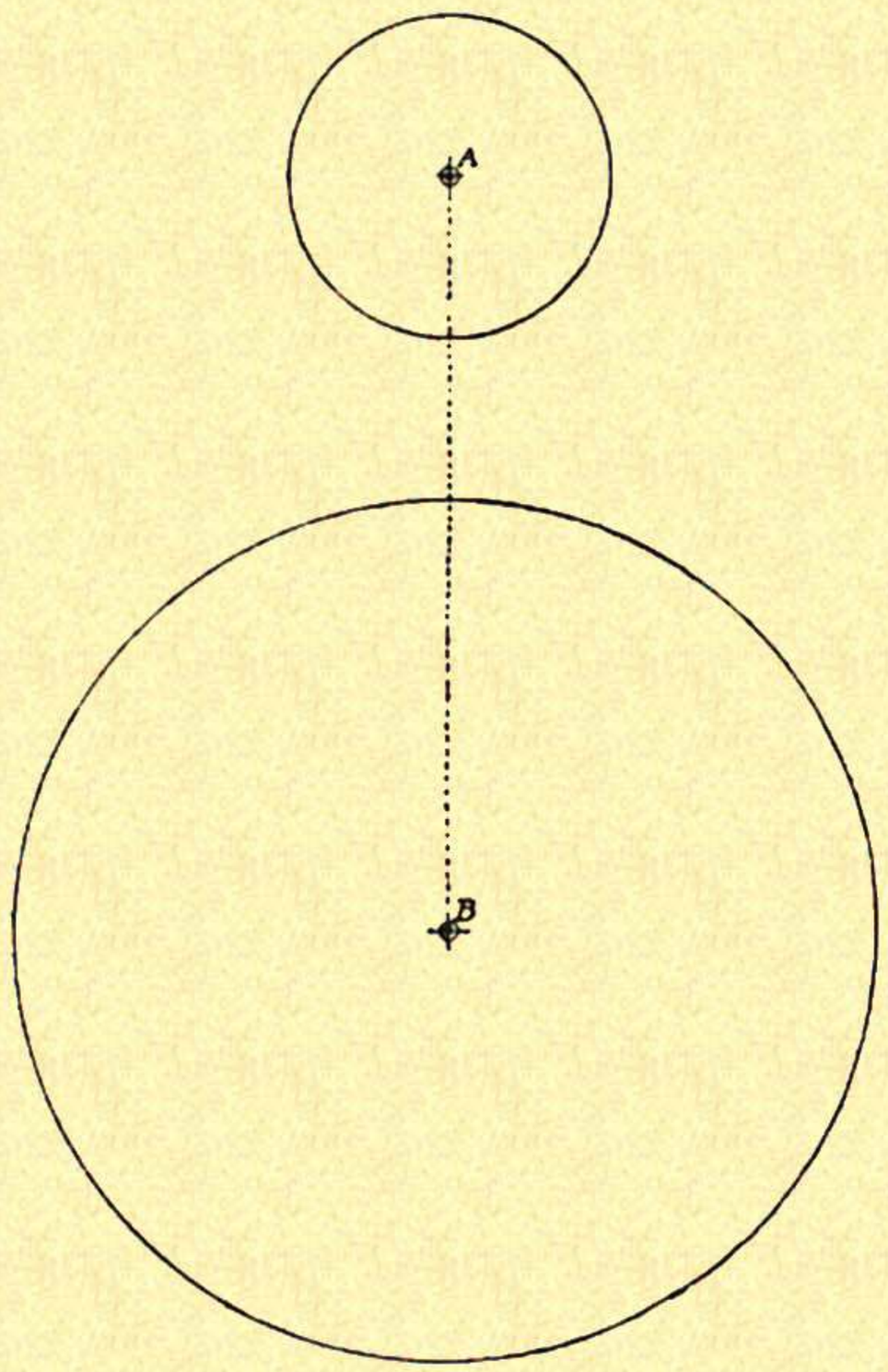
and a rod BD is hinged to them at B and D). Draw the gear in the configuration in which CD has its lowest position. Then suppose AB raised to its highest position, and show this configuration. Also trace the path followed by the point Q during this motion.



5. Find a triangle equal in area to the figure above. Reduce the triangle to a square of equal area.

6. The two pulleys, centres A and B. are to be connected (i) by an open belt, (ii) by a crossed belt, both of negligible thickness. Draw the two belts in position, marking the points at which they begin to bend round the pulleys in each case.

The crossed belt passes between the pulleys, the open belt does not.



N.B.—The five figures in this paper are drawn on half the true scale.

MILITARY ENTRANCE EXAMINATION,

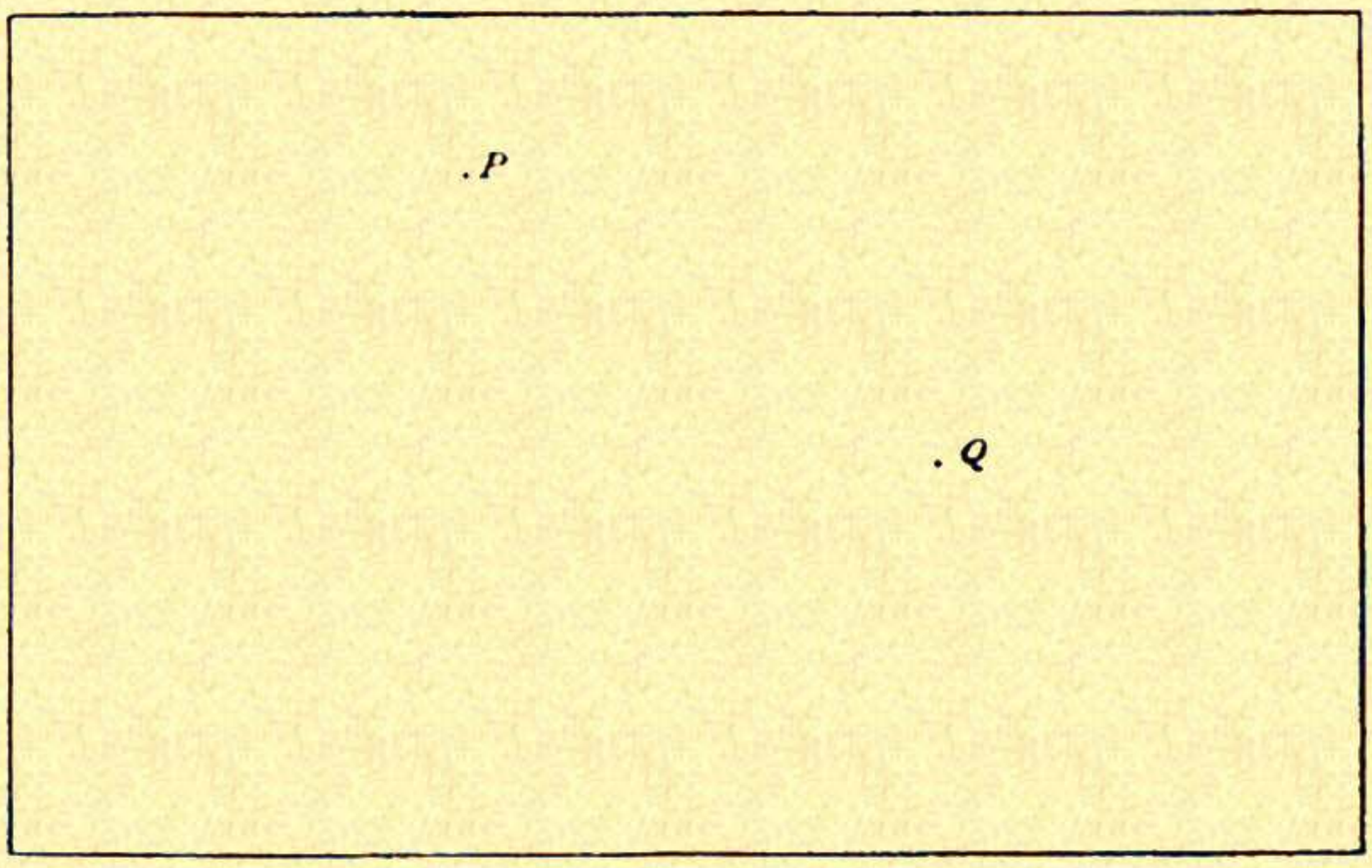
June and July 1903.

(Including the construction of scales, and practical plane and solid geometry.)

N.B.—*All lines of construction must be shown.*

In the absence of special instructions, protractors, set-squares, and parallel rules may be used. No credit will be given for trial methods.

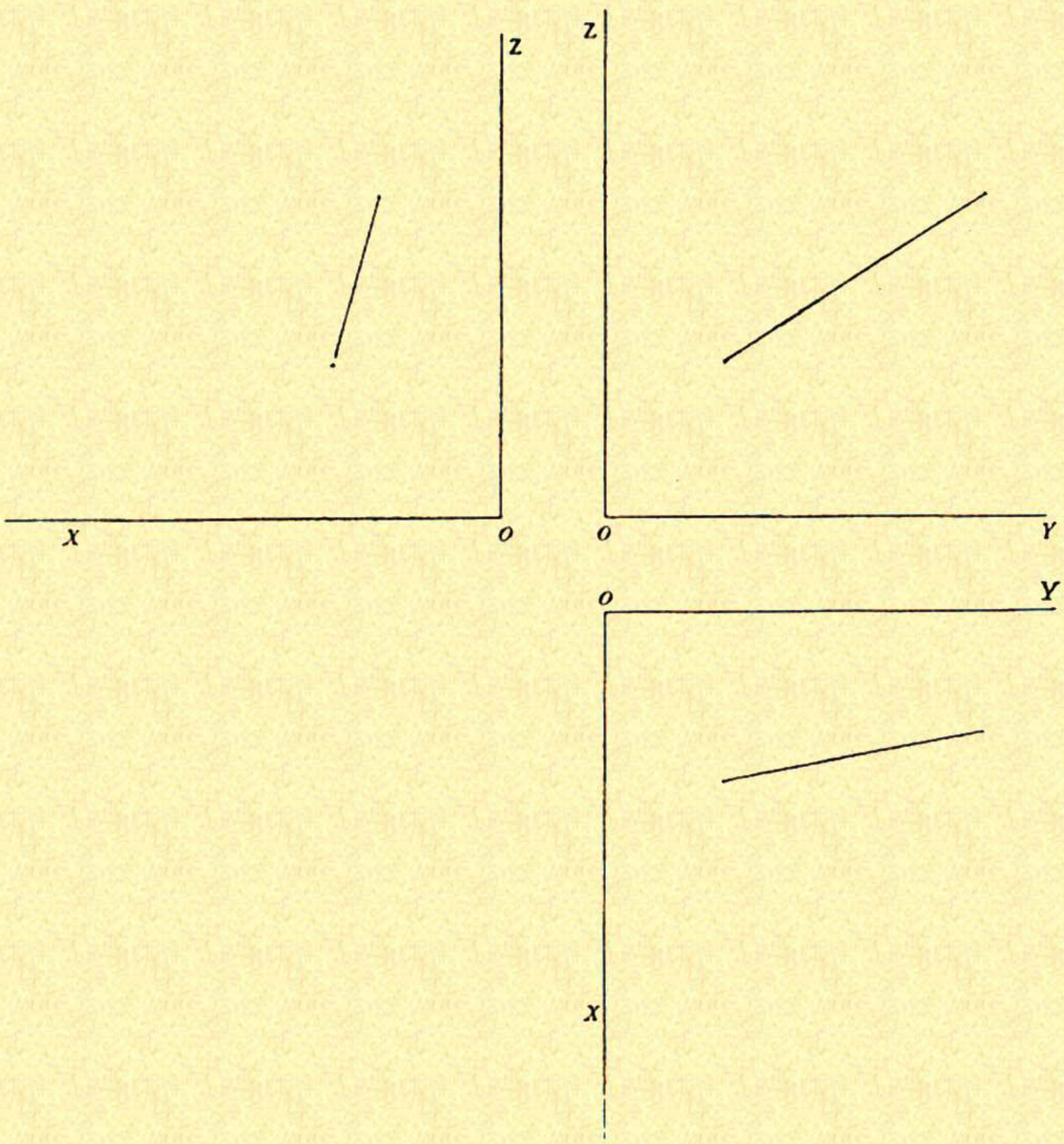
1. The diagram below represents an area of which the long side measures 16 miles. Make a plain scale for the diagram, to measure distances up to 15 miles, and showing furlongs.



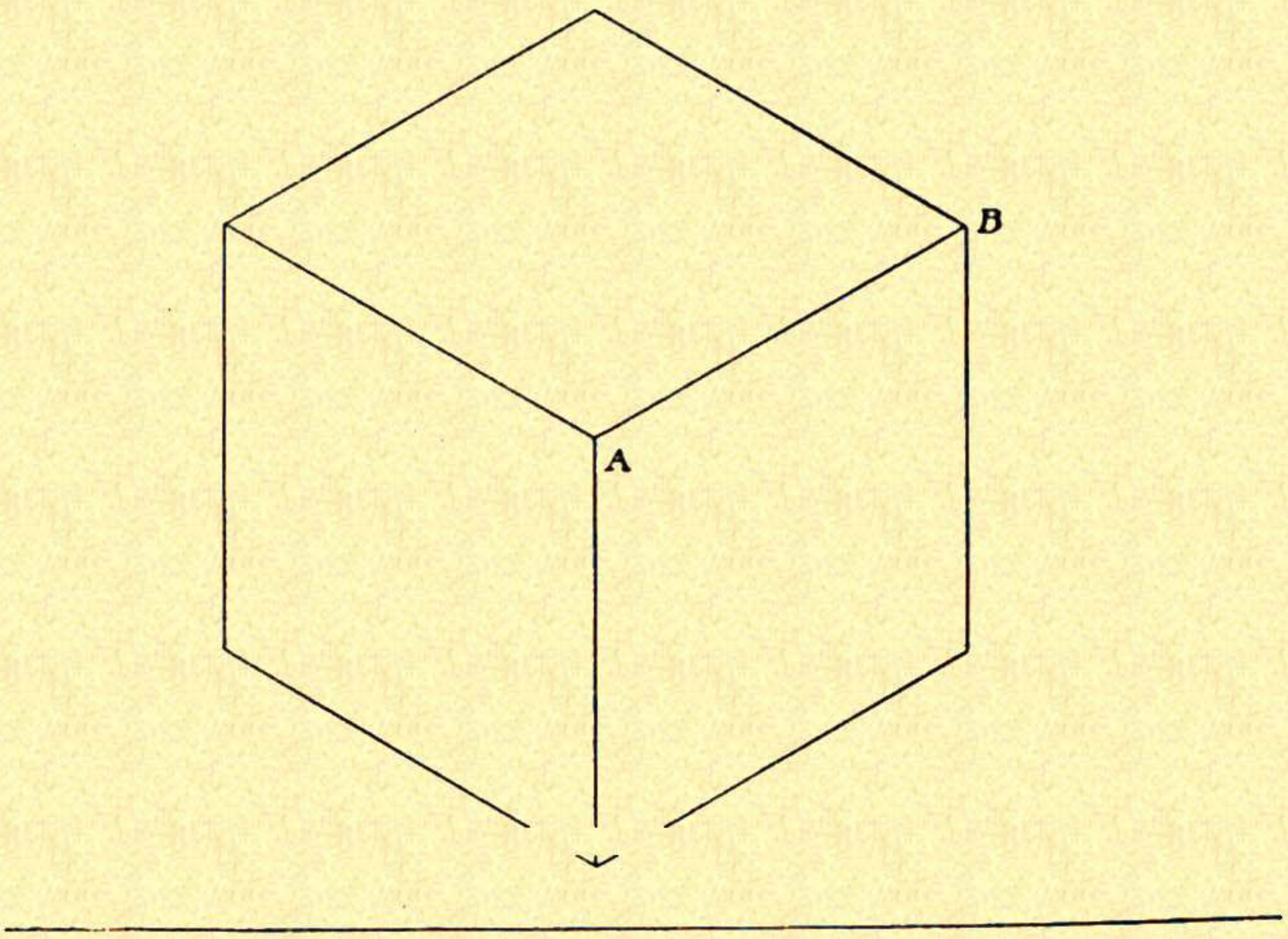
A man wishing to fix his position on the map finds that, from where he stands, a church at P is N. 23° W., and a tower at Q is N. 76° E. Mark his position R on the map. The long sides of the map run east and west.

["N. 23° W." means a direction between north and west, and making 23° with the north.]

2. Three perpendicular planes XOZ, ZOY and YOX meet along the lines OX, OY and OZ. The diagram shows the projections of a rod on these three perpendicular planes. Find its length and the angle it makes with each plane.

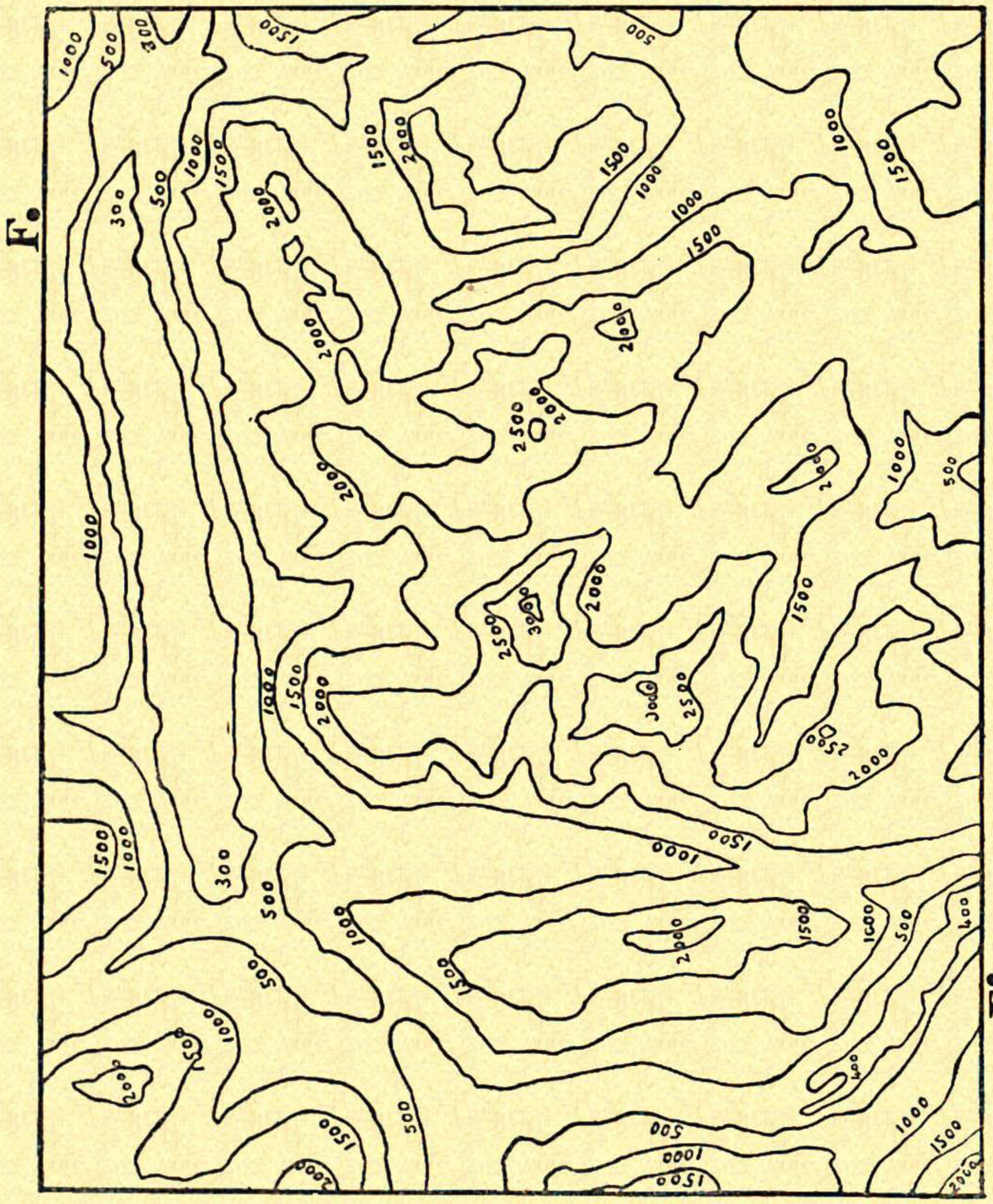


3. The diagram shows an elevation of a cube. Find the true length of the edge AB and its inclination to the vertical plane. Also draw the cube in plan.

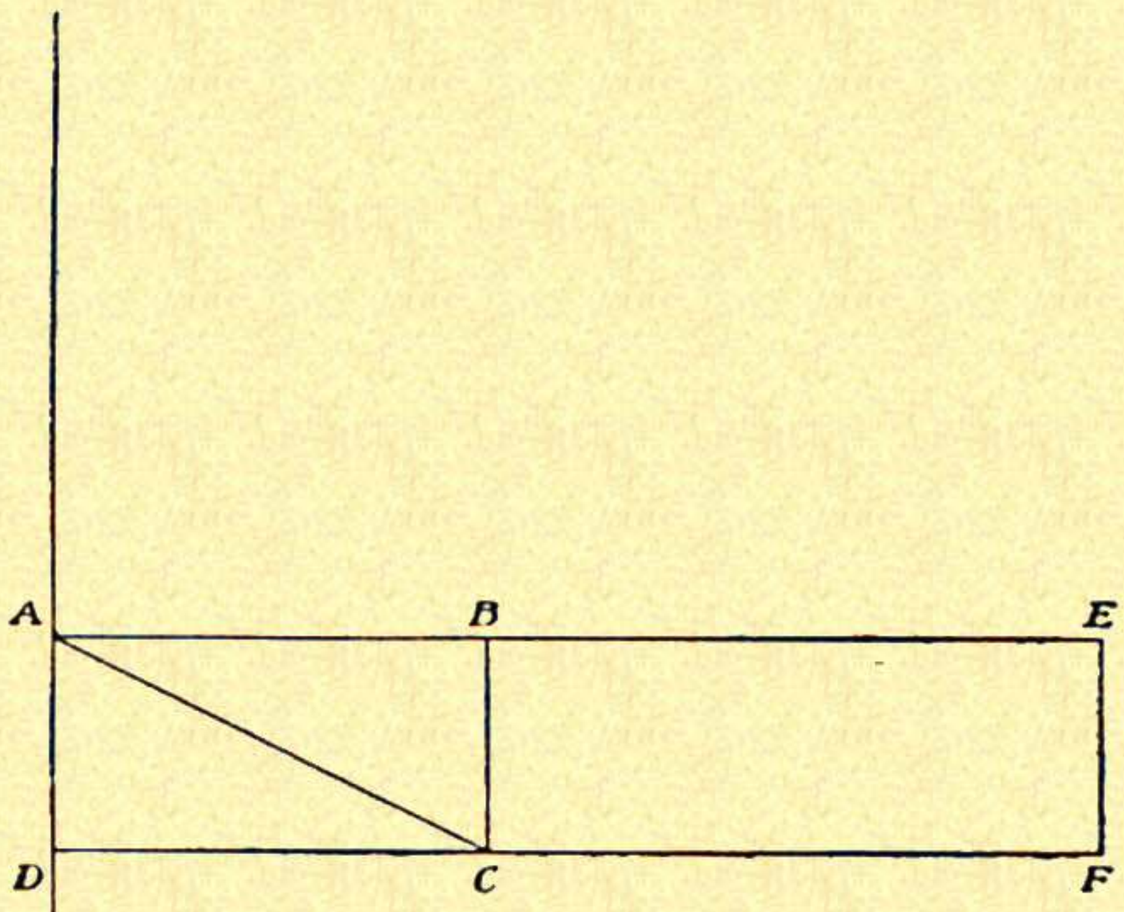


4. A copper can has the form of a part of cone, and is 12 inches high, 9 inches in diameter at the bottom and 4 inches in diameter at the top. Draw, opened out and on a scale of 1 inch to 4 inches, the piece of copper that forms the curved surface.

5. The map below shows sections of a piece of country by horizontal planes at various heights, the height of each being given in feet. Draw a vertical section of the country from E to F, using the horizontal scale of the map, and a vertical scale of 1 inch to 1000 feet.



6. AB, AC and DC are three rods hinged to a vertical wall at A and D and hinged to the rectangular block BCFE at B and C. Show the path that F will describe (1) if the rod AB breaks, (2) if AC breaks, and (3) if DC breaks; and mark these paths with the numbers 1, 2, 3.



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